TEST OF OPTICAL STOCHASTIC COOLING AT FERMILAB

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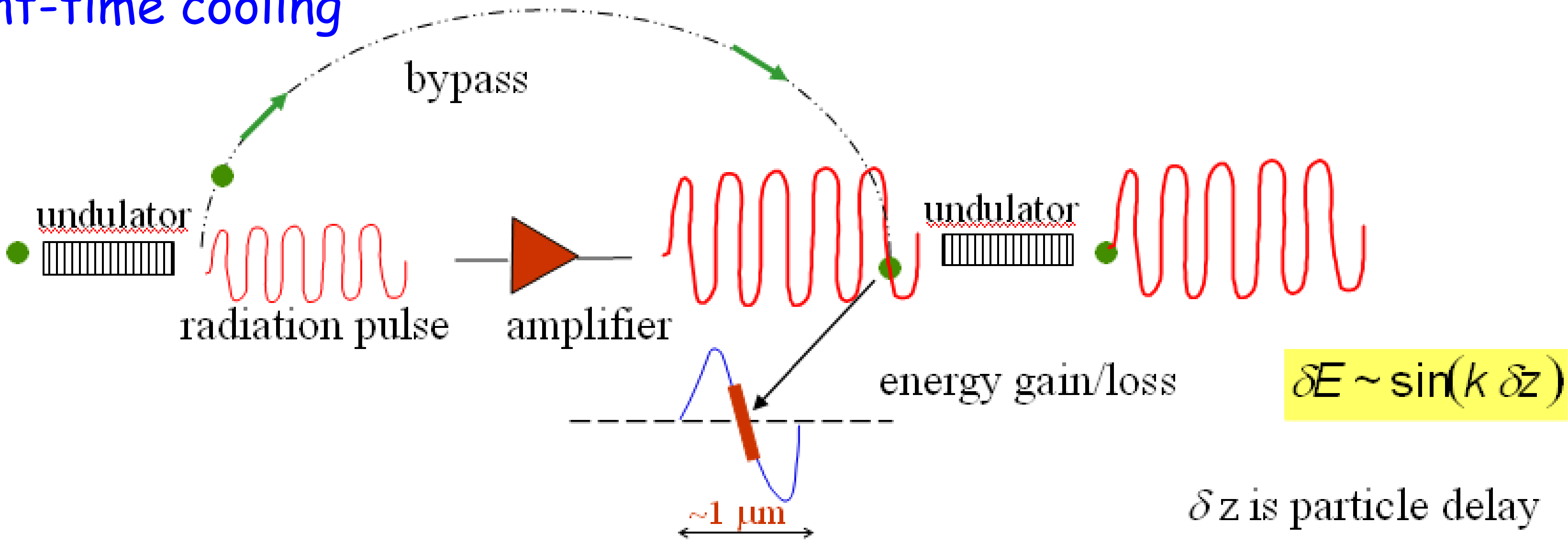
Basics of Optical Stochastic Cooling

- The damping rate of stochastic cooling:

$$\boxed{\lambda_{opt} \approx \frac{2\pi^2 W}{N n_\sigma^{\;2}} \frac{\sqrt{\pi}\sigma_s}{C}, \quad \begin{cases} W = f_{\max} - f_{\min}\ , & \text{for Rectangular gain shape} \\ W = 2\sqrt{\pi}\sigma_f\ , & \text{for Gaussian gain shap} \end{cases}}$$

- *OSC was suggested by Zolotorev, Zholents and Mikhailichenko (1994)*
- Transition from the microwave SC to OSC increases the bandwidth by about 3 orders of magnitude ($\lambda \to 10^{-4}\lambda$, $\Delta f/f=50\% \to \Delta f/f=10\%$)
- Pickup and kicker must work in the optical range and support the same bandwidth as the amplifier
  - Undulators were suggested for both pickup and kicker
  - Bandwidth: OA bandwidth & inverse number of wiggles
  - Transient-time cooling

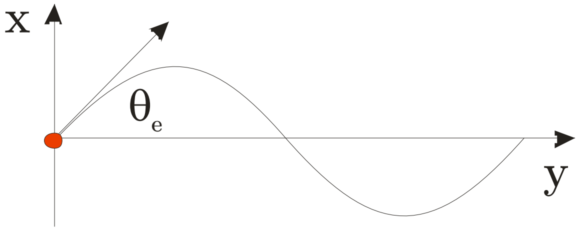


$\delta E \sim \sin(k\,\delta z)$

δz is particle delay

Basics of Optical Stochastic Cooling (continue)

- OSC can operate only with ultra-relativistic particles
  - Slow particles do not radiate at optical frequencies
- Radiation wave length



$$\lambda = \frac{\lambda_{wgl}}{2\gamma^2}\left(1+\gamma^2\left(\frac{1}{2}\theta_e^{\,2}+\theta^2\right)\right) - \text{flat undulator}$$

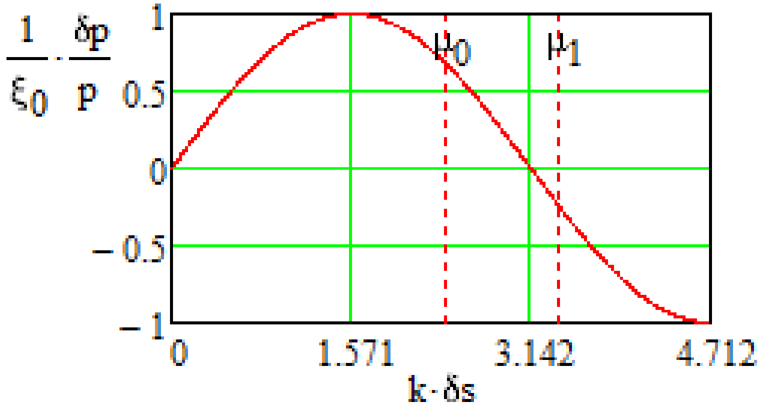
- Radiation is concentrated in the angle $1/\gamma$
- Correction signal is proportional to a longitudinal position change on the travel from pickup to kicker
- Only longitudinal kicks are effective
  - Requires s-x coupling for hor. cooling and x-y coupling for vert. cooling
- Non-zero dispersion in OSC pickup introduces difference between M_{56} and partial slip-factor ($\tilde{M}_{56}$), and, consequently x-s coupling

⇨ Cooling rates (per turn)

$$\frac{\delta p}{p} = \xi_0 \sin\left(k\delta s\right), \quad \delta s = \tilde{M}_{56}\frac{\Delta p}{p}$$

$$\boxed{\begin{aligned}\lambda_x &= \frac{\xi_0}{2}k\left(M_{56}-\tilde{M}_{56}\right)\\ \lambda_s &= \frac{\xi_0}{2}k\tilde{M}_{56}\end{aligned}} \quad => \quad \boxed{\lambda_x+\lambda_s = \frac{\xi_0}{2}kM_{56}}$$

Basics of OSC - Cooling Range



- Cooling force depends on Δs nonlinearly

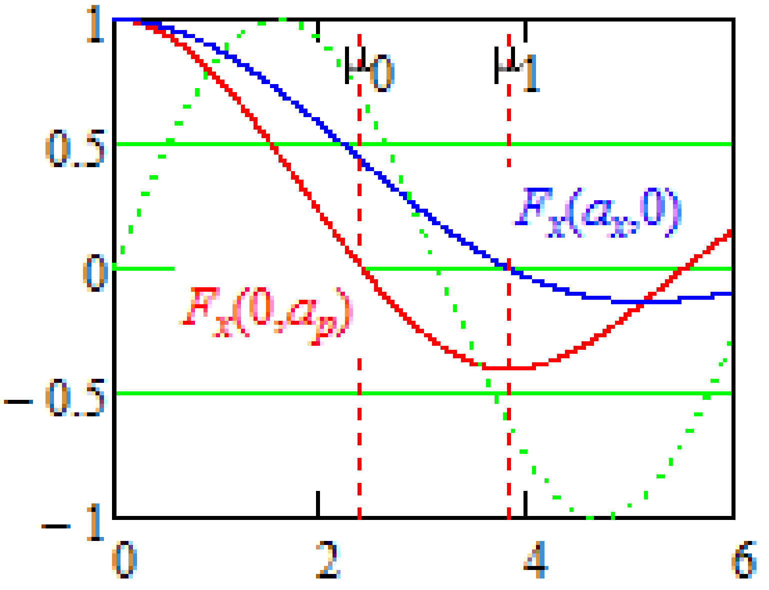
$$\frac{\delta p}{p} = k\xi_0\,\Delta s \quad \Rightarrow \quad \frac{\delta p}{p} = \xi_0 \sin\left(k\delta s\right)$$

where $k\delta s = a_x \sin(\psi_x) + a_p \sin(\psi_p)$

and a_x & a_p are the amplitudes of longitudinal displacements in cooling chicane due to ⊥ and L motions measured in units of laser phase

