Helical Undulator Magnetic Field in BMAD

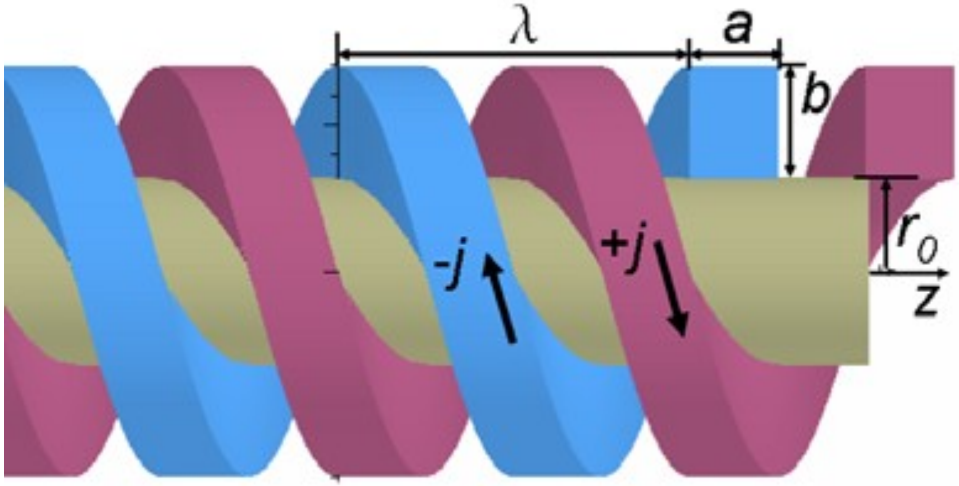
S.H. Kim, Magnetic Field Analysis of Helical Undulators, ANL/APS/LS-331

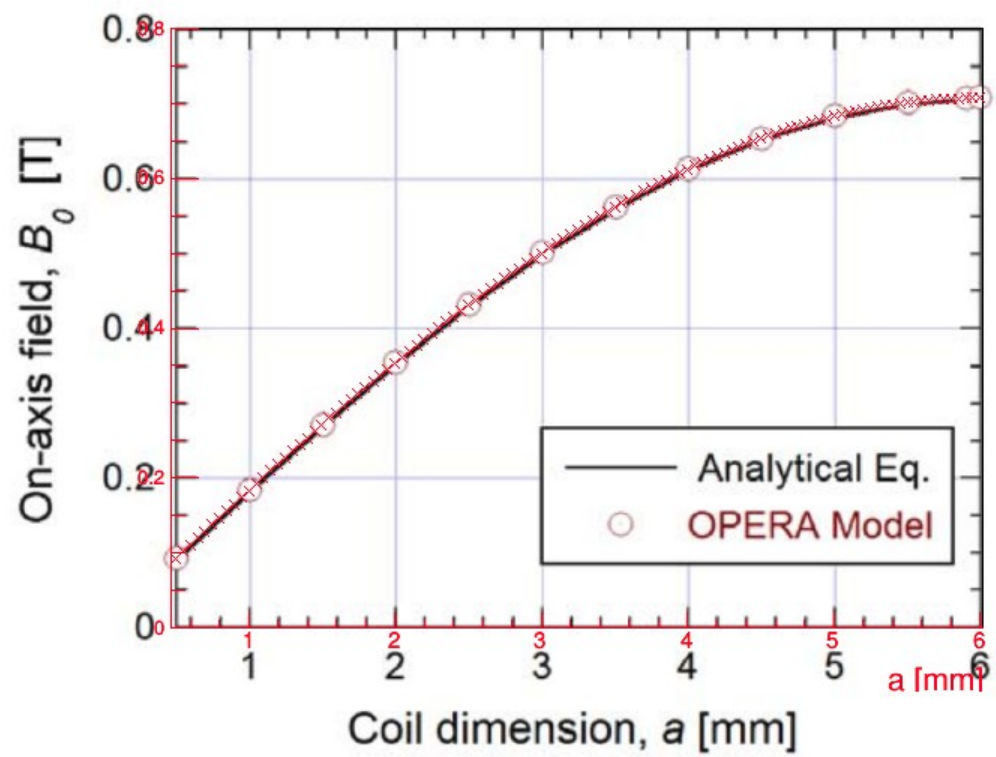
$$\mathbf{B}_< = \sum_{n=1}^{\infty} B_0^n \cdot \sin(\frac{n\pi}{2})\{\hat{r}[I_{n-1}(nkr)+I_{n+1}(nkr)]\cos n(kz-\phi) + \hat{\phi}\left[\frac{2}{kr}I_n(nkr)\right]\sin n(kz-\phi) + \hat{z}[-2I_n(nkr)]\sin n(kz-\phi)\},$$

