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Damping dynamics of optical stochastic cooling

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Abstract

A necessary condition for transverse phase space damping in the optical stochastic cooling (also applicable in the microwave stochastic cooling) with transit-time method for both the longitudinal and transverse damping dynamics is studied. An optimal laser focusing condition for laser-beam interaction in the correction undulator was also obtained. The required laser amplification power can be large for hadron colliders at very high energies.

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1. Introduction

High-energy charged particles emit photons in dipoles. The photon emission is a random process. Using the photons instead of microwave signals in beam cooling would solve the problem of coherent signal contamination, and may dramatically enhance the cooling rate. The optical stochastic cooling (OSC) was proposed by Mikhailichenko and Zolotorev [1], in which a quadrupole wiggler and a longitudinal kicker system at a high dispersion location were applied to damp betatron and synchrotron motions via the synchro-betatron coupling. Subsequently, Zolotorev and Zholents applied transit-time method to optical stochastic cooling [2]. This paper follows the transit-time method of optical stochastic cooling to derive a necessary condition for the beam transport system, and study the OSC cooling dynamics [3].

A typical stochastic cooling system consists of a pickup, an amplifier, and a kicker. The electromagnetic (EM) wave radiated by a charged particle in the first undulator is amplified by the optical amplifier, while the particle travels in the beam-bypass. The amplified EM wave and the particle are brought together to interact in the second undulator. This will change the particle's energy. The amount of energy change depends on the magnitude of the EM wave and the relative phase between the particle's and the EM wave's transit times from the first to the second undulators.

We consider a test particle with a momentum deviation $\delta_i = \Delta P_i/P$, and the betatron phase space coordinates (x_i, x_i'). In the Frenet-Serret

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coordinate system, the path length of the test particle in the bypass section is [4]

$$\ell_i = \int_{s_1}^{s_2} \sqrt{\tilde{x}'^2 + \tilde{z}'^2 + \left(1 + \frac{\tilde{x}}{\rho}\right)^2}\, \mathrm{d}s \approx \int_{s_1}^{s_2} \left(1 + \frac{\tilde{x}}{\rho}\right) \mathrm{d}s = \ell_0 + x_{i1} I_1 + x'_{i1} I_2 + \delta_i I_D \tag{1}$$

where $\hat{x}, \hat{s}$ and $\hat{z}$ form a curvilinear coordinate system with a horizontal bending radius ρ, the coordinates $\tilde{x}$ and $\tilde{z}$ are the deviation from a reference orbit, and s is the longitudinal coordinate along the reference orbit, x_{i1}, x'_{i1} are the conjugate phase space coordinates for the ith particle at the location s_1, and the integrals I_1, I_2, and I_D are

$$I_1 = \int_{s_1}^{s_2} \frac{M_{11}(s, s_1)\, \mathrm{d}s}{\rho(s)}, \quad I_2 = \int_{s_1}^{s_2} \frac{M_{12}(s, s_1)\, \mathrm{d}s}{\rho(s)}, \quad I_D = \int_{s_1}^{s_2} \frac{D(s)\, \mathrm{d}s}{\rho(s)} \tag{2}$$

where the integrals are carried out from the first undulator at s_1 to the second undulator at s_2 via the particle beam bypass.

In the first undulator, a test particle radiates an EM wave propagating in the s-direction: $\mathscr{E}_i = \mathscr{E}_0 \sin(ks - \omega t + \phi_i)$ with electric field amplitude $\mathscr{E}_0$ and phase ϕ_i. The wave number and frequency are $k = 2\pi/\lambda$ and $\omega = kc$. With proper pathlength adjustment, the fractional change of its momentum is [5]:

$$\delta P_i/P = -[\mathrm{sgn}(I_D)] G \sin(\Delta\phi_i), \quad \Delta\phi_i = k(\ell_i - \ell_0) = k[x_i I_1 + x'_i I_2 + \delta_i I_D] \tag{3}$$

where $\mathrm{sgn}(I_D)$ is the sign of I_D, $G = gq\,\mathscr{E}_0 N_\mathrm{u} \lambda_\mathrm{u} K[\mathrm{JJ}]/(2c\gamma P)$ is the amplitude of the fractional momentum gain-factor, q is the magnitude of the particle charge, N_u is the number of undulator periods, g is the optical–amplifier amplification factor, and δP_i is the amount of momentum change related to the coherent longitudinal kick $\Delta\delta_i = \delta P_i/P$.

