Linearized Helical Undulator Focusing

- Known that planar undulator provides transverse focusing in non-oscillatory plane

- How does this translate to the helical case?

Magnetic Field

$$B_r^1 = B_0\left\{1+\frac{3(kr)^2}{8}+\frac{5(kr)^4}{192}+\ldots\right\}\cos(kz-\phi),$$

$$B_\phi^1 = B_0\left\{1+\frac{(kr)^2}{8}+\frac{(kr)^4}{192}+\ldots\right\}\sin(kz-\phi),$$

$$B_z^1 = -B_0\left\{kr+\frac{(kr)^3}{8}+\ldots\right\}\sin(kz-\phi).$$

S.H. Kim, “Magnetic Field Analysis of Helical Undulators”

Magnetic Field (cont.)

- $B_x = B_0 [\cos(kz) + k^2/8 (r^2 \cos(kz) + 2x^2 \cos(kz) + 2xy \sin(kz))]$

- $B_y = B_0 [\sin(kz) + k^2/8 (r^2 \sin(kz) + 2y^2 \sin(kz) + 2xy \cos(kz))]$

- $B_z = B_0 [(k + k^3 r^2/8)*(y \cos(kz) - x \sin(kz))]$

0^{th} Order Solution

- $x'' = -e/(\bar{v}^2 \gamma m)\,(v_y B_z - v_z B_y)$
- $y'' = -e/(\bar{v}^2 \gamma m)\,(v_z B_x - v_x B_z)$
- $z'' = -e/(\bar{v}^2 \gamma m)\,(v_x B_y - v_y B_x)$

0^{th} Order Solution

- $x = -e B_0 / (\bar{v} \gamma m k^2) \sin(kz) + \Delta x'_0 z + \Delta x_0$
- $y = e B_0 / (\bar{v} \gamma m k^2) \cos(kz) + \Delta y'_0 z + \Delta y_0$
- $z = \bar{v} z$

1st Order Fields

- $B_x = B_0 \cos(kz)$
- $B_y = B_0 \sin(kz)$
- $B_z = B_0 k (y \cos(kz) - x \sin(kz))$

1^{st} Order Forces

- $x'' = -e/(\bar{v}^2\gamma m)\ \{[-eB_0/(\gamma mk)\sin(kz) + \Delta y'_0\,\bar{v}]\ *$

  $[eB_0^2/(\bar{v}\gamma mk) - B_0k(\Delta x'_0\,z + \Delta x_0)\sin(kz) + B_0k(\Delta y'_0\,z + \Delta y_0)\cos(kz)]$

  $-\ \bar{v}B_0\sin(kz)\}$

- $y'' = -e/(\bar{v}^2\gamma m)\ \{\bar{v}B_0\cos(kz)\ -$

  $[-eB_0/(\gamma mk)\cos(kz) + \Delta x'_0\,\bar{v}]\ *$

  $[eB_0^2/(\bar{v}\gamma mk) - B_0k(\Delta x'_0\,z + \Delta x_0)\sin(kz) + B_0k(\Delta y'_0\,z + \Delta y_0)\cos(kz)]\}$

1^{st} Order Forces (Averaged over One Period)

- $x'' = -e^2B_0^2/(\bar{v}^2\gamma^2m^2k)\,\Delta y'$

  $- e^2B_0^2/(2\bar{v}^2\gamma^2m^2)\,(\Delta x' z + \Delta x_0)$

- $y'' = e^2B_0^2/(\bar{v}^2\gamma^2m^2k)\,\Delta x'$

  $- e^2B_0^2/(2\bar{v}^2\gamma^2m^2)\,(\Delta y' z + \Delta y_0)$

- I make the assumption that z changes slowly relative to sin(kz)/k
- $<(\Delta y' z + \Delta y_0)>$ is Δy at the center of the undulator

Conclusions

- Focusing force exists, and is equal to that of the planar undulator in each plane:

$$\frac{dp_y}{dz} = k_1 \left(y + \frac{2}{3}\, k_z^2\, y^3 \right)$$

$$k_1 = \frac{-1}{2} \left(\frac{e\, B_{\max}}{P_0\, (1 + p_z)} \right)^2$$

D. Sagan, BMAD Manual.

- Additional forces act to couple x and y motion

- Can continue process to get higher-order terms...