- Averaging yields the form-factors for damping rates

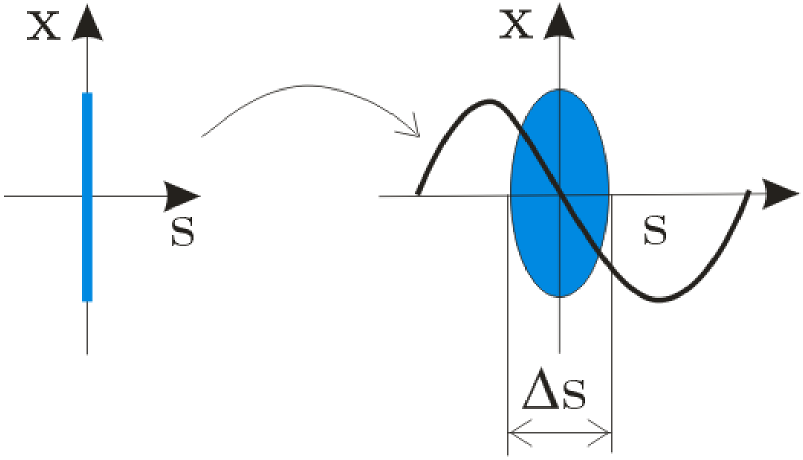
$$\boxed{\begin{aligned} &\lambda_{s,x}(a_x,a_p) = F_{s,x}(a_x,a_p)\lambda_{s,x} \\ &F_x(a_x,a_p) = \frac{2}{a_x}\mathrm{J}_0(a_p)\mathrm{J}_1(a_x) \\ &F_p(a_x,a_p) = \frac{2}{a_p}\mathrm{J}_0(a_x)\mathrm{J}_1(a_p) \end{aligned}}$$



- Damping requires both lengthening amplitudes (a_x and a_p) to be smaller than μ_0≈2.405

Basics of OSC - Sample Lengthening

- On the way from pickup to kicker a zero length sample lengthens on its way from pickup-to-kicker
  - Both Δp/p and ε contribute to the lengthening



$$\sigma_{\Delta s}{}^2 = \sigma_{\Delta s\varepsilon}{}^2 + \sigma_{\Delta sp}{}^2$$

$$\sigma_{\Delta s\varepsilon}{}^2 = \varepsilon\left(\beta_p M_{51}{}^2 - 2\alpha_p M_{51} M_{52} + \gamma_p M_{52}{}^2\right)$$

$$\sigma_{\Delta sp}{}^2 = \sigma_p{}^2 \tilde{M}_{56}{}^2$$

where $\tilde{M}_{56} \equiv M_{51} D_p + M_{52} D'_p + M_{56}$

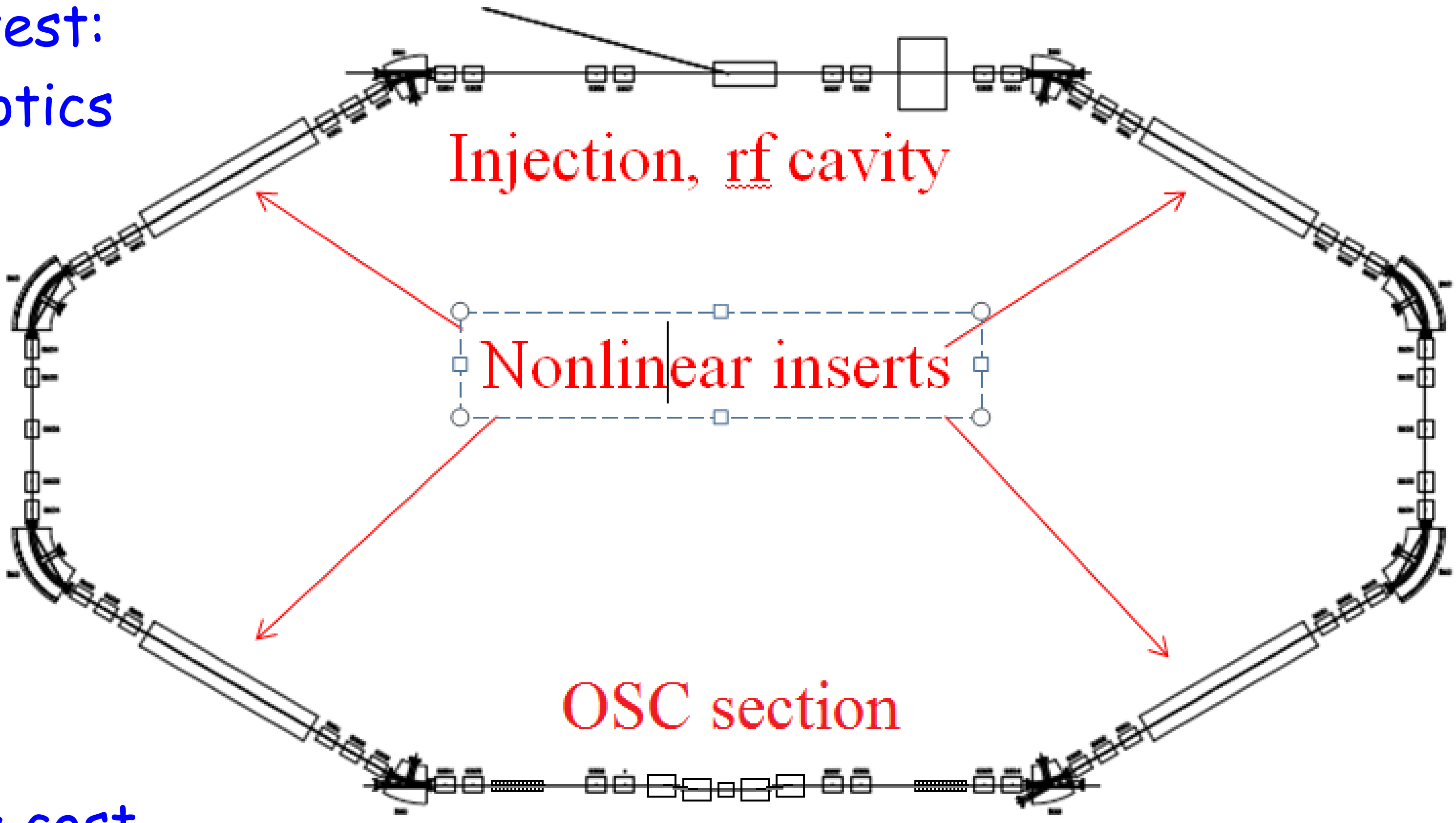
- While in linear approximation β_p and α_p do not affect damping rates they affect sample lengthening due to beam horizontal emittance and, consequently, the horizontal cooling range

$$n_{\sigma\varepsilon}\sigma_{\Delta s\varepsilon} k \le \mu_0 \qquad \mu_0 \approx 2.405$$

$$n_{\sigma p}\sigma_{\Delta sp} k \le \mu_0$$

Test of OSC in Fermilab

- First attempt to test the OSC in BATES, ~2007
  - Existing electron synchrotron
  - Did not get sufficient support
- Presently Fermilab is constructing a dual purpose small electron ring called IOTA to test:
  - Integrable optics
  - OSC
- Part of ASTA program
  - Full energy injection from SC linac
- Test in a small electron ring is a cost effective way to test the OSC



Injection, rf cavity

Nonlinear inserts

OSC section

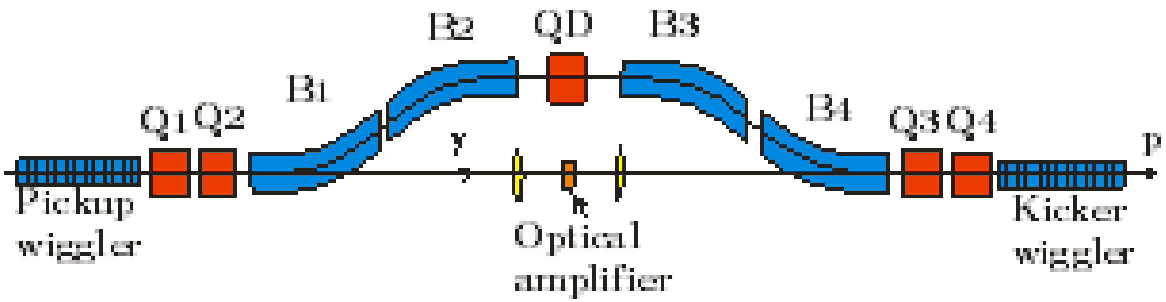
OSC Limitations on IOTA Optics

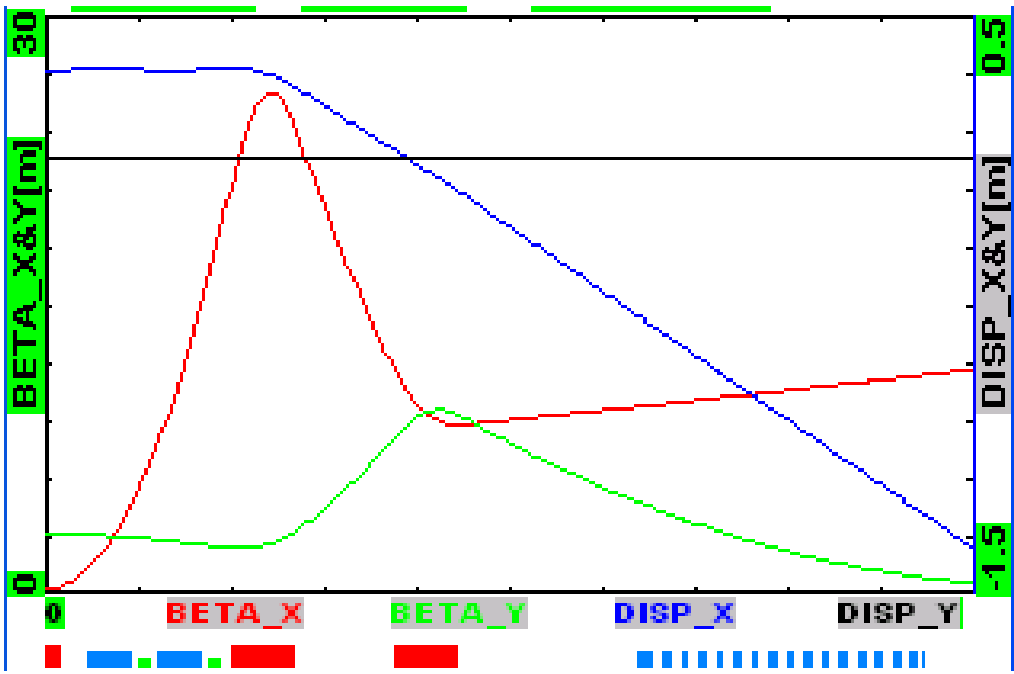
- Delay in OA amplifier determines delay in the chicane (Δs) => $M_{56} \approx 2\Delta s$
- D quad in the center and non-zero dispession introduce xs-coupling: $\tilde{M}_{56} \approx M_{56} - \Phi D^* h$

  h - orbit offset in the chicane
- Sample lengthening minimization due to betat. motion requires collider type optics with small β^*