n=1 (the only non-vanishing terms on the z-axis)

$$B_0 = \frac{2\mu_0 j\lambda}{\pi}\sin(k\frac{a}{2})\int_{r_0}^{r_0+b}\{krK_0(kr)+K_1(kr)\}\frac{dr}{\lambda}$$

λ = 12 mm
r_0 = 3.15 mm
b = 3.84 mm
a=0.5÷5.99 mm
j = 1 kA/mm^2





D. Segan, The Bmad Reference Manual.

$$
\begin{aligned}
B_\rho &= \mathrm{Re}\left[\sum_{n=-N/2}^{N/2-1} \frac{1}{2}\, e^{i\,k_n\,z}\, \cos(m\,\theta - \theta_{0m})\, b_m(n) \left[I_{m-1}(k_n\,\rho) + I_{m+1}(k_n\,\rho)\right]\right] \\
B_\theta &= \mathrm{Re}\left[\sum_{n=-N/2}^{N/2-1} \frac{-1}{2}\, e^{i\,k_n\,z}\, \sin(m\,\theta - \theta_{0m})\, b_m(n) \left[I_{m-1}(k_n\,\rho) - I_{m+1}(k_n\,\rho)\right]\right] \qquad (14.43) \\
B_z &= \mathrm{Re}\left[\sum_{n=-N/2}^{N/2-1} i\, e^{i\,k_n\,z}\, \cos(m\,\theta - \theta_{0m})\, b_m(n)\, I_m(k_n\,\rho)\right]
\end{aligned}
$$

Field 1	*Field 2*
$\theta_0 = 0$	$\theta_0 = \pi/2$
$b_m = B0$	$b_m = -i\ B0$

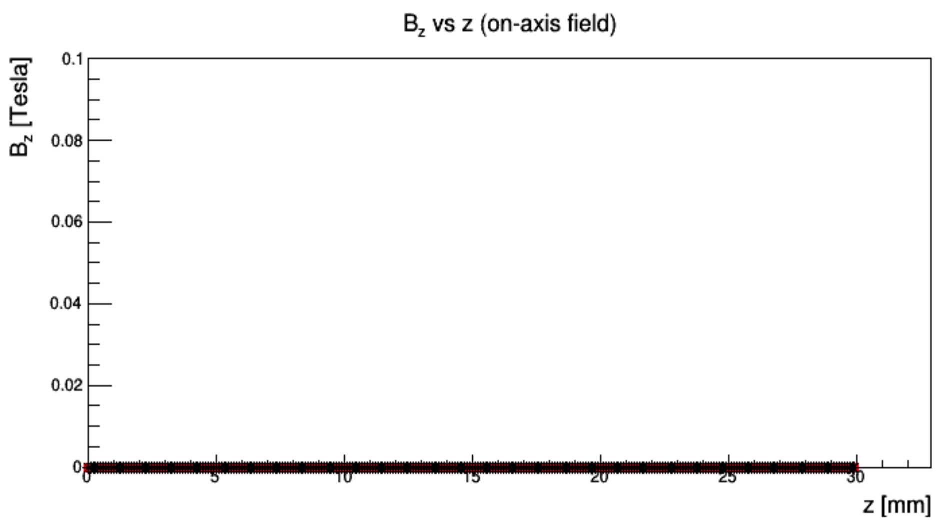
$$
\begin{aligned}
\mathbf{B}_< = \sum_{n=1}^{\infty} B_0^n \cdot \sin\left(\frac{n\pi}{2}\right)\Big\{\hat{r}\left[I_{n-1}(nkr) + I_{n+1}(nkr)\right]\cos n(kz-\phi) \\
+\hat{\phi}\left[\frac{2}{kr} I_n(nkr)\right]\sin n(kz-\phi) + \hat{z}\left[-2I_n(nkr)\right]\sin n(kz-\phi)\Big\},
\end{aligned}
$$

