OSC simulation update

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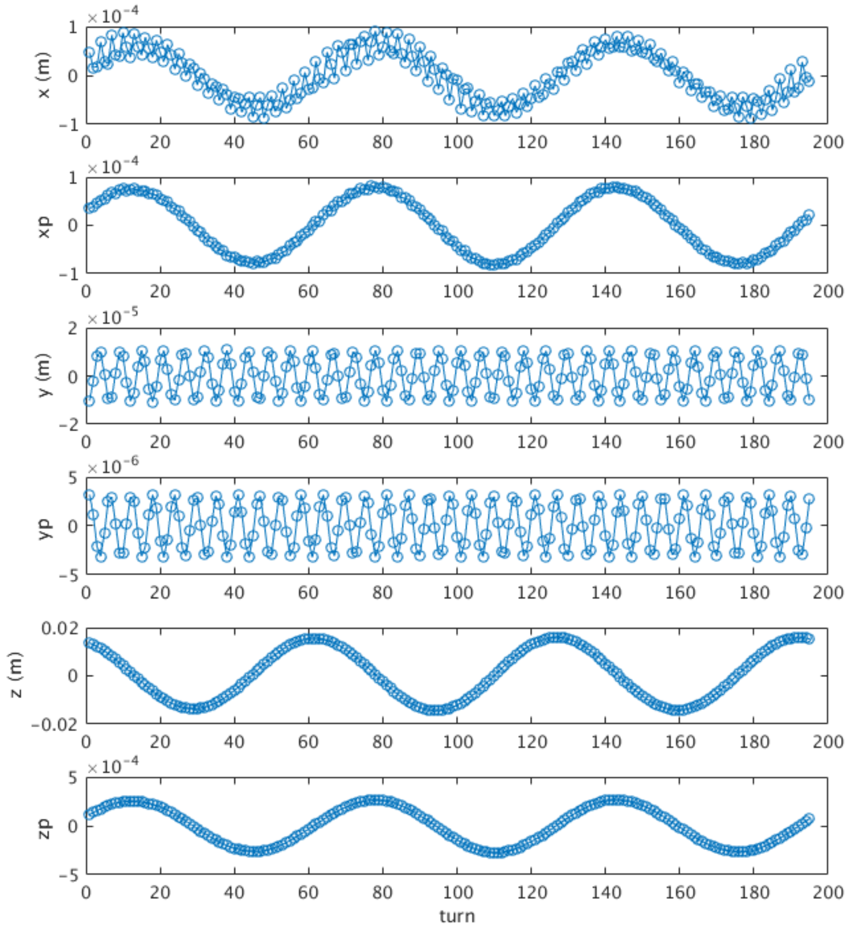
1. Particle longitudinal mixing
2. Use Gaussian distribution function to generate incoherent kicks

1/23/2018

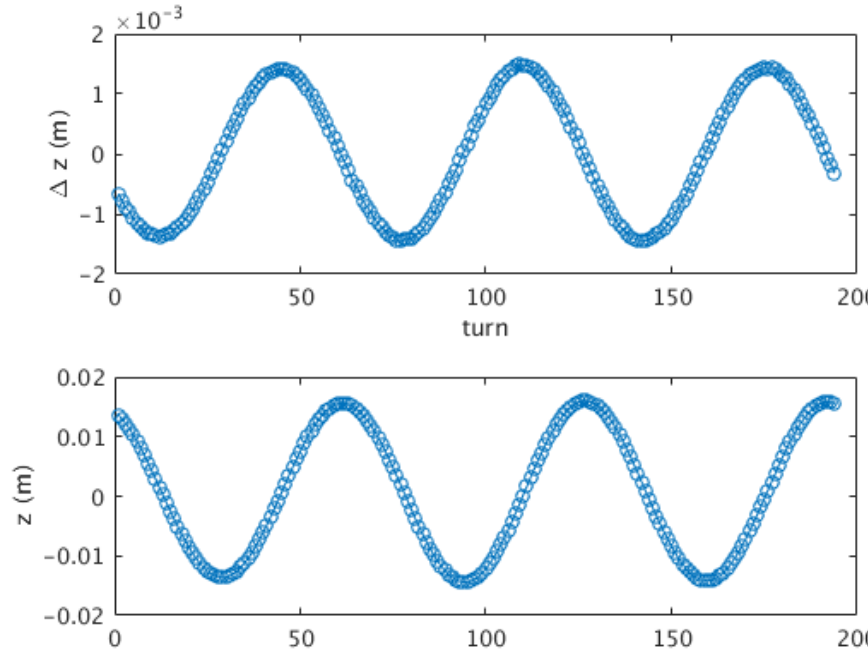
Particles mixing

Track 1E4 particles for 200 turns with incoherent kicks added
Record the 6D coordinates