Let D_2 and D'_2 be the dispersion function and its derivative at the second undulator. The changes of the particle betatron coordinates at the exit of the second undulator are $\Delta x_{i2} = -D_2(\delta P_i/P)$ and $\Delta x'_{i2} = -D'_2(\delta P_i/P)$, where x_{i2} and x'_{i2} are the phase space coordinates of the ith particle at the second undulator location.

2. Cooling rates

A test particle interacts with the EM wave of its own radiation and sample particles within a distance less than $N_\mathrm{u}\lambda$. These interactions constitute the incoherent component of the kick received by the particle. Assume that a test particle interacts with N_s electro-magnetic waves (including its own wave) in a sample. The change of the particle's momentum at the exit of the cooling insertion becomes $\delta_{ic} = \delta_i - [\mathrm{sgn}(I_D)] G \sum_j^{N_\mathrm{s}} \sin(\Delta\phi_i + \psi_{ij})$, where δ_{ic} is the relative momentum of the ith particle after the longitudinal kick, N_s is the number of particles in the sample, $\psi_{ij} = \Delta\phi_j - \Delta\phi_i$.

A test particle interacts with the electromagnetic waves radiated from the sample of N_s particles. Carrying out the ensemble average (assuming $I_D > 0$), we find the longitudinal and transverse damping decrements

$$\alpha_\delta \equiv -\frac{\langle \delta_{ic}^2 - \delta_i^2 \rangle}{\sigma_\delta^2} = 2GkI_D \mathrm{e}^{-u} - \frac{G^2 N_\mathrm{s}}{2\sigma_\delta^2},$$

$$u = \frac{1}{2} k^2 [(\beta_1 I_1^2 - 2\alpha_1 I_1 I_2 + \gamma_1 I_2^2)\varepsilon_x + I_D^2 \sigma_\delta^2]$$

$$\alpha_x = -\frac{\langle P_{x2c}^2 + x_{2c}^2 - (P_{x2}^2 + x_2^2) \rangle}{\sigma_{x2}^2} = 2GkI_\perp \mathrm{e}^{-u} - \frac{G^2 N_\mathrm{s} \mathscr{H}_2}{2\varepsilon_x}$$

where u is a measure of the total beam thermal energy, and

$$\begin{aligned} I_\perp = -\frac{\beta_1}{\beta_2} \Bigg\{ & P_{D2} \Big[\Big((\beta_2 M_{21} + \alpha_2 M_{11}) - \frac{\alpha_1}{\beta_1} (\beta_2 M_{22} + \alpha_2 M_{12}) \Big) \left(I_1 - \frac{\alpha_1}{\beta_1} I_2 \right) \\ & + \frac{1}{\beta_1^2} (\beta_2 M_{22} + \alpha_2 M_{12}) I_2 \Big] \\ & + D_2 \left[\left(M_{11} - \frac{\alpha_1}{\beta_1} M_{12} \right) \times \left(I_1 - \frac{\alpha_1}{\beta_1} I_2 \right) + \frac{1}{\beta_1^2} M_{12} I_2 \right] \Bigg\}. \end{aligned} \tag{4}$$

The transverse cooling requires the condition $I_\perp > 0$ (for $I_D > 0$).