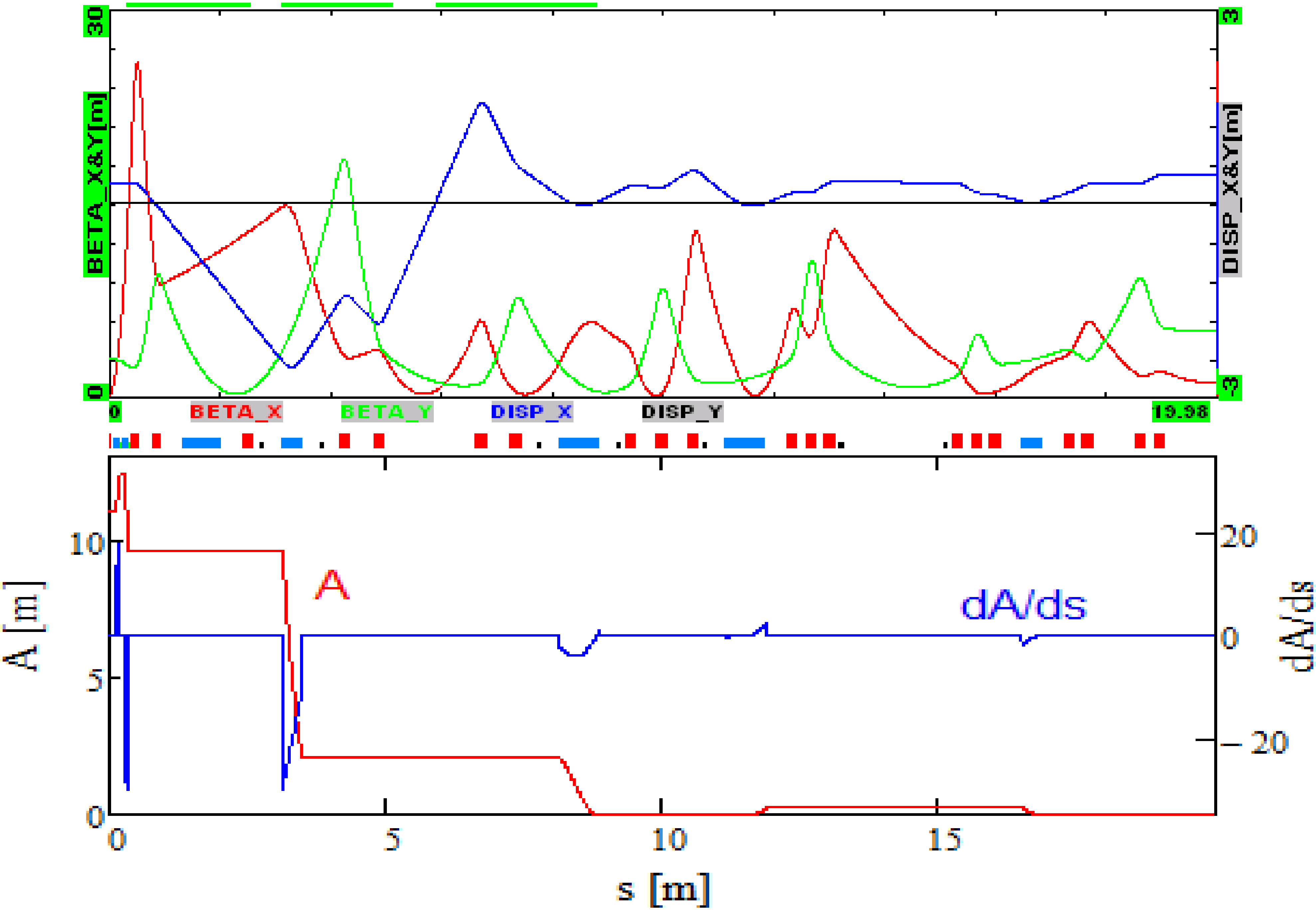
  ⇨ large dispersion invariant

  $A^* = D^{*2} / \beta^*$



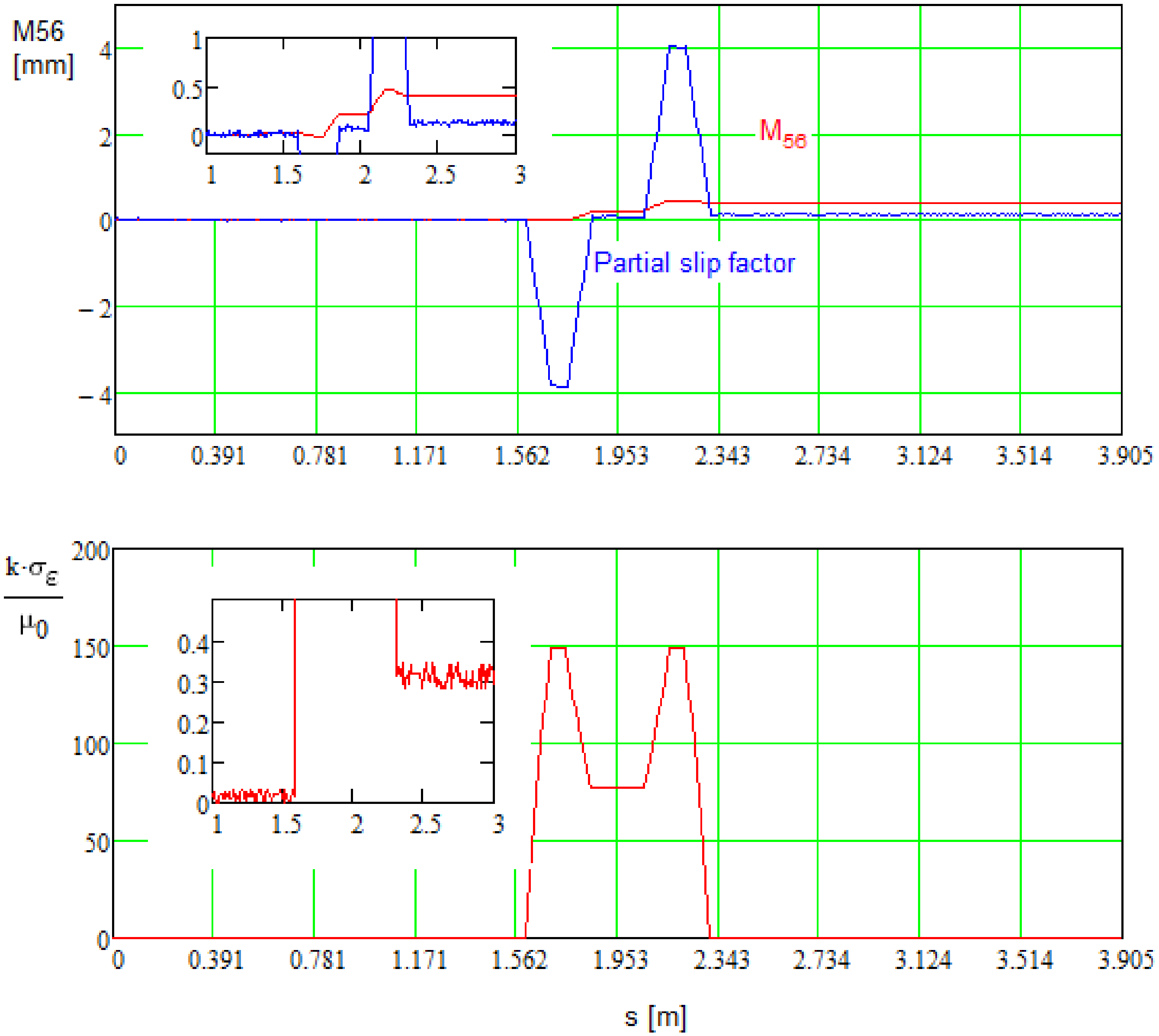


- An average value of A in dipoles determines the equilibrium emittance.
  - A^* is large and A needs to be reduced fast to get an acceptable value of the equilibrium emittance



Optics functions and dispersion invariant for IOTA half ring

Sample Lengthening on the Travel through Chicane



Sample lengthening due to momentum spread (top) and due to betatron motion (bottom)

- Very large sample lengthening on the travel through chicane
- High accuracy of dipole field is required to prevent uncontrolled lengthening, $\Delta(BL)/(BL)_{dipole}<10^{-3}$

Second Order Contributions to Sample Lengthening

- Linear part of long. displacement due to bet. motion:

$$\Delta L_{\mathrm{maxX}} = \left(a_x / \sigma_x\right) \times 97\,\mathrm{nm}$$