coordinates vs turn for 1 particle



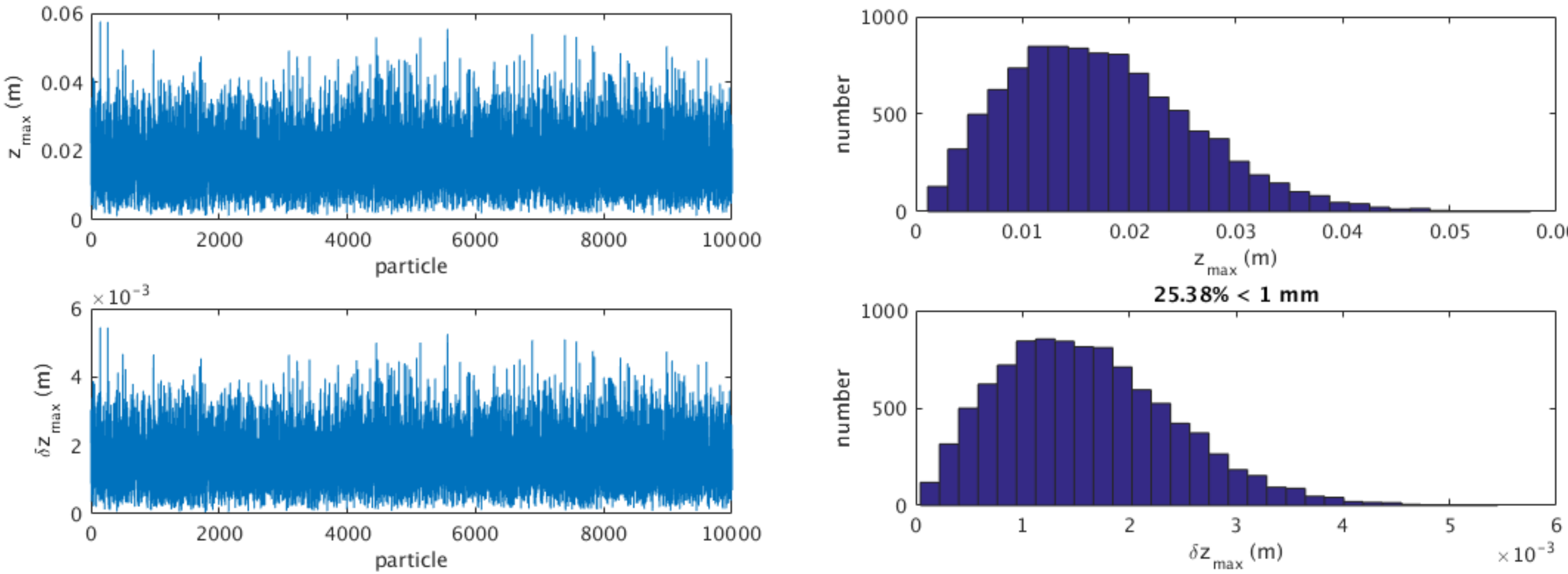
$\delta z = z_{k+1} - z_k$, k=1...200 turn



- z oscillate at synchrotron tune ~6 kHz (~66 turns).
- δz is the largest at the bunch center (z=0) and zero at bunch tails.
- The larger the oscillation amplitude (z_{max}) is, the larger δz will b.

Particles mixing

z_{max} and δz_{max} of 1E4 particles



With 1E4 particles, at 1 turn, 25.4% particles' z movement is < 1 mm.

The particle longitudinal mixing is not very good within z_slice = 1 mm.
However, it is perfect mixing within 1 μm.