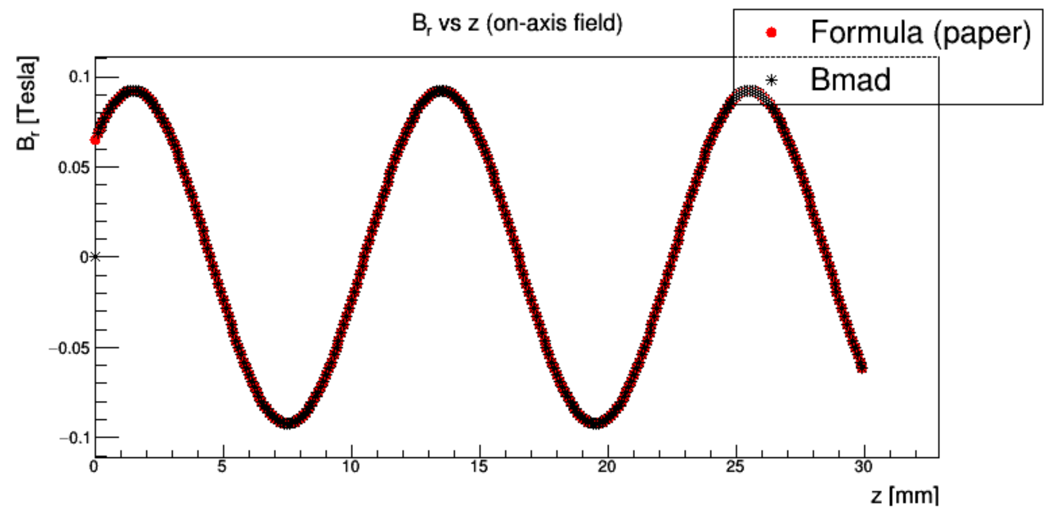
Lattice file:

