OPTICALLY BASED DIAGNOSTICS FOR OPTICAL STOCHASTIC COOLING*

M. B. Andorf[1], V. A. Lebedev[2], P. Piot[1,2], J. Ruan[2]
[1] Department of Physics and Northern Illinois Center for Accelerator & Detector Development, Northern Illinois University DeKalb, IL, USA
[2] Fermi National Accelerator Laboratory, Batavia, IL, USA

Abstract

An Optical Stochastic Cooling (OSC) experiment with electrons is planned in the Integrable Optics Test Accelerator (IOTA) ring currently in construction at Fermilab. OSC requires timing the arrival of an electron and its radiation generated from the upstream pickup undulator into the downstream kicker undulator to a precision on the order of less than a fs. The interference of the pickup and kicker radiation suggests a way to diagnose the arrival time to the required precision.

INTRODUCTION

Fermilab is currently preparing a proof-of-principle of Optical Stochastic Cooling (OSC) using 100 MeV electrons at the Integrable Optics Test Accelerator (IOTA). In OSC a particle emits radiation in a pickup undulator. The light from the pickup is then transported, and in the presence of an optical amplifier, amplified, to a kicker undulator located downstream and identical to the pickup undulator. Therefore the particle follows a curved path in a magnetic bypass chicane such that it arrives at the entrance of the kicker just as the light from the pickup is also arriving [1, 2]. For the reference particle the arrival time of radiation and particle in the kicker is such that it receives no kick. Particles with a nonzero synchrotron or betatron coordinate are displaced relative to the reference particle by an amount $\Delta s = M_{51}x + M_{52}\theta + M_{56}(\Delta p/p)$ and receive a longitudinal kick given as

$$\frac{\delta p}{p} = -\xi_0 \sin(k\Delta s). \tag{1}$$

In the above M_{ij} are the elements of the 6×6 transfer matrix from pickup to kicker, $k \equiv 2\pi/\lambda$ is the radiation wavenumber, and $(\delta p)/p$ is the relative momentum change of the particle after a single pass through the cooling section. The kick normalized amplitude is $\xi_0 = 2E_0\sqrt{G}/(pc)$ where E_0 corresponds to, in the case of a sufficiently small undulator parameter, the energy radiated in a single undulator, G is the gain in power from the optical amplifier (if present), p the design momentum and c the velocity of light. When the dispersion function is non vanishing at the kicker undulator location a horizontal kick is also applied thereby resulting in horizontal cooling. Likewise, introducing coupling between the two transverse phase spaces provides a path for cooling along all three degrees of freedom.

* Work supported by the by the US Department of Energy (DOE) contract DE-SC0013761 to Northern Illinois University. Fermilab is operated by the Fermi research alliance LLC under US DOE contract DE-AC02-07CH11359.

Table 1: Undulator Parameters for the OSC in IOTA

parameter	value	unit
U_o	100	MeV
K	1.04	-
period	11.06	cm
λ (at zero angle)	2.2	µm
Total length	77.4	cm

In IOTA, the OSC proof-of-principle experiment will be accomplished in two phases. In a first stage, we will not have an amplifier and rely on passive cooling: the optical system will consists of a telescope composed of three lenses chosen so that the optical transfer matrix between the pickup and kicker undulators is $-I$ where I is identity matrix. The telescope can transport $\sim$ 90% of the energy from the pickup to the kicker. Parameters for the undulators can be found in Table 1 and yield a value for the kick amplitude, $\xi_0 = 8.5\times10^{-10}$. Simulations show that the kick is further reduced by roughly 10 % due to dispersion in telescope [3]. In the second stage, a 7-dB Cr:ZnSe optical amplifier (with an operating band from 2.2 – 2.9 µm) will be installed and is expected to double the cooling rates over the passive cooling scheme.

In this paper we explore how to measure the timing error between the arrival of the pickup signal and the particle in the kicker. The timing precision needs to be < 1 fs in order to fully optimize the cooling rates. We propose a technique that uses optical interference between the pickup and kicker radiation signals. Before detailing the measurement technique we first consider in the impact of arbitrary timing error on the OSC process.