3. Stochastic cooling dynamics

The cooling process can be expressed as

$$\frac{\mathrm{d}\varepsilon_x}{\mathrm{d}t} = -\frac{2GkI_\perp \varepsilon_x}{T_0}\mathrm{e}^{-u} + \frac{G^2 N_\mathrm{s}\mathcal{H}_2}{2T_0},$$
$$\frac{\mathrm{d}\sigma_\delta^2}{\mathrm{d}t} = -\frac{2GkI_D\sigma_\delta^2}{T_0}\mathrm{e}^{-u} + \frac{G^2 N_\mathrm{s}}{2T_0} \tag{5}$$

where T_0 is the revolution period. The momentum gain factor G is set by the laser amplifier [6]. If the optimal gain factors for the momentum and transverse cooling are the same, we can set the laser gain factor to obtain an optimal momentum gain factor. The condition for equal optimal gain factors is $I_D\mathcal{H}_2\sigma_\delta^2 = I_\perp \varepsilon_x$. In this case, the ratio of the damping decrements becomes $\alpha_\delta/\alpha_\perp = I_D/I_\perp$. However, if $I_D \neq I_\perp$, the equal gain condition cannot be fulfilled at all time.

At the equal decrement condition, the particle bypass line should be designed with the condition: $I_\perp = I_D$. The beam will maintain the equilibrium condition with $\varepsilon_x = \mathcal{H}_2\sigma_\delta^2$. Let G_0 be an initial gain factor. The equation of damping dynamics becomes

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{2G_0kI_D}{T_0}u\mathrm{e}^{-u} + \frac{G_0^2 N_\mathrm{s} v}{2T_0},$$
$$v = \frac{1}{2}k^2[(\beta_1 I_1^2 - 2\alpha_1 I_1 I_2 + \gamma_1 I_2^2)\mathcal{H}_2 + I_D^2]. \tag{6}$$

The equilibrium emittance is reached when $\mathrm{d}u/\mathrm{d}t = 0$. Fig. 1 shows the right-hand side of Eq. (6). Note that cooling is possible when $u_\mathrm{eq} \leqslant u \leqslant u_\mathrm{th}$, where u_eq is the equilibrium thermal energy and u_th is the cooling threshold energy.

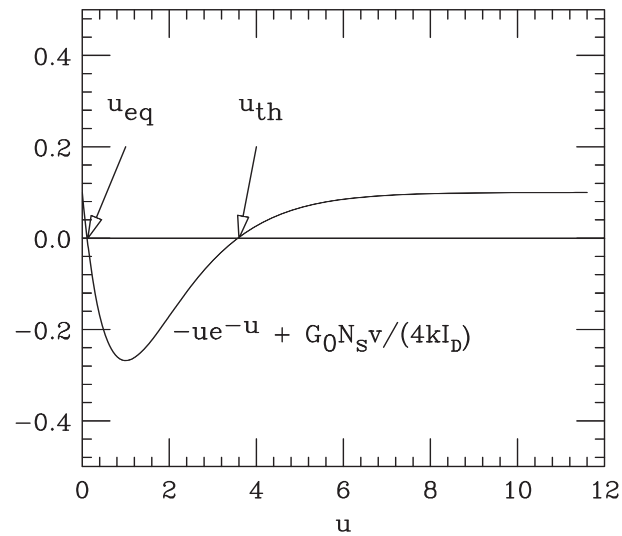

Fig. 1. The cooling dynamical function for a fixed gain G_0 with $G_0N_\mathrm{s}v/4kI_D = 0.1$ (used only for an illustrative example), is shown as a function the parameter u. Note that beam cooling occurs only when $u \leqslant u_\mathrm{th}$, and the cooling stops when $u = u_\mathrm{eq}$ is reached. In this case, cooling appears to be possible for $u \leqslant u_\mathrm{th} \approx 3.6$. However, the sinusoidal nature of the momentum kick in Eq. (3) renders this parametric region not applicable.