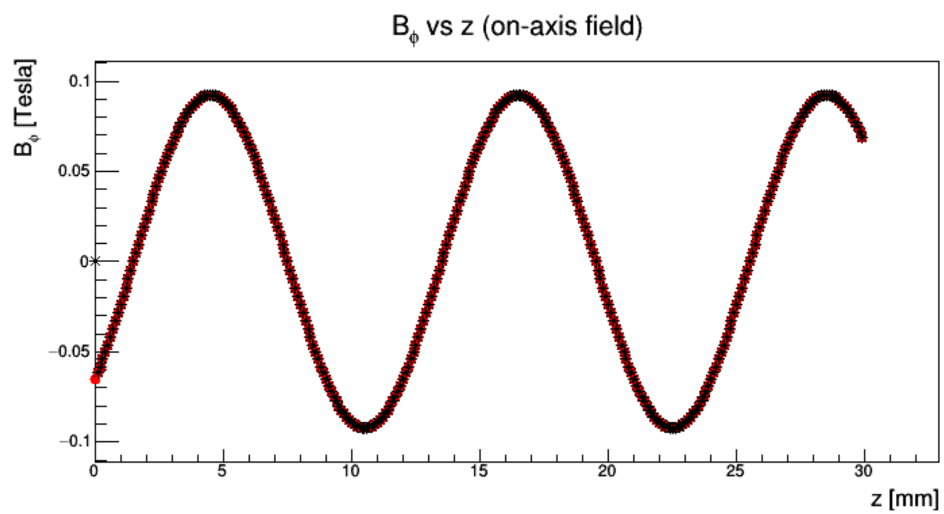
```
pillbox_cav0: em_field, tracking_method = runge_kutta, mat6_calc_method = tracking, superimpose,
field_calc = fieldmap, offset = 0.6, aperture = 0.08826348295016159, L = 1.2,

 cylindrical_map = {
   m = 1,
   theta0_azimuth = 0.0,
   harmonic = 0,
   dz = 0.012,
   ele_anchor_pt = center,
   field_scale = 2,
   e_coef_re = (0.0),
   e_coef_im = (0.0),
   b_coef_re = (0.0923295),
   b_coef_im = (0.0) }

pillbox_cav1: em_field, tracking_method = runge_kutta, mat6_calc_method = tracking, superimpose,
field_calc = fieldmap, offset = 0.6, aperture = 0.08826348295016159, L = 1.2,

 cylindrical_map = {
   m = 1,
   theta0_azimuth = 1.5707963267948965,
   harmonic = 0,
   dz = 0.012,
   ele_anchor_pt = center,
   field_scale = 2,
   e_coef_re = (0.0),
   e_coef_im = (0.0),
   b_coef_im = (0.0923295),
   b_coef_re = (0.0) }
```