- Major non-linear contribution comes from angle: $\Delta L = \int \left(\theta^2(s)/2\right) ds$
- Integration over trajectory yields:

$$I_1 = \int \left(1+\alpha^2\right) d\mu$$

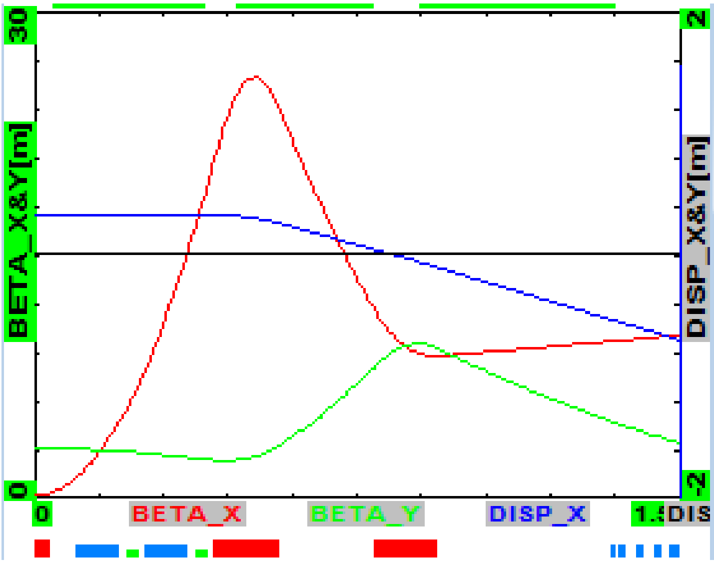
$$\Delta L = \frac{\varepsilon}{4}\left(I_1 + \left(I_2 + I_3\right)\cos\left(2\psi\right) + \left(I_4 + I_5\right)\cos\left(2\psi\right)\right), \quad I_2 = 2\int \alpha \sin\left(2\mu\right) d\mu\,, \quad I_3 = -\int \left(1-\alpha^2\right)\cos\left(2\mu\right) d\mu$$

$$I_4 = 2\int \alpha \cos\left(2\mu\right) d\mu\,, \quad I_5 = \int \left(1-\alpha^2\right)\sin\left(2\mu\right) d\mu$$

ψ – the phase of betatron motion

- Performing numerical integration from pickup to kicker results in:

$$\Delta L_{\mathrm{maxX}} = \left(\frac{a_x}{\sigma_x}\right)^2 \times 690\,\mathrm{nm}\,, \quad \Delta L_{\mathrm{maxY}} = \left(\frac{a_y}{\sigma_y}\right)^2 \times 140\,\mathrm{nm}$$



- These values are too large and need to be compensated
- Sextupole correction
  - 1-st Sext – correction ΔL
  - 2-nd sext – major correction non-linearity
  - … correction of chromaticity & RDT – work in progress (next presentation)

IOTA Optics and Parameters

Main Parameters of IOTA storage ring for OSC

Circumference	40 m
Nominal beam energy	100 MeV
Bending field	4.79 kG
Transverse emittances, $\varepsilon=\varepsilon_x=\varepsilon_y$, rms	11 nm
Rms momentum spread, σ_p	$1.21\cdot10^{-4}$
SR damping times (ampl.), $\tau_s/(\tau_x=\tau_y)$	1.36 / 1.58 s

Main parameters of cooling chicane

Delay in the chicane, Δs	2 mm
Horizontal beam offset, h	20.1 mm
M_{56}	3.95 mm
D^* / β^*	307 mm / 8.59 mm
Cooling rates ratio, $(\lambda_x=\lambda_y)/\lambda_s$	1.18
Cooling ranges (before OSC), $n_{\sigma x}/n_{\sigma s}$	2.1 / 3.2
Dipole: magnetic field *length	4.22 kG * 10 cm
Strength of central quad, GdL	1.58 kG

- Energy is reduced 150→100 MeV to reduce ε, σ_p and undulator period and length
- Operation on coupling resonance Q_x/Q_y= 5.83/3.83 reduces horizontal emittance and introduces vertical OSC damping

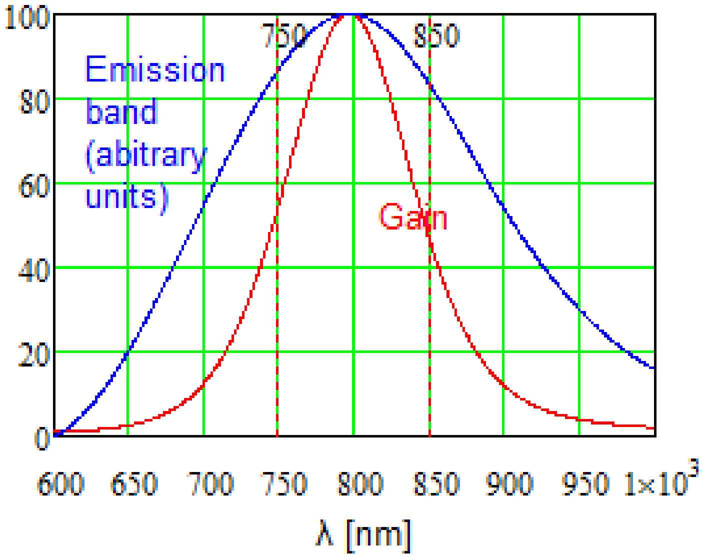
- Small β^* is required to minimize sample lengthening due betatron motion

RF, Touschek, IBS and Gas scattering

Slip-factor	-0.065
RF harmonic	4
RF voltage	100 V
SR loss	14 V/turn
RF bucket height	$1.4\cdot10^{-3}$
Rms bunch length,	25 cm
Number of particles	$2.5\cdot10^{6}$
Emittance growth rate due to SR (H)	36 nm/s
Emittance growth rate due to IBS (H) ($\varepsilon_x=\varepsilon_y$)	3.6 nm/s
Growth rate for $(\Delta p/p)^2$ due to SR	$2.1\cdot10^{-8}$
Growth rate for $(\Delta p/p)^2$ due to IBS ($\varepsilon_x=\varepsilon_y$)	$2.7\cdot10^{-9}$
Touschek lifetime (set by bucket height)	4.3 hour
Machine acceptance (set by dynamic aperture)	1 μm
Average vacuum (H_2 equivalent)	$1.5\cdot10^{-10}$
Emittance growth due to gas scattering (H/V)	2.5/1.8 nm
Gas scattering lifetime	17 min.

- Number of particles per bunch is set so that IBS would be ~10% of growth rates set by SR
- Very good vacuum is required to support good lifetime in the presence of strong limitation of dynamic aperture