Debugging of BMAD Code

- Vardan had found that the peak frequency found by BMAD does not always agree with what we would expect from analytic formula

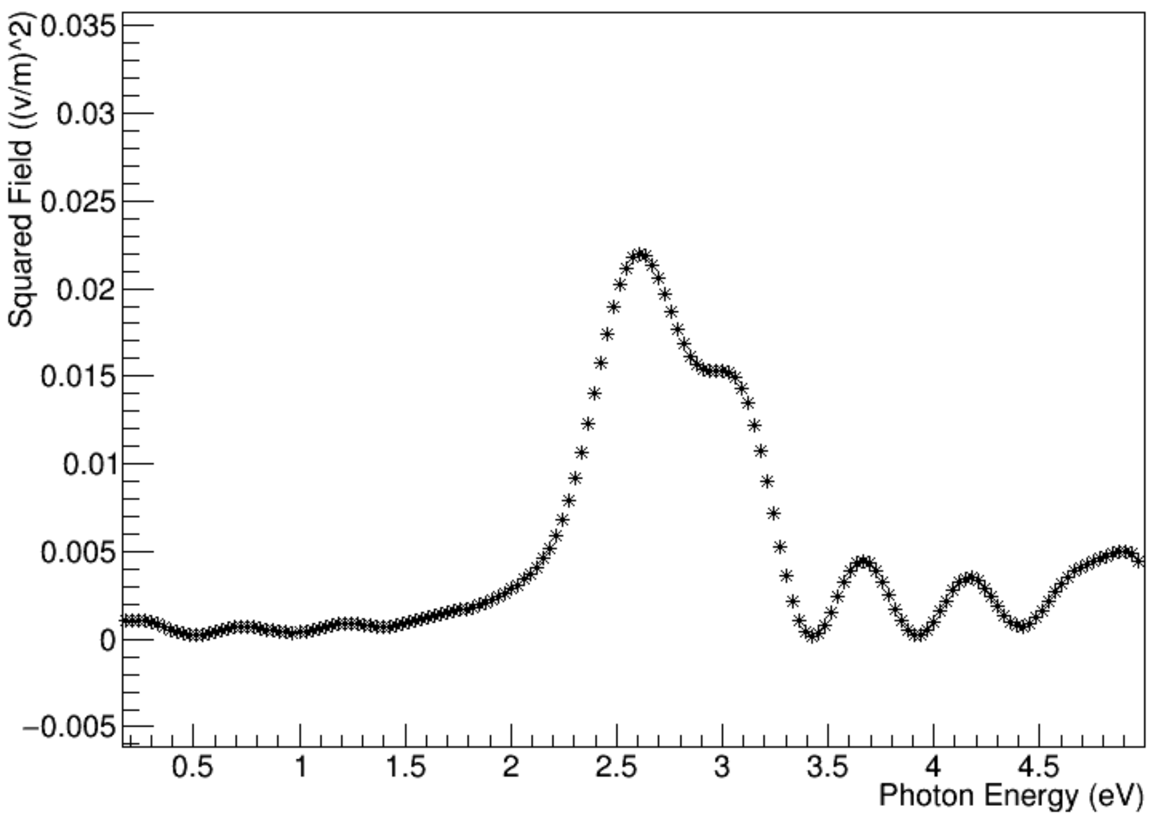
  $$\omega = (2\,\omega_u\,\gamma^2)/(1 + K^2/2 + (\gamma\theta)^2)$$

- Why?