Control plots

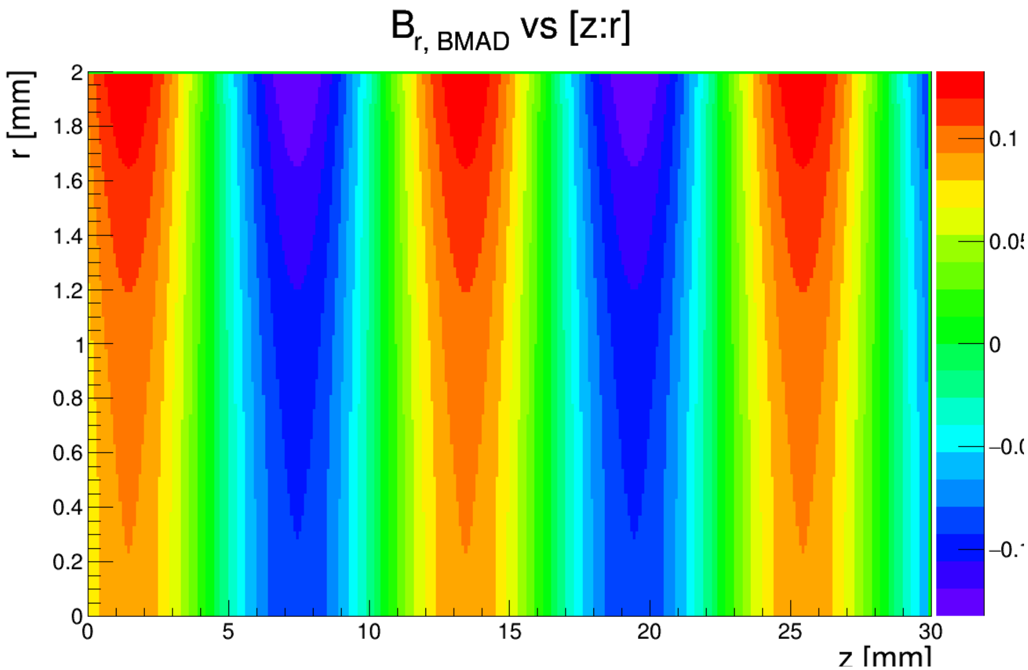
B field on z-axis, $(x \to 0, y \to 0)$