4. Amplification factor

The total energy of the photon emission in the first undulator is

$$W_1 = \frac{1}{2}\varepsilon_0\mathcal{E}_1^2 A_1 c\Delta t_\mathrm{R} = \frac{1}{4\pi\varepsilon_0}\pi\xi k q^2[\mathrm{JJ}]^2 \tag{7}$$

where $\mathcal{E}_1$ is the peak electric field amplitude produced in the first undulator, A_1 is the cross-section area of the coherent radiation,[2] and $\Delta t_\mathrm{R} = N_\mathrm{u}\lambda/c$ is the duration of the radiation pulse. We also use the fact that a particle with a charge q emits about $(\pi q^2\xi/4\pi\varepsilon_0\hbar c)[\mathrm{JJ}]^2$ coherent photons at the energy $\hbar\omega$ during one pass of the undulator.

The input and output peak powers of the laser amplifier are

$$\hat{P}_1 = \frac{W_1}{\Delta t_\mathrm{R}}N_\mathrm{s} = \frac{1}{2}\varepsilon_0\mathcal{E}_1^2 A_1 c N_\mathrm{s}, \quad \hat{P}_2 = g^2\hat{P}_1$$

[2] If we assume that the photon beam be distributed as *bi-Gaussian* radially but uniformly along the longitudinal *s*-direction, the total energy of the photons can be written as

$$W_0 = \int \frac{W_0/\Delta s}{2\pi\sigma_x\sigma_z}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{z^2}{2\sigma_z^2}\right)\mathrm{d}x\,\mathrm{d}z\Delta s$$

where the energy density is $\varepsilon_0\mathcal{E}^2 = (W_0/\Delta s)/(2\pi\sigma_x\sigma_z)$. Here, $\mathcal{E}$ is the peak field at $r = 0$. Now, we can write $W_0 = \varepsilon_0\mathcal{E}^2 A\Delta s$, i.e. the effective photon beam area is $A = 2\pi\sigma_x\sigma_z$. For a photon beam with cylindrical symmetry, we find $A = 2\pi\sigma_r^2$.

where g^2 is the power gain from the laser amplifier, $N_s = N_B N_u \lambda / 2\sqrt{6} c\sigma_\tau$ is the number of particles in a sample within a bandwidth of $\Delta\omega|_{FWHM} = \omega/N_u$. Here, we have assumed 100% photon transmission in the optical amplifier, and assume that the bandwidth of the laser amplifier is larger than that of the undulator radiation.

The peak electric field at the second undulator depends on the amplifier gain factor and focusing property through conservation of energy, i.e. $\mathscr{E}_2^2 A_2 = g^2 \mathscr{E}_1^2 A_1$, where $\mathscr{E}_2$ and A_2 are the peak electric field amplitude and the photon beam area at the waist (see footnote 2), presumably at the mid-point, of the second undulator. The momentum gain factor G is

$$G = \frac{q\langle \mathscr{E} \rangle_2 N_u \lambda_u K[\mathrm{JJ}]}{2c\gamma P},$$
$$\langle \mathscr{E} \rangle_2 = \frac{2\mathscr{E}_2}{L} \int_0^{L/2} \frac{\mathrm{d}s}{\sqrt{1 + (s/\beta_*)^2}} \tag{8}$$

where $L = N_u \lambda_u$ is the length of the second undulator and β_* is the betatron amplitude function for the photon beam at the waist.

For a given momentum gain factor, the peak power becomes

$$\hat{P}_2 = G^2 \frac{N_s (E_b/q)^2}{Z_0 \xi N_u [\mathrm{JJ}]^2} \mathscr{F}_2,$$
$$\mathscr{F}_2 = \frac{A_0/A_2}{8[\ln(A_0/A_2 + \sqrt{1 + (A_0/A_2)^2})]^2} \tag{9}$$

where E_b is the beam energy, Z_0 is the impedance in vacuum, $A_2 = 2\pi\sigma_*^2$ is the rms photon beam area at the waist of the second undulator (see footnote 2), and $A_0 = N_u \lambda_u \lambda / 4$. Minimum laser amplifier occurs when $A_2 = 0.3012 A_0$, where $\mathscr{F}_2 = 0.1132$.[3]

[3] The emittance of the photon beam, $\lambda/(4\pi)$, may substantially differ from the emittance of the charged particle beams, e.g. 3.3 nm for Tevatron at 1 TeV and 16 nm for RHIC beam at 100 GeV/amu. The efficiency of the cooling may be reduced by the overlap area between the charged particle and the photon beams. The optimal energy gain at the second undulator for the charged particle beams is equivalent to the minimum in the laser power.