Gaussian distribution for incoherent kicks

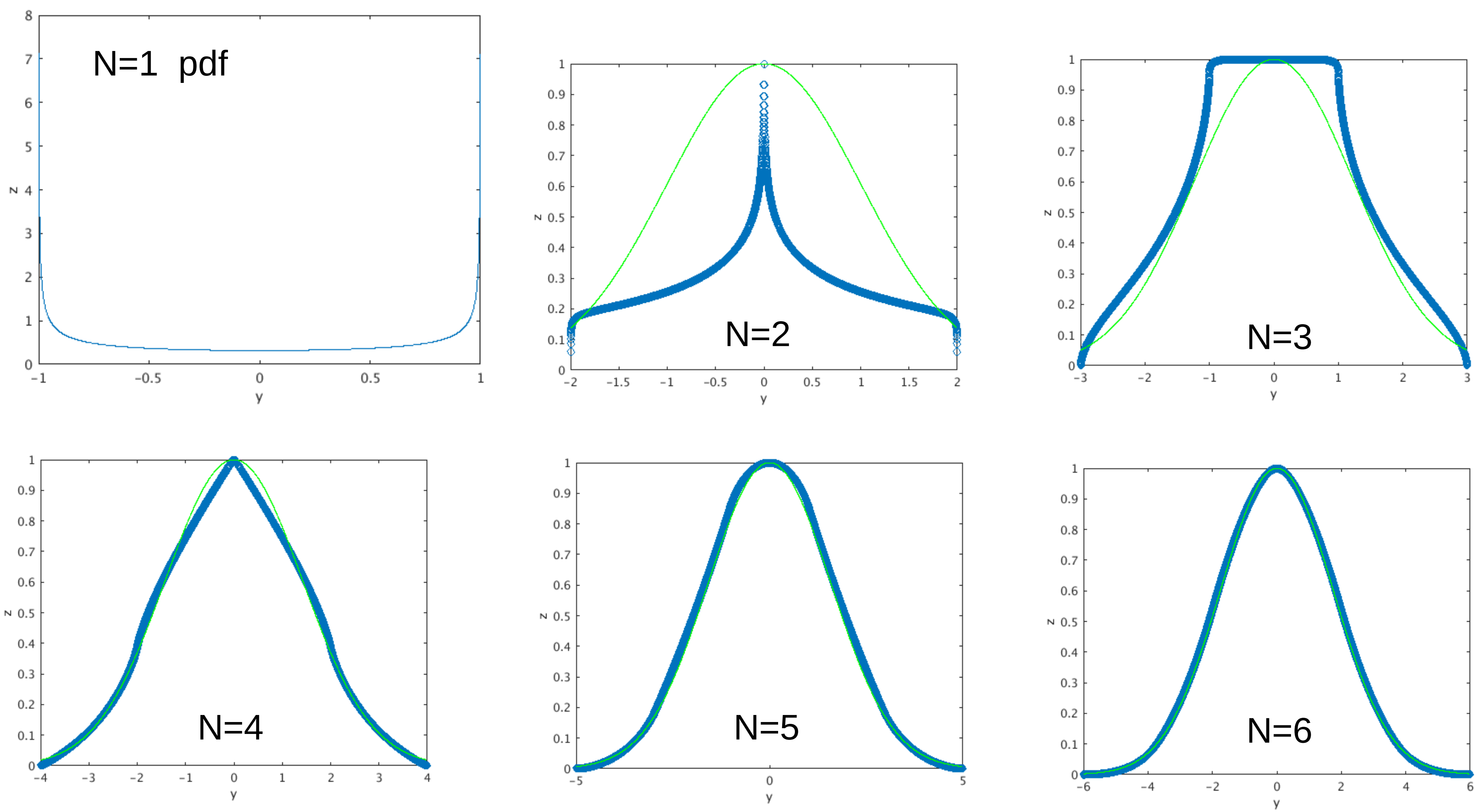
$$\delta_{ic} = \delta_i + G\sin(\Delta\phi_i) + G\sum_{k\neq i}^{N_s}\sin(\Delta\phi_i + \psi_{ik}) .$$

x: random numbers within range [0, 2π] (or, [-2π,0], [-N_u2π,0]

y=sin(x): probability distribution function f(y)=1/(πsqrt(1-y^2)), y=[-1,1]

z=N Sum (sin(x)): N convolutions of f(y)

When N> 6, the pdf of z is Gaussian function with a sigma proportional to sqrt(N).