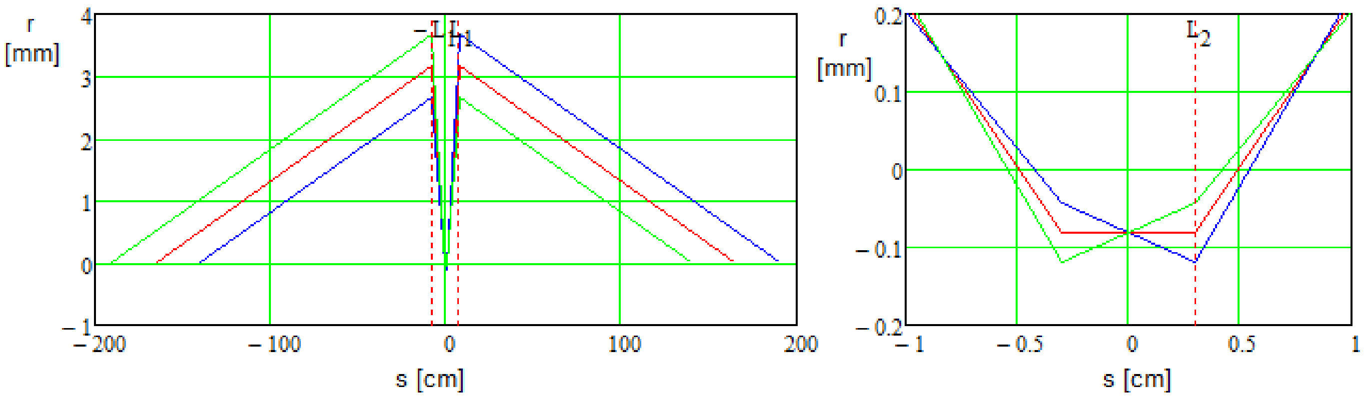
Optical Amplifier



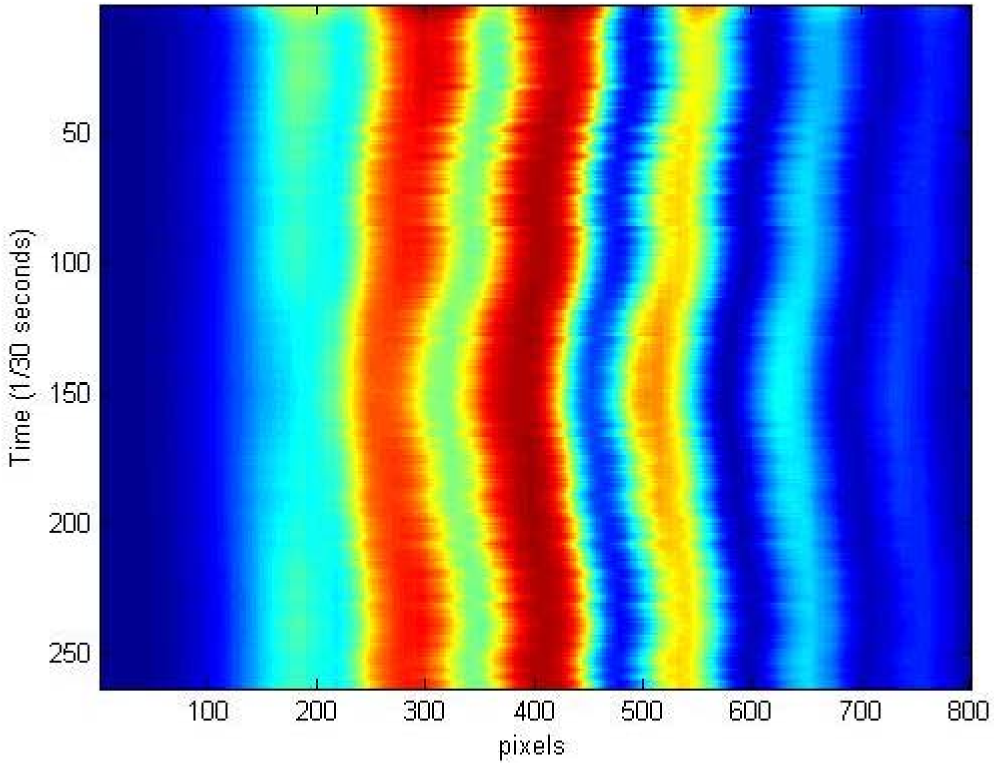
- Ti:Sapphire Optical Amplifier has a few advantages
  - Quite wide bandwidth
    - 10% FWHM at G_0=100
  - Allows operation in the CW regime
    - Decay time due to sp. rad. ~3.15 μs
  - Can deliver significant amplification with only ~1 mm signal delay.
- We bought a highly doped (0.5%wt Ti_2O_3) 2 mm thick Ti: Sapphire crystal from GT Crystal Systems for a prototype of OA
- An estimated low power gain is ~100 (20 Db) with pumping power density of 1.8 MW/cm^2
- Pumping along the direction of amplified radiation
  - *P* = 50 W, square profile with *r* = 30 μm
- Cooling the OA to the liquid nitrogen temperature is required.
  - It increases the crystal thermal conductivity
    ⇒ an acceptable ΔT across the crystal (~8K) and thermal stress
  - It reduces dn/dT ⇒
    reduces optics distortions related to high pumping power

Focusing of Beam Radiation to OA and Kicker

- Two lens system (F=8 cm, radius – 3.5 mm)
  - Reasonable compromise between major requirements
  - The spot size in OA to be sufficiently small: r<30 μm
    - diffraction limited size in OA: HWHM=6 μm or total size r≈15 μm
    - size due to beam convergence/divergence at OA input/exit ≈25 μm
  - Requirements to suppress Depth of field effects in kicker wiggler
    - diffraction limited size in kicker wiggler: HWHM=120 μm or total size r≈300 μm
    - Size increase due to the depth of field for radiation radiated at the entrance or exit of pickup wiggler: 170 μm
  - To mitigate the depth of field effects the wigglers are moved from the chicane by ~50 cm
- For OSC tests without OA the 4 lens telescope will be used
  - Complete suppression of depth of field
  - Larger bandwidth



Test of Optical Amplifier Prototype (continue)



Interference picture displacements on time#

- Interferometer is assembled
  - first tests started
  - Working on
    - the stabilization of interference picture
    - electronics to measure displacement of interference pattern with wave-length change

Courtesy of Matt Andorf

Cooling Rates

Main parameters of OSC

Undulator parameter, K	0.6
Undulator period	4.92 cm
Radiation wavelength at zero angle	750 nm
Number of periods, *m*	10
Total undulator length, L_w	0.50 m
Length from OA to undulator center	1.65 m
Amplifier gain (amplitude)	10
Telescope aperture, 2*a*	7 mm
Lens focal length, *F*	80 mm
Damp. rates (*x*=*y*/*s*)	160/140 s^{-1}

- Undulator period was chosen so that $\lambda|_{\theta=0}$=750 nm
- Cooling rates were computed using earlier developed formulas(HB2012)
  - Averaging over amplifier band yielded additionally ~20% reduction of rates.
- 2 mrad angular acceptance of optical system (aperture r=3.5 mm)

  ⇒ upper boundary of the band = 850 nm
- E.-m. wave dispersion in the OA amplifier is included into the gain
  - *G* = 10 implies an amplitude amplification of 10

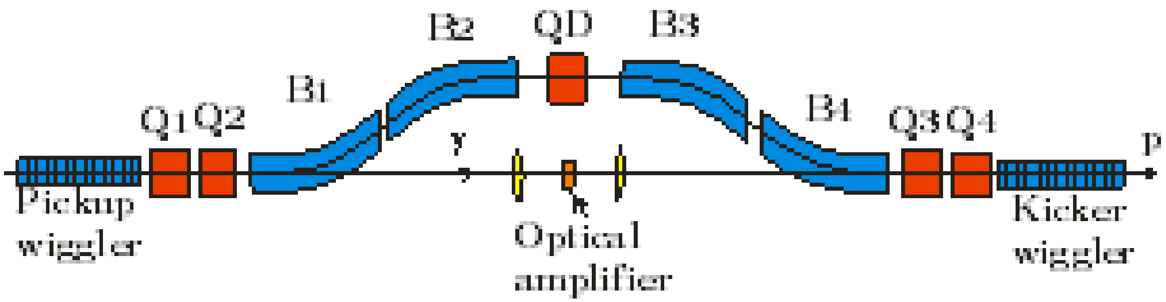
    ⇒ Dispersion makes the power gain to be somewhat larger than G^2.
- Undulator parameter K=0.6 is close to the optimal for chosen bandwidth and aperture

Conclusions

- Optical stochastic cooling looks as a promising technique for future hadron colliders
- Experimental study of OSC in Fermilab is in its initial phase
  - It is aimed to validate cooling principles and to demonstrate cooling with and without optical amplifier
    - Even in the absence of amplification (passive system, $G = 1$) the OSC damping exceeds SR damping by more than an order of magnitude
- The beam intensity ranges from a single electron to the bunch population limited by operation at the optimum gain (10^8)
  - Single electron cooling - localization of electron wave function and essence of quantum mechanics
    - Quantum noise for passive cooling
  - Cooling at the optimal gain (ultimate cooling) gets us to otherwise hidden details of OSC, in particular, to signal suppression

Backup Slides

OSC Limitations on IOTA Optics



- In the first approximation the orbit offset in the chicane (h), the path lengthening (δs) and the defocusing strength of chicane quad (Φ) together with dispersion and beta-function in the chicane center (D^*, β^*) determine the entire cooling dynamics
- δs is set by delay in the amplifier => M_{56}
- $\Phi D^* h$ is determined by the ratio of decrements => for known ε we obtain the dispersion invariant (A^*)

$$M_{56} \approx 2\Delta s \,,$$

$$\tilde{M}_{56} \approx 2\Delta s - \Phi D^* h \,,$$

$$\lambda_x / \lambda_s \approx \Phi D^* h / \left(2\Delta s - \Phi D^* h\right),$$

$$n_{\sigma p} \approx \mu_0 / \left(\left(2\Delta s - \Phi D^* h\right) k \sigma_p\right),$$

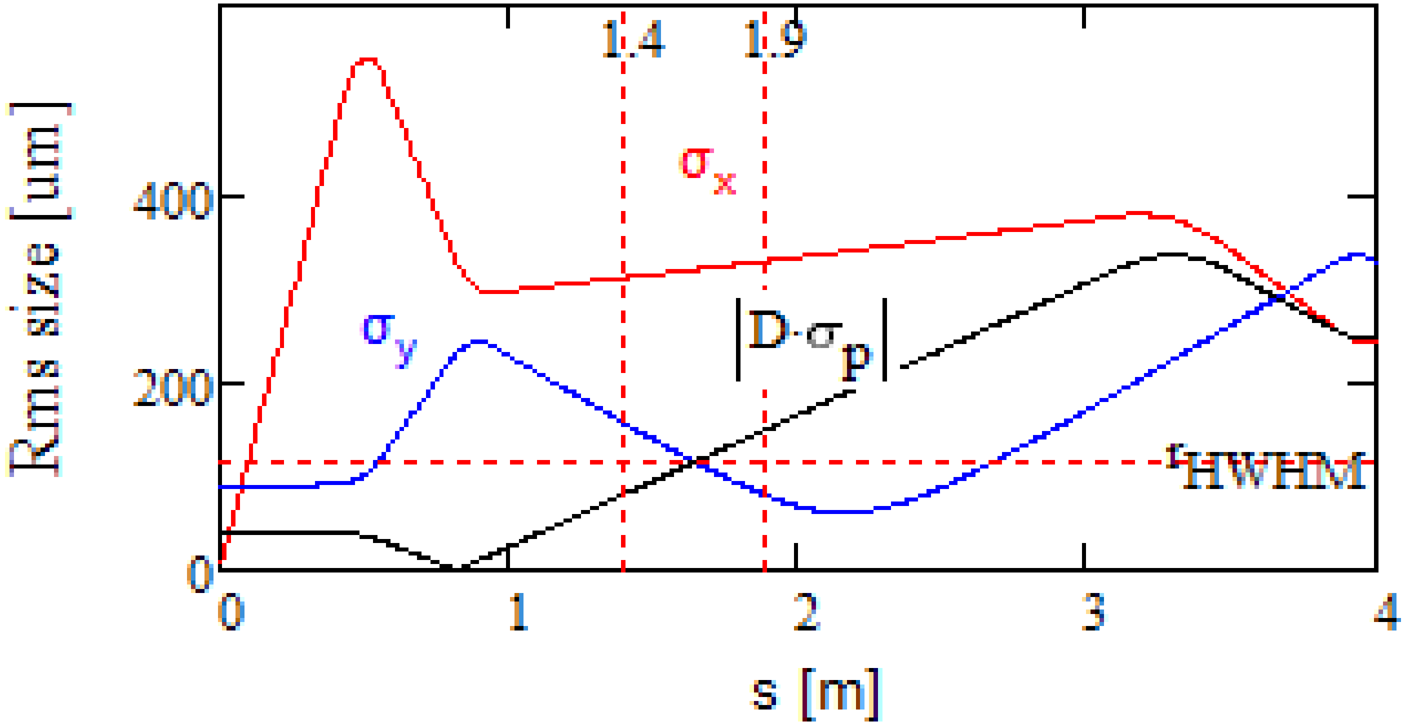
$$n_{\sigma x} \approx \mu_0 / \left(2kh\Phi\sqrt{\varepsilon\beta^*}\right),$$

$$\Rightarrow \Phi D^* h \approx \frac{\mu_0}{2kn_{\sigma x}}\sqrt{\frac{A^*}{\varepsilon}} \,, \quad A^* \equiv \frac{D^{*2}}{\beta^*}$$