LONGITUDINAL TIMING

Cooling Rates and Transit Time Error

The damping decrements λ_i ($i \in [x, s]$) for small betatron and synchrotron amplitudes are given by [4]

$$\begin{bmatrix}\lambda_x\\ \lambda_s\end{bmatrix} = \frac{k\xi_o}{2}\begin{bmatrix}M_{56}-S_P\\ S_P\end{bmatrix}, \tag{2}$$

where $S_p = M_{51}D_p + M_{52}D_P' + M_{56}$ is the partial slip factor such that a particle with a momentum deviation $\Delta p/p$ will be

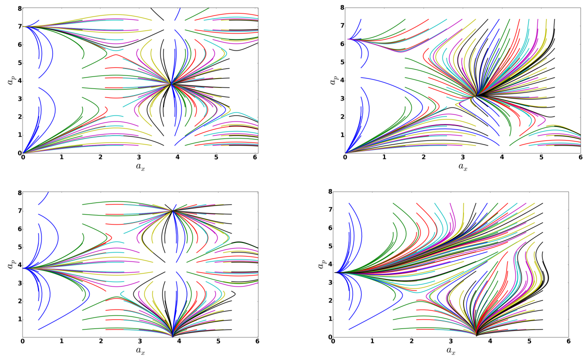

Figure 1: Trajectories in a_x,a_p space. Top figures are for $ks_d = 0$. The left shows trajectories from OSC without diffusion from synchrotron damping, while the right figure includes both. The bottom figure is set for $ks_d = \pi$ with again the left figure showing trajectories from OSC and the right including diffusion from synchrotron radiation.

longitudinally displaced by the amount $S_p\Delta p/p$ while traveling from the pickup to kicker undulators (without accounting the additional displacement due to betatron motion). The quantities D_P is the dispersion in the pickup and $D_P^{'} = \frac{dD_P}{ds}$.

As can be seen from Eq. (1) the corrective kick becomes nonlinear for large particle amplitudes and so the damping decrements depend on amplitude. This is accounted for in [4] by averaging kicks over betatron and synchrotron oscillations. Here we follow what was done in the latter reference but include an additional phase term in Eq. (1), ks_d, where s_d/c is the difference between between the arrival times measured at the kicker between the reference particle and its radiation emitted (at a retarded time) from the pickup. Including this additional term yields the damping decrements

$$\begin{aligned}\begin{bmatrix}\lambda_x(a_x,a_p)\\ \lambda_p(a_x,a_p)\end{bmatrix} &= \frac{L_p-|s_d|}{L_p}\cos(ks_d)\\ &\times\begin{bmatrix}\lambda_x 2J_o(a_p)J_1(a_x)/a_x\\ \lambda_p 2J_o(a_x)J_1(a_p)/a_p\end{bmatrix}, \qquad (3)\end{aligned}$$

where the left most factor has been introduced to account for the decrease in the kick amplitude due to the finite length of the undulator pulse, $L_p \approx N_{und}\lambda$, and N_{und} is the number of undulator periods. The parameters a_x and a_p are the amplitudes of longitudinal displacement from pickup to kicker, expressed in units of the wave phase, respectively due to betatron and synchrotron oscillations

$$\begin{aligned} a_x &= k\sqrt{\tilde{\varepsilon}S_x},\\ a_p &= k\left|S_p\right|(\Delta p/p), \qquad (4)\end{aligned}$$

where $\tilde{\varepsilon}$ is the particles Courant-Snyder invariant, $S_x = (\beta_p M_{51}^2 - 2\alpha_p M_{51}M_{52} + (1+\alpha_p^2)M_{52}^2))$, and α_p and β_p are the Courant-Snyder parameters in the pickup.