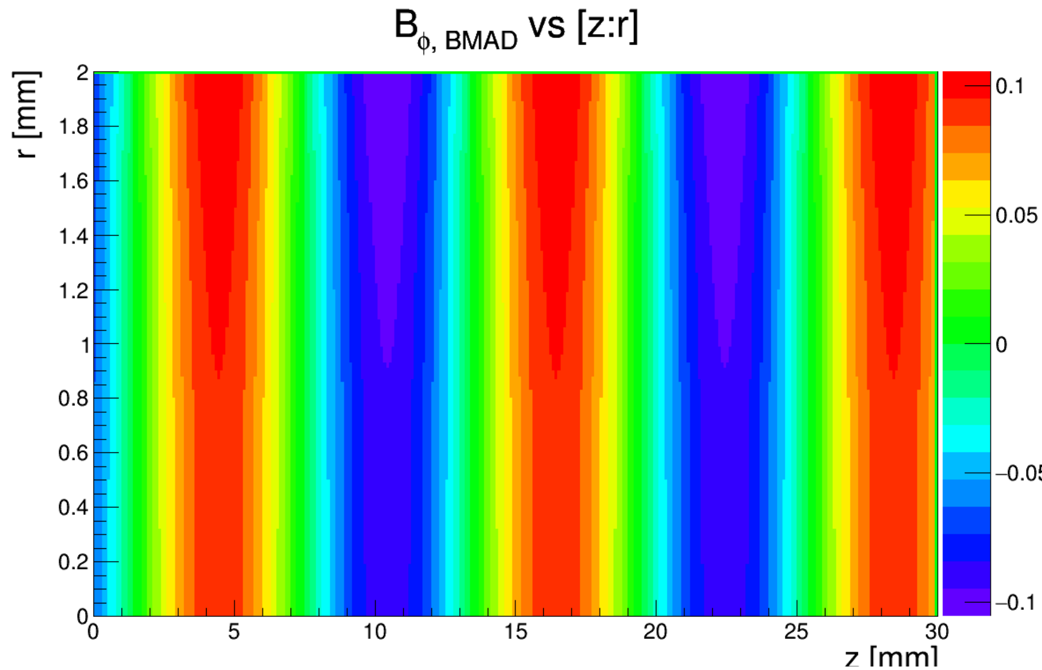


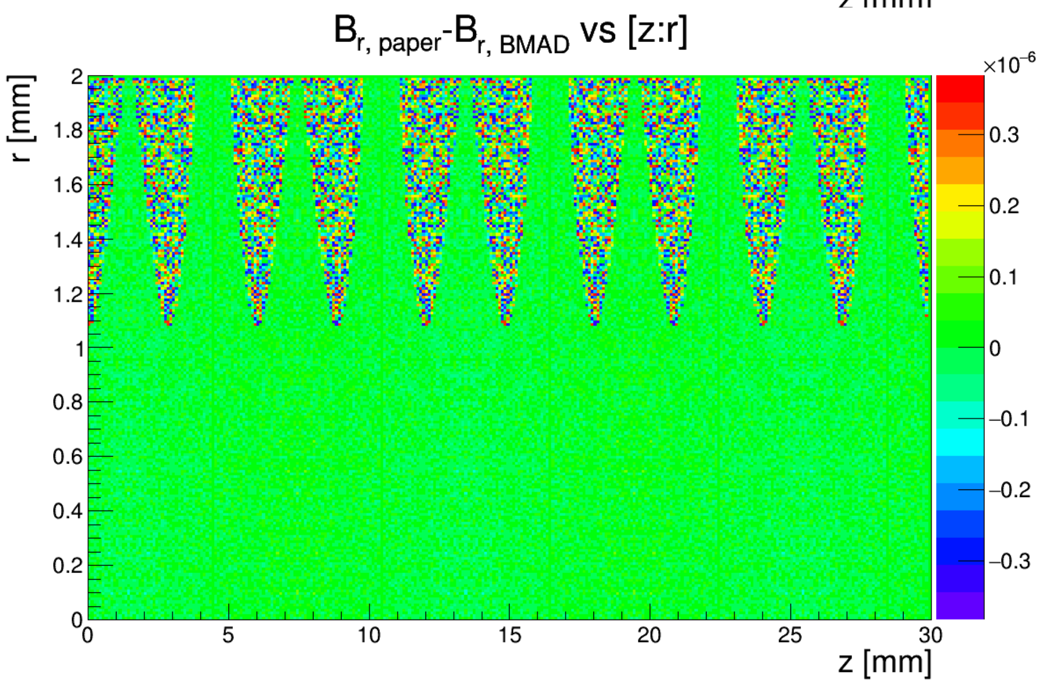


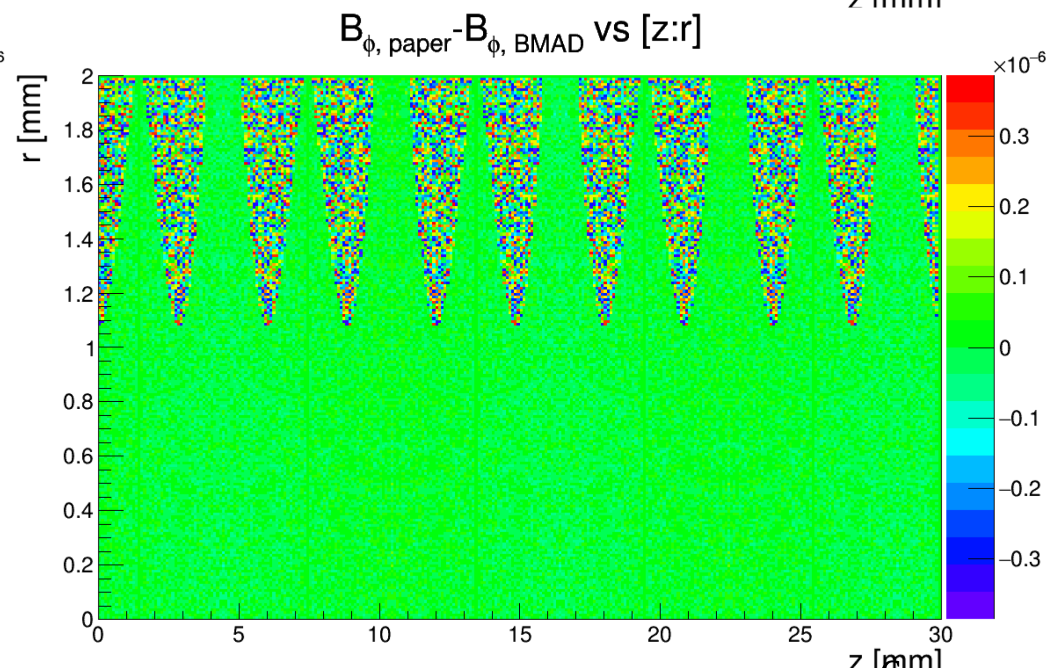