- An average value of A in dipoles determines the equilibrium emittance. A^* is large and A needs to be reduced fast to get an acceptable value of the emittance (ε)

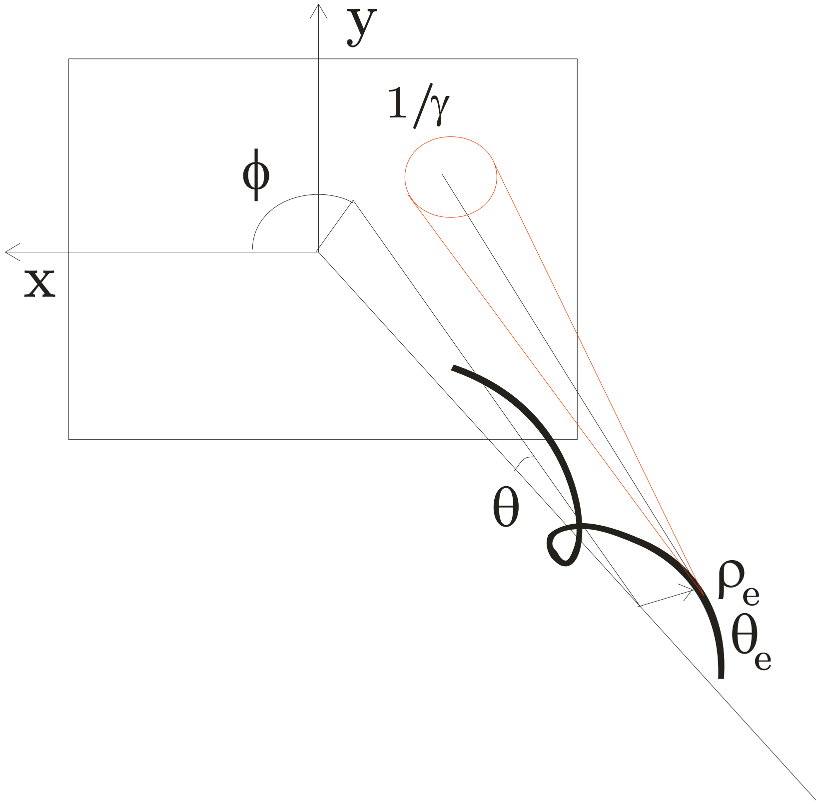
Effect of Beams Overlap on Cooling Rates

- In computation of cooling rates we neglected incomplete overlap of light and particle beams in the kicker undulator at the beginning of cooling process when the e-beam size is determined by SR.
- The problem is negligible for cooled beam
  - Factor of 5 reduction at the cooling beginning



Rms beam sizes (horizontal – σ_x, vertical – σ_y, and due to momentum spread - $|D\sigma_p|$) in vicinity of cooling chicane starting from the center of OSC section

Basics of OSC – Radiation from Undulator



- Radiation of ultra-relativistic particle is concentrated in 1/γ angle
- Undulator parameter:

$$K \equiv \gamma\theta_e = \frac{\lambda_{wgl}}{2\pi}\frac{eB_0}{mc^2}$$

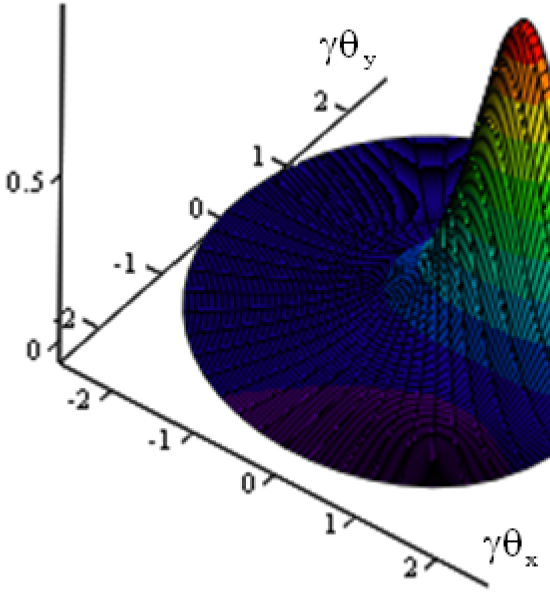
- For K ≥ 1 the radiation is mainly radiated into higher harmonics

- Liénard-Wiechert potentials and E-field of moving charge in wave zone

$$\begin{cases} \varphi(\mathbf{r},t) = \left.\dfrac{e}{(R-\boldsymbol{\beta}\cdot\mathbf{R})}\right|_{t-R/c} \\ \mathbf{A}(\mathbf{r},t) = \left.\dfrac{e\mathbf{v}}{(R-\boldsymbol{\beta}\cdot\mathbf{R})}\right|_{t-R/c} \end{cases} \Rightarrow$$

$$\mathbf{E}(\mathbf{r},t) = \frac{e}{c^2}\left.\frac{(\mathbf{R}-\boldsymbol{\beta}\cdot R)(\mathbf{a}\cdot\mathbf{R}) - \mathbf{a}R(R-\boldsymbol{\beta}\cdot\mathbf{R})}{(R-\boldsymbol{\beta}\cdot\mathbf{R})^3}\right|_{t-R/c}$$

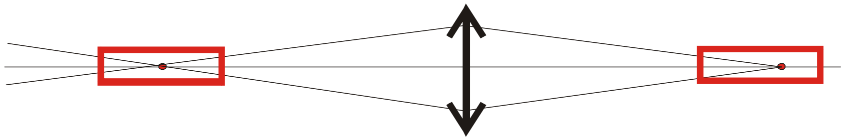
where $\mathbf{a} \equiv \dfrac{d\mathbf{v}}{dt}$



E_x for K=1

Basics of OSC – Radiation Focusing to Kicker Undulator

- Modified Kirchhoff formula

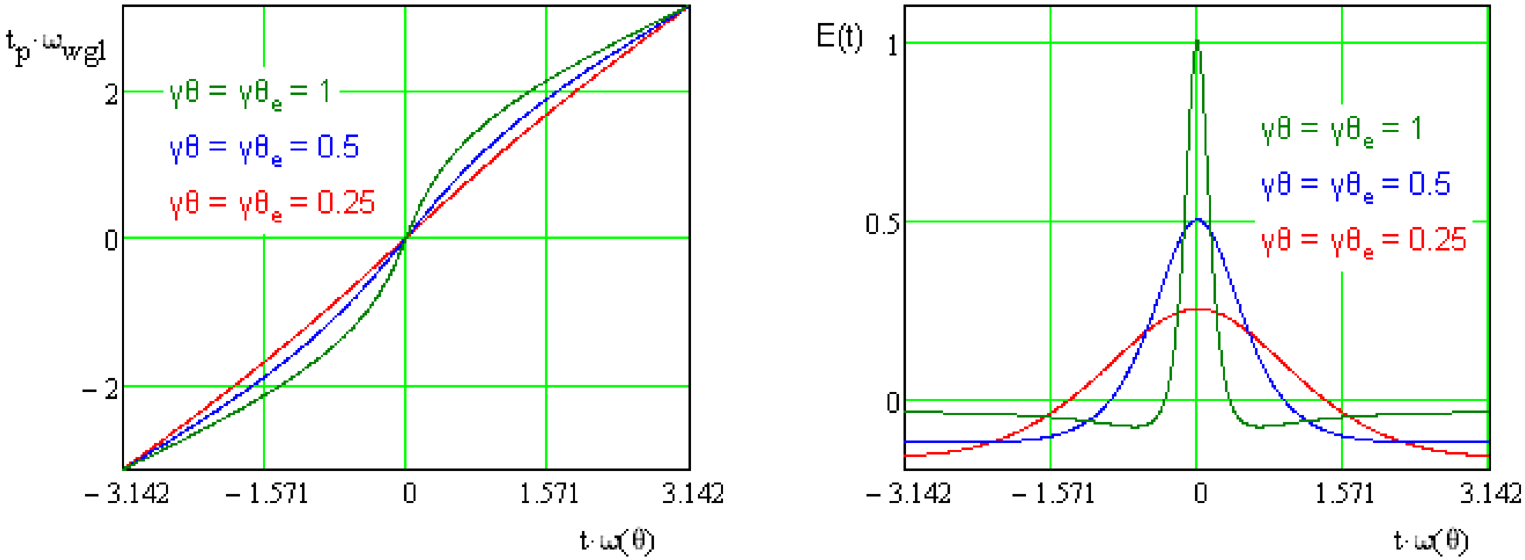


$$E(r)=\frac{\omega}{2\pi ic}\int_S \frac{E(r')}{|r-r'|}\mathrm{e}^{i\omega|r-r'|}\,ds'$$