We can now write a system of coupled rate equations for $\tilde{\varepsilon}$ and $\Delta p/p$, where in addition to the effects of OSC we include a term to approximately describe both synchrotron damping and excitation

$$\begin{aligned}\frac{d\sqrt{\tilde{\varepsilon}}}{dt} &= \lambda_x(a_x,a_p)\frac{\sqrt{\tilde{\varepsilon}}}{\tau_s}\\ &+\frac{\lambda_{x,damp}}{\tau_s}(\sqrt{\tilde{\varepsilon}}-\sqrt{\varepsilon_o})\\ \frac{d\Delta p/p}{dt} &= \lambda_p(a_x,a_p)\frac{\Delta p/p}{\tau_s}\\ &+\frac{\lambda_{p,damp}}{\tau_s}(\Delta p/p-\sigma_p), \qquad (5)\end{aligned}$$

where τ_s is the revolution period of the accelerator, $\lambda_{x,damp}$, $\lambda_{p,damp}$ are the synchrotron cooling decrements in amplitude per turn, ε_0 is the equilibrium beam emittance and σ_p is equilibrium momentum spread, both without OSC.

The above equations exclude fluctuations. Thus, they describe an evolution of average amplitudes for an ensemble of particles having the same initial amplitudes. They do not imply, in the absence of OSC, that all particles are driven to have the values $\tilde{\varepsilon} = \varepsilon_0$ and $\Delta p/p = \sigma_p$, which is only statistically true for the whole beam. In reality particles make random walks about these values, reflecting the random process of photon emission. Table 2 summarizes the OSC chicane and beam parameters used in this contribution.

Table 2: OSC and Beam Parameters

parameter	value	unit
delay	2	mm
S_p	1.38	mm
S_x	2.24	μm
σ_p	1.1×10^{-4}	-
ε_o	9.1	nm
$\lambda_{p,damp}$	1.2×10^{-7}	-
$\lambda_{x,damp}$	7.2×10^{-8}	-

Momentarily neglecting the synchrotron radiation terms we look for the fixed points of Eq. (5). It is easily seen that such points occur only when both $J_0(a_x)$ and $J_0(a_p)$ are zero, or when $J_1(a_x)$ and $J_1(a_p)$ are zero. Computation of the eigenvalues of the Jacobian reveals that the J_0 fixed points correspond to saddle points while the J_1 fixed points are either stable or unstable nodes.

There are three particularly important fixed points $[0, \mu_{1,1}]$, $[\mu_{0,1}, \mu_{0,1}]$,and $[\mu_{1,1}, 0]$ where $\mu_{0,1} \approx 2.405$ is the first zero of J_o and $\mu_{1,1} \approx 3.832$ is the second zero for J_1. These three points roughly map out the cooling boundary and enclose most particles. When $\cos(ks_d) > 0$ all three points are unstable and all particles within this region are attracted to the stable node $[0, 0]$, a $J_1 = 0$ fixed point.

When $\cos(ks_d) < 0$ the stability of the nodes are switched while the saddle points remain unstable. Thus all particles within the cooling range are attracted to either $[0, \mu_{1,1}]$ or $[\mu_{1,1}, 0]$ depending on their initial coordinate. Therefore longitudinal timing errors correspond to only three modes of behavior, the two just described which are determined only by the sign of $\cos(ks_d)$. And a 3rd intermediate case with $\cos(ks_d) = 0$ where damping rates become zero.

The two left plots of Fig. 1 show the trajectories of the $[a_x, a_p]$ phase space when diffusion due to synchrotron radiation is neglected. The plots on the right in Fig. 1 include diffusion due to synchrotron radiation. They indicate the cooling boundary is expanded when $\cos(ks_d) > 0$ and when $\cos(ks_d) < 0$ particles are expelled out, as before, but to fixed points with smaller amplitudes with the exact location depending on the magnitude of $\cos(ks_d)$.

Method to Determine ks_d

Here we demonstrate a simple method for measuring ks_d, and so the timing error, by observing the light intensity of the pickup and kicker downstream of the cooling insertion at the first harmonic. Note that the bunch length is much longer than the wavelength and so the kicker and pickup pulses of different particles add incoherently. If we consider a bunch with N electrons the total loss of energy for the nth electron in the bunch is given as

$$\Delta U(\delta p/p, \tilde{\varepsilon})_n = -U_o \xi_0 \Big[1 + \sin(ks_n + ks_d) \frac{L_p - |s_d|}{L_p}\Big], \quad (6)$$

