## Modeling and simulation

In preparation for a demonstration of optical stochastic cooling in the Cornell Electron Storage Ring (CESR) we have developed a particle tracking simulation to study the relevant beam dynamics. Optical radiation emitted by an electron in the pickup undulator gives a momentum kick to that same particle in the kicker undulator. The optics of the electron bypass from pickup to kicker couples betatron amplitude and momentum offset to path length, so that the momentum (coherent) kick reduces betatron amplitude and momentum spread. Nearby electrons contribute an incoherent noise. The simulation enables determination of cooling rates and their dependence on bunch and lattice parameters of bypass optics, effects of coherent kicks and incoherent noise, as well as time dependent (and static) magnet alignment and field errors.

The particle tracking code makes extensive use of Bmad, a subroutine library for relativistic charged-particle dynamics simulations \cite{bmad:2006}. The bunch is modeled as a distribution of 1000 macroparticles with some initial emittance (typically the equilibrium emittance as determined by radiation damping and/or intra-beam scattering), and tracked through a CESR lattice for many turns. At every turn the 6-dimensional phase space of each particle is recorded at the location of the pickup, to construct the sigma matrix, from which the normal mode emittances in three planes are computed \cite{wolski:2006}. Quantum excitation and damping (stochastic emission of photons) can be turned on or off as desired. On every turn, each macroparticle receives both coherent and incoherent kicks when exiting the kicker undulator. The incoherent contribution depends on the number of real particles (as opposed to macroparticles), in the bunch. We have developed a computationally efficient method for including the incoherent contribution.

The momentum kick received by each particle in the kicker undulator is represented as(ttosc:1994) where G is the gain, the relative phase of electron and light for the ith electrons, and e relative phase of the j nearby electrons.

 

## Coherent Kicks

The first term on the right hand side of Eq.~\ref{eq:kicks} is the coherent kick. It is straightforward to apply this coherent kick to the macroparticles. The path lengthening of each particle from the the pickup to the kicker (center to center) is computed: $\Delta s=z\_k-z\_p$, where $z\_k$ and $z\_p$ are the same-turn longitudinal coordinates of the particle at the middle of the kicker and pickup undulator, respectively. Then the phase shift was calculated based on Eq.



The coherent kick $\Delta p\_{z\textrm{-}co}=-G\sin(k\Delta s)$ where the system gain G depends on undulator parameters, bypass beam optics and amplifier gain.

## Incoherent Kicks

The second term in the right of Eq.~\ref{eq:kicks} describes the incoherent kicks that are contributed by the radiation from nearby particles. The most straightforward way to implement the incoherent kicks would be to count the number of particles trailing the reference particle within the range $\leq N\_u\lambda$, (Radiation of particles outside of that range (slice) will not arrive in the kicker undulator in time to interact with the reference particle.), and sum their contributions directly. This method is however impractical, as it requires tracking the entire population the bunch ($10^7$ or $10^8$ particles) .

Since $\psi\_{ij}=k(z\_i-z\_j)$ and $0 \leq z\_i-z\_j \leq N\_u \lambda$, $\psi\_{ij}$ is within the range $[0, 2N\_u\pi]$. The particles are randomly distributed longitudinally within the slice $[z\_i-N\_u\lambda, z\_i]$ so that $\psi\_{ij}$ is likewise randomly distributed within $[0, 2N\_u\pi]$. If f(y) is the probability distribution function of y=sin(x), then the total incoherent kick , z is given by N\_s convolutions of f(y) where N\_s is the number of particles in the slice. When N\_s is large (> 6), the probability distribution of z is Gaussian with width proportional to sqrt(N\_s). The details are determined numerically. The incoherent kick is then given by

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\begin{equation}

\Delta p\_{z\textrm{-}in} = -G R \sigma\_{in} \textrm{,}

\end{equation}

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where $R$ is a random number generated for a normal distribution. The total longitudinal kick $\Delta p\_z = \Delta p\_{z\textrm{-}co} + \Delta p\_{z\textrm{-}in}$ is then applied to\

the particle at the exit of the kicker undulator every turn to simulate the OSC cooling process.

The simulation has been used to quantify cooling rates and ranges and to determine dependencies for the lattice with chicane bypass. The layout of the chicane bypass is shown in Fig. Recall that the differential path length of light and electrons in the chicane bypass is only 2mm.



Figure . Layout of the chicane bypass. Note the very different horizontal and vertical scales. The differential path length for electrons and light is 2mm.

The results of a simulation of the cooling for the chicane bypass is shown in Fig. The simulation includes radiation damping and excitation, coherent as well as incoherent kicks, and dipole field variation. We assume 10 million electrons in a 10mm long bunch. The dipole field variation is a combination of a 60Hz modulation and white noise and is based on measurements of the CESR dipole. Each curve in the Fig shows the horizontal emittance versus turn number, for a different value of the gain system gain. The gain depends on the undulator characteristics, the chicane beam optics, and amplification. In the absence of amplification (passive cooling), the gain for the chicane bypass configuration is about 2e-10. Beam energy is 1.0 GeV.



Figure Simulated horizontal emittance versus turn number for a bunch of 1e7 electrons, for a range of system gain. The simulation is for the chicane bypass configuration. Beam energy is 1GeV.

Preliminary simulations for the lattice configured with the longer arc bypass, (see below) are shown in Fig. and cooling is indeed evident. Work is underway to improve the passive gain by further optimization of the beam optics and undulator parameters, and as noted above, the arc bypass configuration is characterized by differential electron/light path length of 63cm (as compared to 2mm in the chicane bypass), thus affording space for active amplification.



Figure . Simulated damping of horizontal emittance for the lattice configure with arc bypass (63cm differential delay)