=>

$$E(r)=\frac{1}{2\pi ic}\int_S \frac{\omega(r')E(r')}{|r-r'|}\mathrm{e}^{i\omega|r-r'|}\,ds'$$

- Effect of higher harmonics
  - Higher harmonics are normally located outside window of optical lens transparency and are absorbed in the lens material

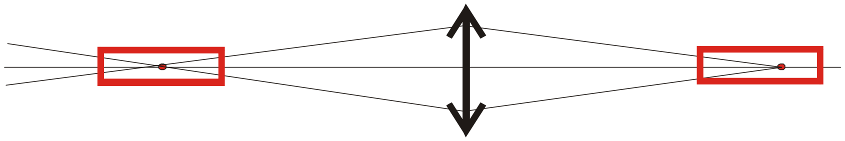


Dependences of retarded time (t_p) and E_x on time for helical undulator

- Only first harmonic is retained in the calculations presented below

Basics of OSC – Longitudinal Kick for K<<1

- For $K \ll 1$ refocused radiation of pickup undulator has the same structure as radiation from kicker undulator. They are added coherently:

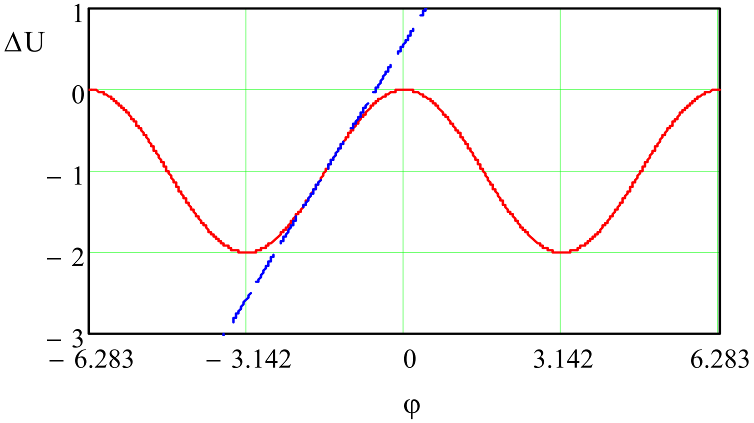


$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 e^{i\phi} \xrightarrow{\mathbf{E}_1=\mathbf{E}_2} 2\cos\left(\phi/2\right)\mathbf{E}_1 e^{i\phi/2}$$

⇨ Energy loss after passing 2 undulators

$$\Delta U \propto \left|E^2\right| = 4\cos\left(\phi/2\right)^2\left|\mathbf{E}_1{}^2\right| = 2\left(1+\cos\phi\right)\left|\mathbf{E}_1{}^2\right| = 2\left(1+\cos\left(kM_{56}\frac{\Delta p}{p}\right)\right)\left|\mathbf{E}_1{}^2\right|$$

- Large derivative of energy loss on momentum amplifies damping rates and creates a possibility to achieve damping without optical amplifier



- ♦ SR damping: $\lambda_{\|\_SR} \approx \frac{2\Delta U_{SR}}{pc} f_0$
- ♦ OSC: $\lambda_{\|\_OSC} \approx f_0 \frac{2\Delta U_{wgl}}{pc}\left(GkM_{56}\right) \xrightarrow{kM_{56}(\Delta p/p)_{\max}=\pi} f_0 \frac{2\Delta U_{wgl}}{pc}\left(\frac{G}{\left(\Delta p/p\right)_{\max}}\right)$

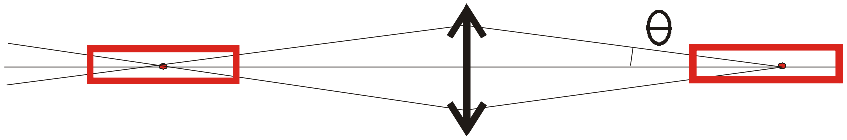
where G - optical amplifier gain, $(\Delta p/p)_{max}$ - cooling system acceptance

⇨ $\lambda_{\|\_OSC} \propto B^2 L \propto K^2 L$ - but cooling efficiency drops with K increase above ~1

Basics of OSC – Longitudinal Kick for K<<1(continue)

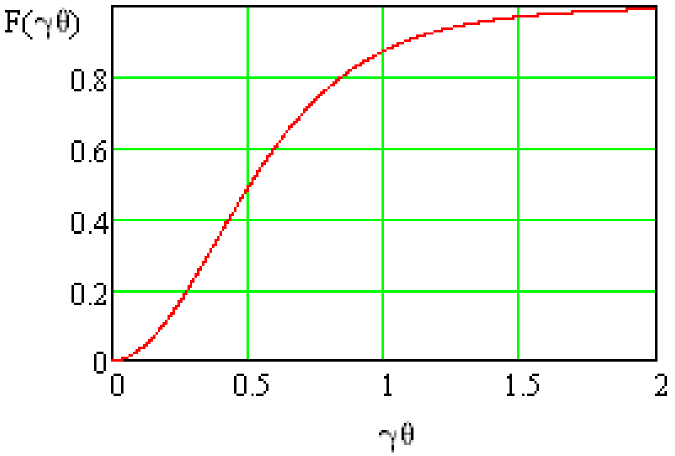
- Radiation wavelength depends on θ as

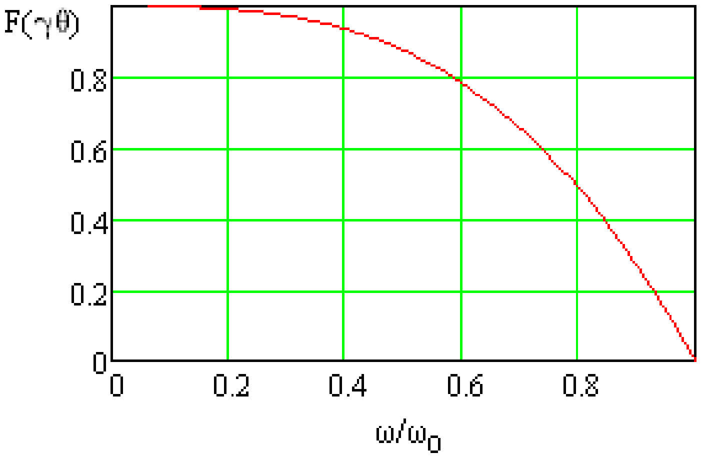
$$\lambda = \frac{\lambda}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$



Limitation of system bandwidth by (1) optical amplifier band or (2) subtended angle reduce damping rate

$$\lambda_{\|\_SR} = \lambda_{\|\_SR0} F\left(\gamma\theta_{\mathrm{m}}\right), \qquad F\left(x\right) = 1 - \frac{1}{\left(1+x^2\right)^3}$$





- For narrow band: $\Delta U_{wgl} = \Delta U_{wgl0}\left(\frac{3\Delta\omega}{\omega}\right), \quad \frac{3\Delta\omega}{\omega} << 1$

where $\Delta U_{wgl0} = \frac{e^4B^2\gamma^2L}{3m^2c^4}\begin{cases} 1, & \text{Flat wiggler} \\ 2, & \text{Helical wiggler} \end{cases}$ the energy radiated in one undulator

Basics of OSC – Radiation from Flat Undulator

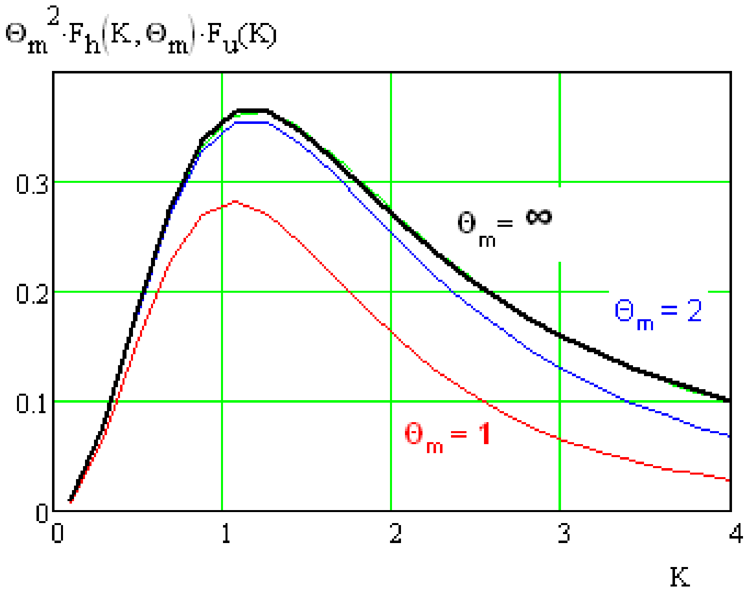
- For arbitrary undulator parameter we have

$$\Delta U_{OSC\_F} = \frac{1}{2}\frac{4e^4 B_0^{\,2}\gamma^2 L}{3m^2c^4} GF_f\left(K,\gamma\theta_{\max}\right)F_u\left(\kappa_u\right)$$

$$F_u\left(\kappa_u\right) = \mathrm{J}_0\left(\kappa_u\right) - \mathrm{J}_1\left(\kappa_u\right), \quad \kappa_u = K^2/\left(4\left(1+K^2/2\right)\right)$$