with $s_n \equiv S_p(\Delta p/p)_n + \sqrt{S_x \varepsilon_n}$. Furthermore if the beam has am rms relative momentum spread σ_p and transverse emittance ε_0 (assumed to be independently Gaussian distributed) then the rms longitudinal displacement is $\sigma_s^2 = (S_p \sigma_p)^2 + \varepsilon_0 S_x$. Integrating over all particles yields the total loss in energy of the beam to be

$$\Delta U_{beam} = -NU_o \xi_o \Big[1 + \frac{L_p - |s_d|}{L_p} \sin(ks_d) \times \exp(-k^2 \sigma_s^2/2)\Big] \quad (7)$$

The energy of the downstream radiation is $-U_{beam}$. Figure 2 shows the radiation energy as a function of timing error recorded downstream of the kicker undulator. A InAs photovoltaic detector can be used for the measurement. A single-staged thermally electric cooled detector has a NEP rating of 6 pw/Hz$^{1/2}$. For an electron bunch with 10^4 particles and rms bunch length of $\sigma_z = 20$ cm this gives a signal to noise ratio of 4 when pickup and kicker signals interfere deconstructively at maximum amplitude. To observe a pattern like that shown in Fig. 2 the cooling and signal detection must be on for only a short time, on the order of ms, before the beam becomes modified. This can be accomplished with two mechanical shutters.

In the above discussion we neglected non-linear contributions to longitudinal motion which mainly come from betatron motion. These nonlinearities are compensated by sextupoles for the center of the distribution. However the compensation is not perfect for the tails [5]. Taking into account that tails do not produce significant contributions to the interference we neglected their effect and considered that the sample lengthening results in a Gaussian distribution in the kicker undulator.

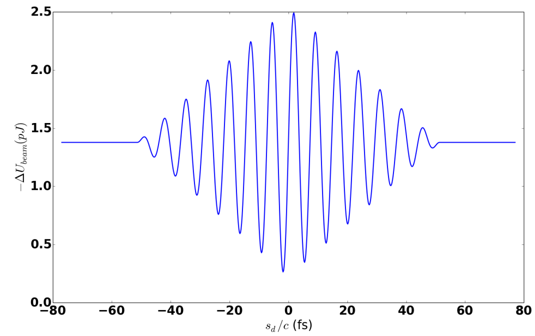

Figure 2: Downstream light energy from the pickup and kicker undulators with N=10^4 electrons as function in the chicane delay achieved with changes of current in the chicane dipoles.

CONCLUSIONS

OSC requires timing of the arrival of the pickup signal and its particle in the kicker to a precision of less than 1 fs. Any attempt to directly measure this arrival time is not practical since, in addition to the extreme precision needed, the signals from single electrons arrive in the form of a bunch with a length on the order of hundreds of picoseconds. The method that has been proposed here does not require a direct measurement of the arrival time but rather takes advantage of the fact that in the cooling insertion particles receive kicks not just on their longitudinal displacement relative to a reference particle, but also their phase relative to the pickup signal. Thus timing information is derived from the light intensity variation with delay of electron bunch in the OSC chicane. The variation originates from the interference of radiations coming from the pickup and kicker undulators. The intereference between radiation produced by two undulators has been observed before in an optical klystron setup [6] and used as a beam diagnostics [7].

REFERENCES

[1] A. A. Mikhailichkenko and M.S. Zolotorev, *Phys. Rev. Lett.* **71** (25), p. 4146, 1993.

[2] M. S. Zolotorev and A. A. Zholents, *Phys. Rev.* E **50** (4), p. 3087, 1994.

[3] M. B. Andorf *et al.*, in *Proc. IPAC'16*, Busan, Korea, p.3028, 2016.

[4] V. A. Lebedev, "Optical Stochastic Cooling," *ICFA Beam Dyn. Newslett.* **65** 100, 2014.

[5] V. Lebedev *et al.*, presented at COOL'15, in press, 2015.

[6] P. Elleaume, *J. Phys. Coll.* **44** (C1), 333, 1983.

[7] R. Bakker *et al.*, in *Proc. EPAC1996*, Barcelona, Spain, p. 667, 1996.