The average laser power is equal to the peak power multiplied by the duty factor, i.e.

$$\langle P \rangle_2 = \hat{P}_2 \frac{n_b 2\sqrt{6}\sigma_\tau}{T_0} = G^2 \frac{(E_b/q)^2}{Z_0 \xi [\mathrm{JJ}]^2} \frac{N_B n_b \lambda}{C} \mathscr{F}_2 \tag{10}$$

where n_b is the number of bunches, σ_τ is the rms bunch length in time, and C is the circumference of the storage ring. Note that the average power is proportional to the total number of particles in the storage ring.

In the low-gain regime, the incoherent heating term is now small and can be neglected. The damping equation becomes $(\mathrm{d}u_\delta/\mathrm{d}t) = -(2GkI_D/T_0)\mathrm{e}^{-u_x} u_\delta \mathrm{e}^{-u_\delta}$. Since $u_\delta \leqslant \pi^2/48$ is small, the damping is almost exponential and becomes more so as the cooling proceeds and will continue until the cooling force is balanced by the heating forces coming from RF noise, intra-beam scattering, etc. This is highly in contrast with the cooling at optimum gain factor discussed in Section 3, where the cooling process becomes more and more inefficient as the beam is cooled. With $u_x = 0$, the cooling time is $\tau_{\mathrm{cool}} \approx (\mathrm{e}_\delta^u / 2GkI_D) T_0$. The resulting peak and average power are

$$\hat{P}_2 = \left(\frac{T_0}{\tau_{\mathrm{cool}}}\right)^2 \frac{N_s (E_b/q)^2 \mathrm{e}^{2u_\delta}}{Z_0 N_u \xi [\mathrm{JJ}]^2 (2kI_D)^2} \mathscr{F}_2,$$
$$\langle P \rangle_2 = \left(\frac{T_0}{\tau_{\mathrm{cool}}}\right)^2 \left(\frac{n_b N_B \lambda}{C}\right) \frac{(E_b/q)^2 \mathrm{e}^{2u_\delta}}{Z_0 \xi [\mathrm{JJ}]^2 (2kI_D)^2} \mathscr{F}_2 \tag{11}$$

where C is the circumference of the storage ring. Note that the average power depends on the total number of particles $n_b N_B$ in the ring and the square of the energy over charge $(E_b/q)^2$.

5. Conclusion

In this paper, we derived a necessary condition for the transverse phase space damping in the optical stochastic cooling. We have also explored the damping rates, the amplification factor, cooling dynamics, and the required peak and average output power of the laser. We derived an optimal laser focusing condition for the charged particle beam and the laser beam interaction in an undulator. With the available optical amplifiers at present, it is rather impractical to use the optical stochastic cooling method to cool proton and heavy ion beams at *very* high energies. However, we find that the cooling method may be beneficial to low-energy electron beams, and around 1 TeV proton beam energy.

We also point out the difficulties of OSC with optimal gain condition. At the optimal gain, the required laser power is usually very large. As the beam is cooled, it is difficult to change the charged particle optics for a larger kI_D to compensate the decrease in emittances. The best solution is to cool beams in the low-gain regime, where the heating term may be negligible. For Tevatron, it seems to be feasible to use the Ti-sapphire $\lambda = 0.78$ μm for OSC at 1 TeV. One needs a shorter wavelength broadband laser for VLHC, and a long wavelength broadband laser for RHIC.

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