Fitting results of numerical integration yields:

$$F_h\left(K,\infty\right) \approx \frac{1}{1+1.07K^2+0.11K^3+0.36K^4}, \quad K \equiv \gamma\theta_e \le 4$$



- Dependence of wave length on θ:

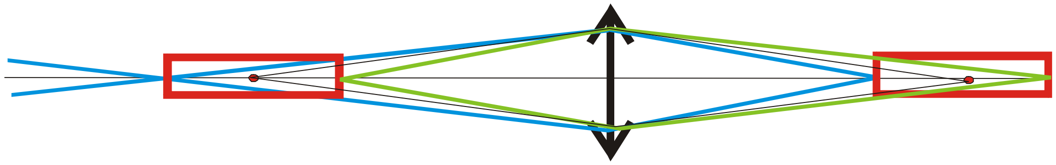
$$\lambda \approx \frac{\lambda_{wgl}}{2\gamma^2}\left(1+\gamma^2\left(\theta^2+\frac{\theta_e^{\,2}}{2}\right)\right)$$

$$K \equiv \gamma\theta_e$$

- Flat undulator is "more effective" than the helical one
- For the same K and λ_{wgl} flat undulator generates shorter wave lengths

- For both cases of the flat and helical undulators and for fixed *B* a decrease of λ_{wgl} and, consequently, λ yields kick increase
  - ♦ but wavelength is limited by both beam optics and light focusing

Basics of OSC – Correction of the Depth of Field

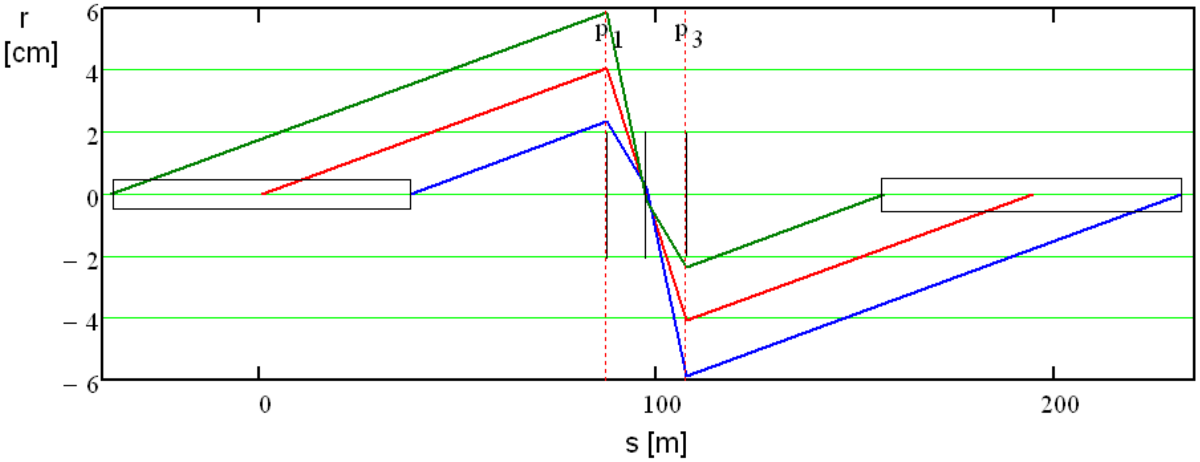


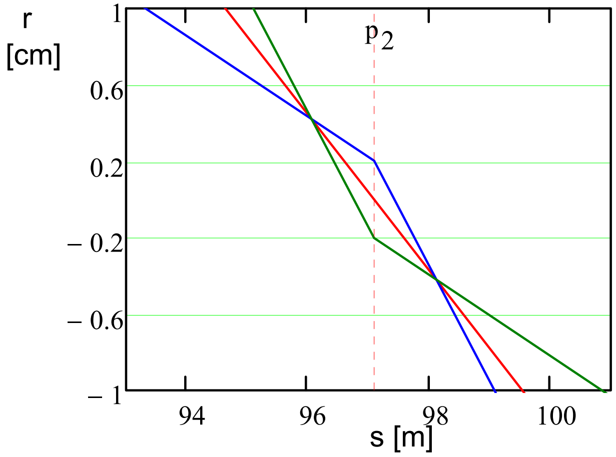
- It was implied above that the radiation coming out of the pickup undulator is focused on the particle during its trip through the kicker undulator
  - It can be achieved with lens located at infinity

$$\frac{1}{2F+\Delta s}+\frac{1}{2F-\Delta s}=\frac{1}{F}\quad\rightarrow\quad\frac{1}{F-\Delta s^2/4F}=\frac{1}{F}\quad\xrightarrow{F\to\infty}\quad\frac{1}{F}=\frac{1}{F}$$

  - but this arrangement cannot be used in practice
- A 3-lens telescope can address the problem within limited space

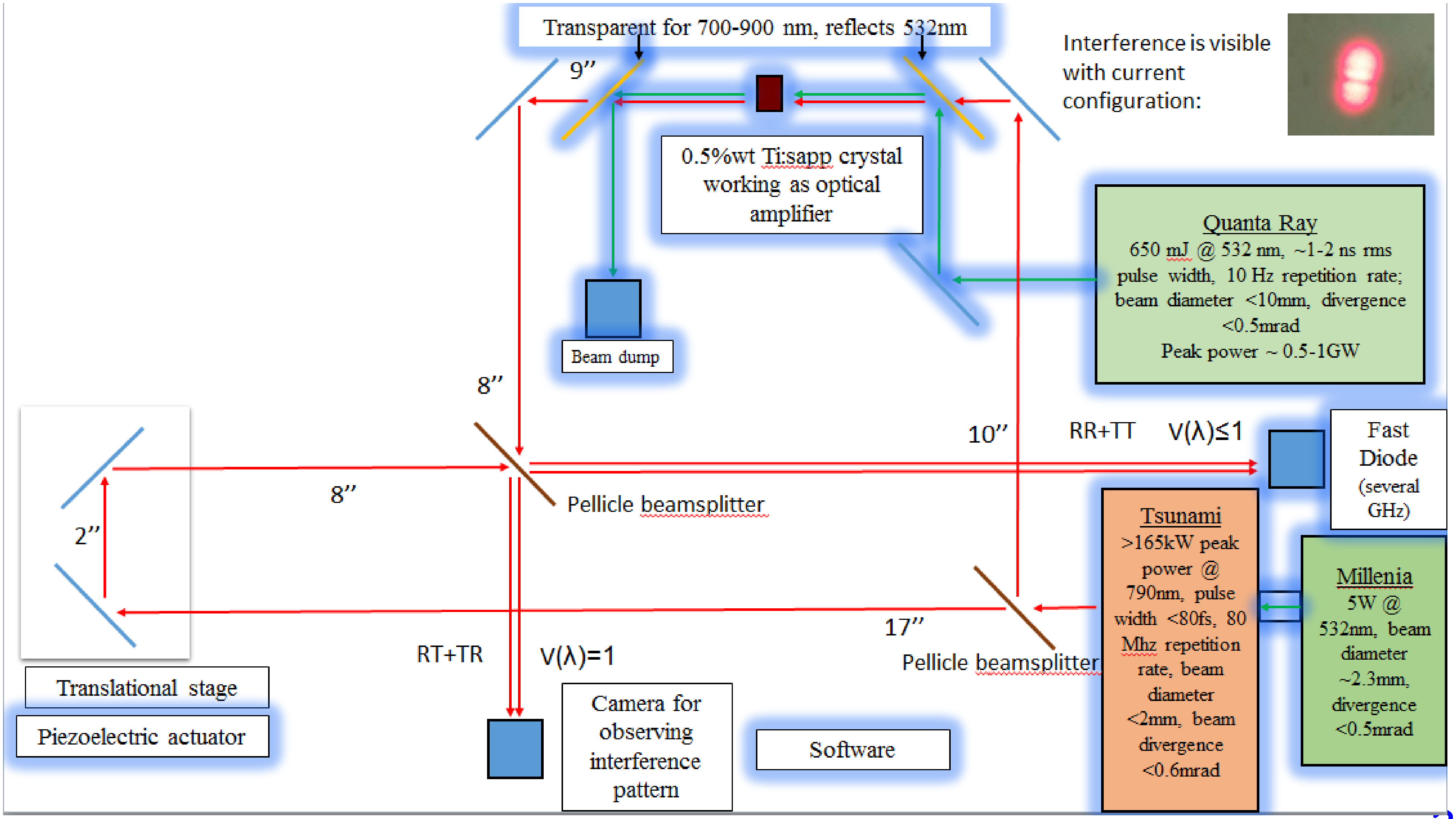
$$\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -F_2^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$





Test of Optical Amplifier Prototype

- OA operation in pulsed regime ⇒ Cooling is not required
- The goal to measure the amplitude and phase of the amplifier gain
  ⇒ Interferometer for phase measurements



Transparent for 700-900 nm, reflects 532nm

Interference is visible with current configuration:

0.5%wt Ti:sapp crystal working as optical amplifier

Quanta Ray
650 mJ @ 532 nm, ~1-2 ns rms pulse width, 10 Hz repetition rate; beam diameter <10mm, divergence <0.5mrad
Peak power ~ 0.5-1GW

Beam dump

Fast Diode (several GHz)

Tsunami
>165kW peak power @ 790nm, pulse width <80fs, 80 Mhz repetition rate, beam diameter <2mm, beam divergence <0.6mrad

Millenia
5W @ 532nm, beam diameter ~2.3mm, divergence <0.5mrad

Translational stage

Piezoelectric actuator

Camera for observing interference pattern

Software