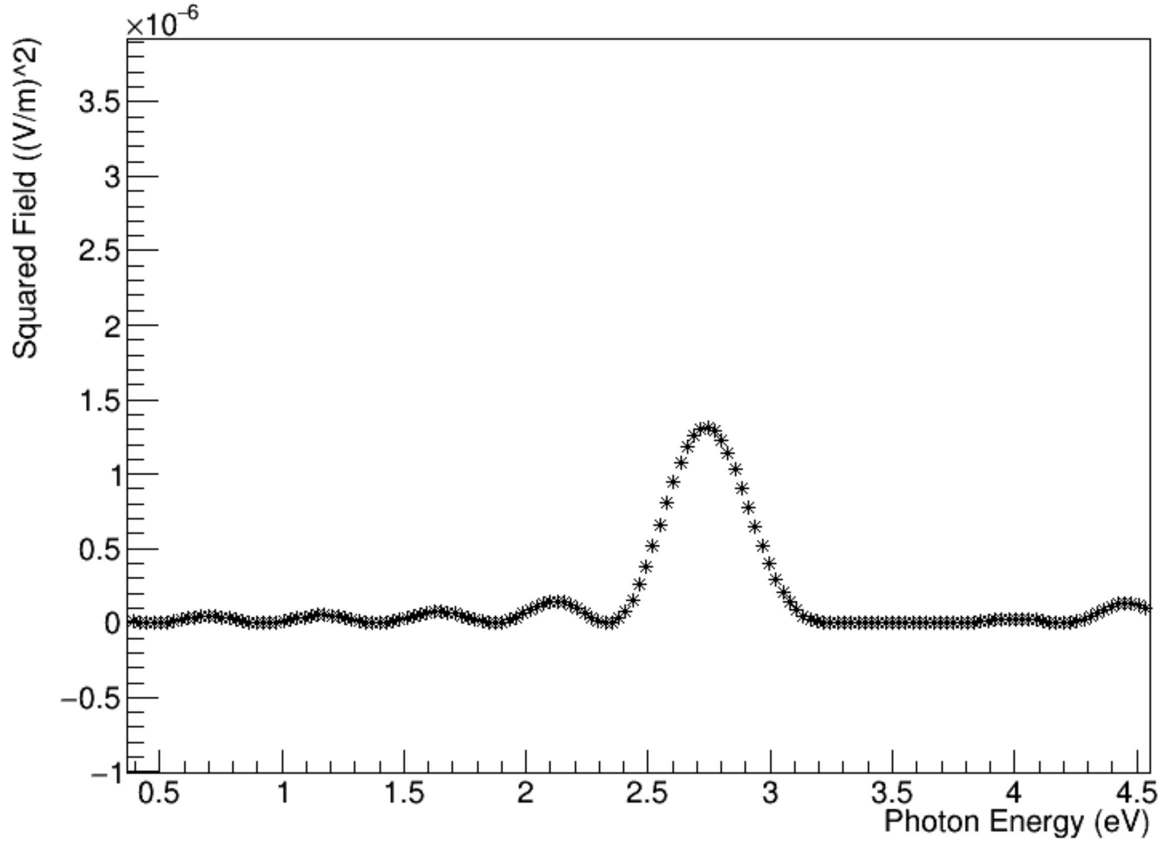
Finite Undulator Length

- 2m-long undulator ~6m from lens – significant change in angle between initial and final electron position

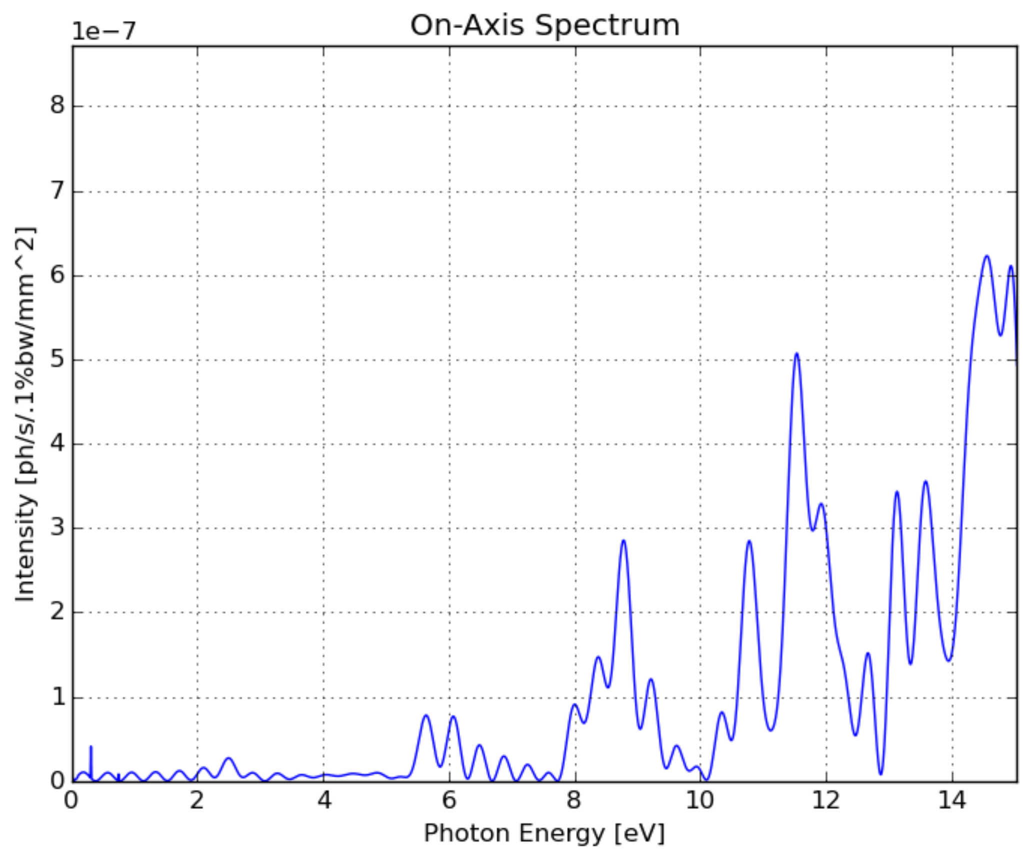
Angle 7.8mm / 6m



Angle 78cm / 600m



SRW 6 meters



On-Axis Spectrum
1e−7
Intensity [ph/s/.1%bw/mm^2]
0
1
2
3
4
5
6
7
8
Photon Energy [eV]
0
2
4
6
8
10
12
14

SRW 600 meters