Gaussian distribution for incoherent kicks

Gaussian distributed bunch: $f(z)=1/\sigma/\mathrm{sqrt}(2\pi)*\exp(-(z-\mu)^2/2/\sigma^2)$

$\mu=0$, $\sigma_0=10$ mm, $\lambda=1$ µm

The probability the particle when z is in $[-N_u\lambda/2, N_u\lambda/2]$ and the largest N_{slice} with 1E9 particles in a bunch are

$N_u=1$, $p=3.989423e-05$, $N_{slice}=p*N=39894$;
$N_u=4$, $p=1.595769e-04$, $N_{slice}=p*N=159577$;

Found the sigmas of the incoherent kicks for these N_{slice}:

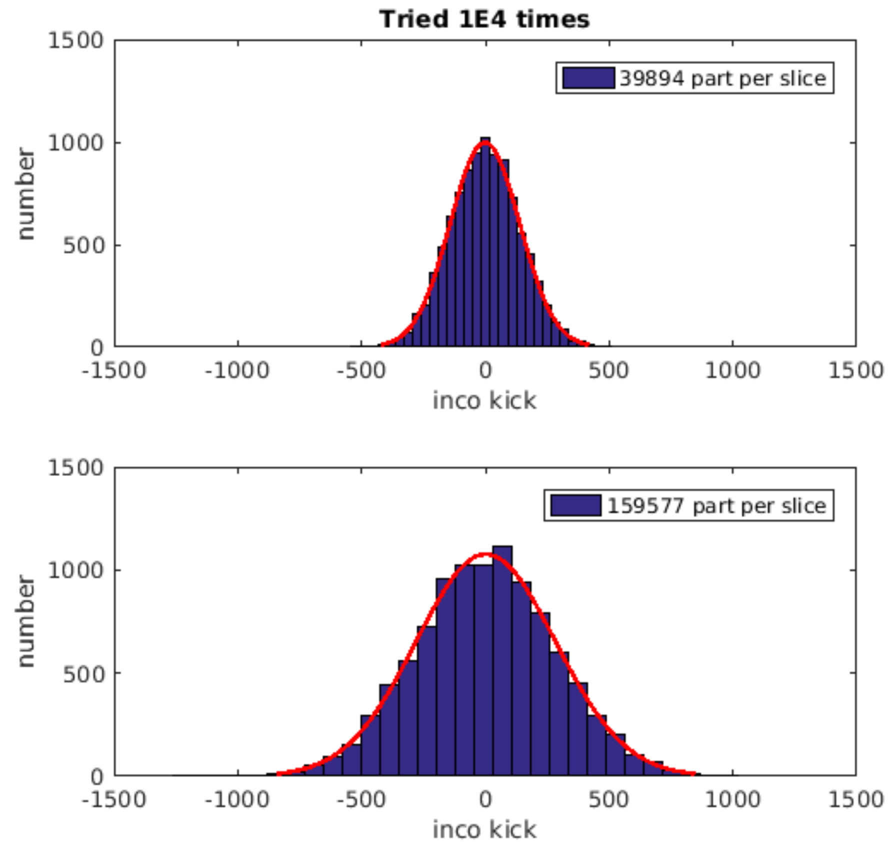
$N_{slice}=39894$, $\sigma_{inco}= 140.4$
$N_{slice}=159577$, $\sigma_{inco}= 280.8$

Gereral case:

$\sigma_{inco}(z)=\sigma_{inco\_max}*\mathrm{sqrt}(N_z/N_{slice\_max})$
$=140.4*\mathrm{sqrt}(N_z/39894)$

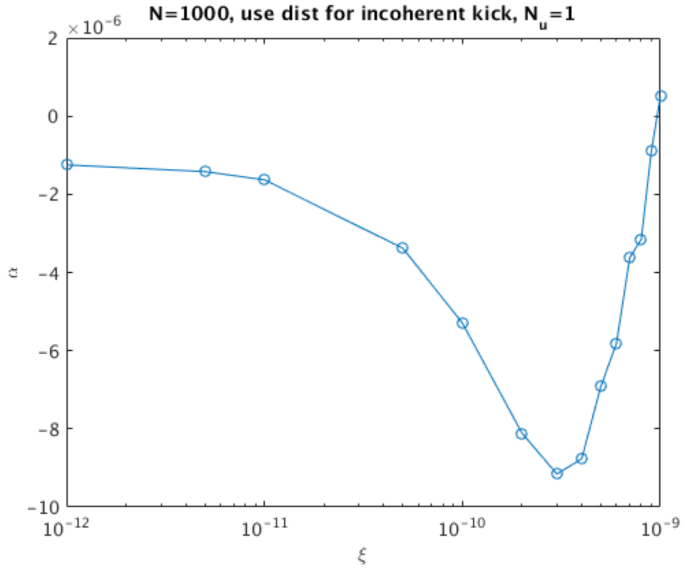
$N_z=f(z)/f(z=0)*N_{slice\_max}*(\sigma_0/\sigma_z)$
$=(\sigma_0/\sigma_z)*\exp(-z^2/2/\sigma_z^2)*N_{slice\_max}$

$\sigma_{inco}(z)=\sigma_{inco\_max}*\exp(-z^2/4/\sigma_z^2)*\mathrm{sqrt}(\sigma_0/\sigma_z)$
$=140.4*\exp(-z^2/4/\sigma_z^2)*\mathrm{sqrt}(\sigma_0/\sigma_z)$

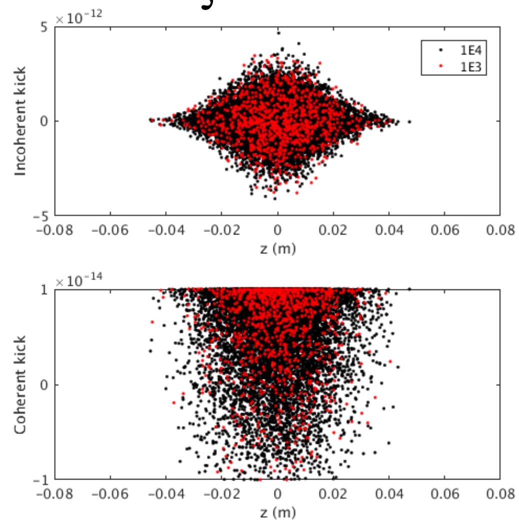


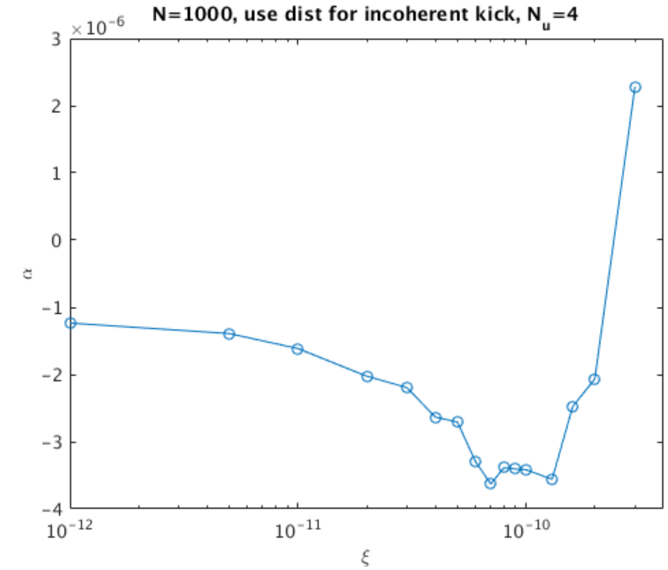
Gaussian distribution for incoherent kicks

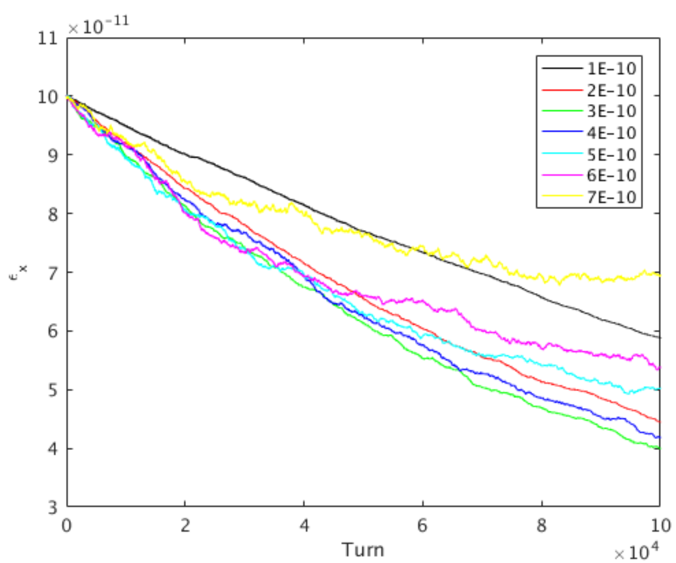
Track 1E3 particles for 1E5 turns
Each particle got a coherent and incoheren kick



ξ=1E-14







α=9.16E-6, N_u=1 @ ξ=3E-10
α=3.63E-6, N_u=4 @ ξ=1.3E-10

> α_x=6.0E-7 (ring)