Control plots

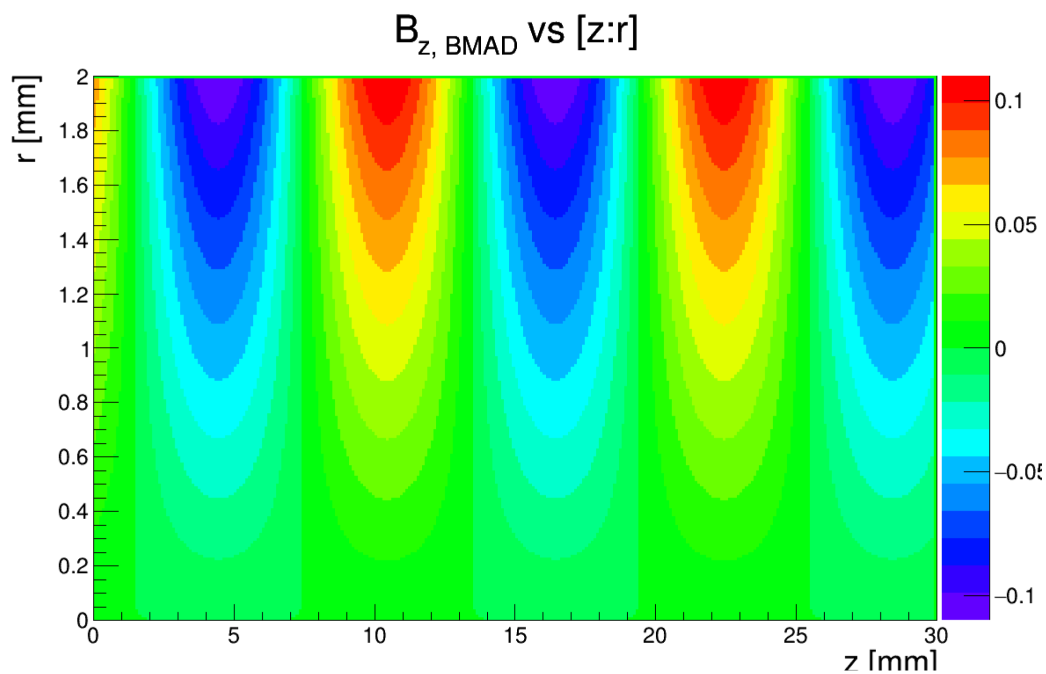


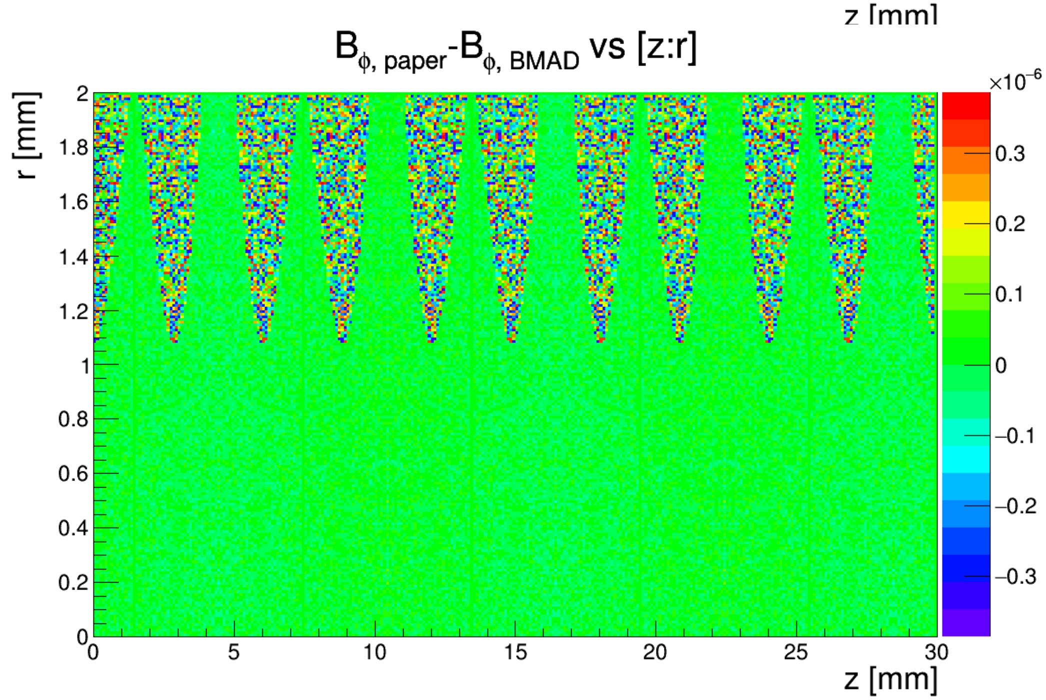






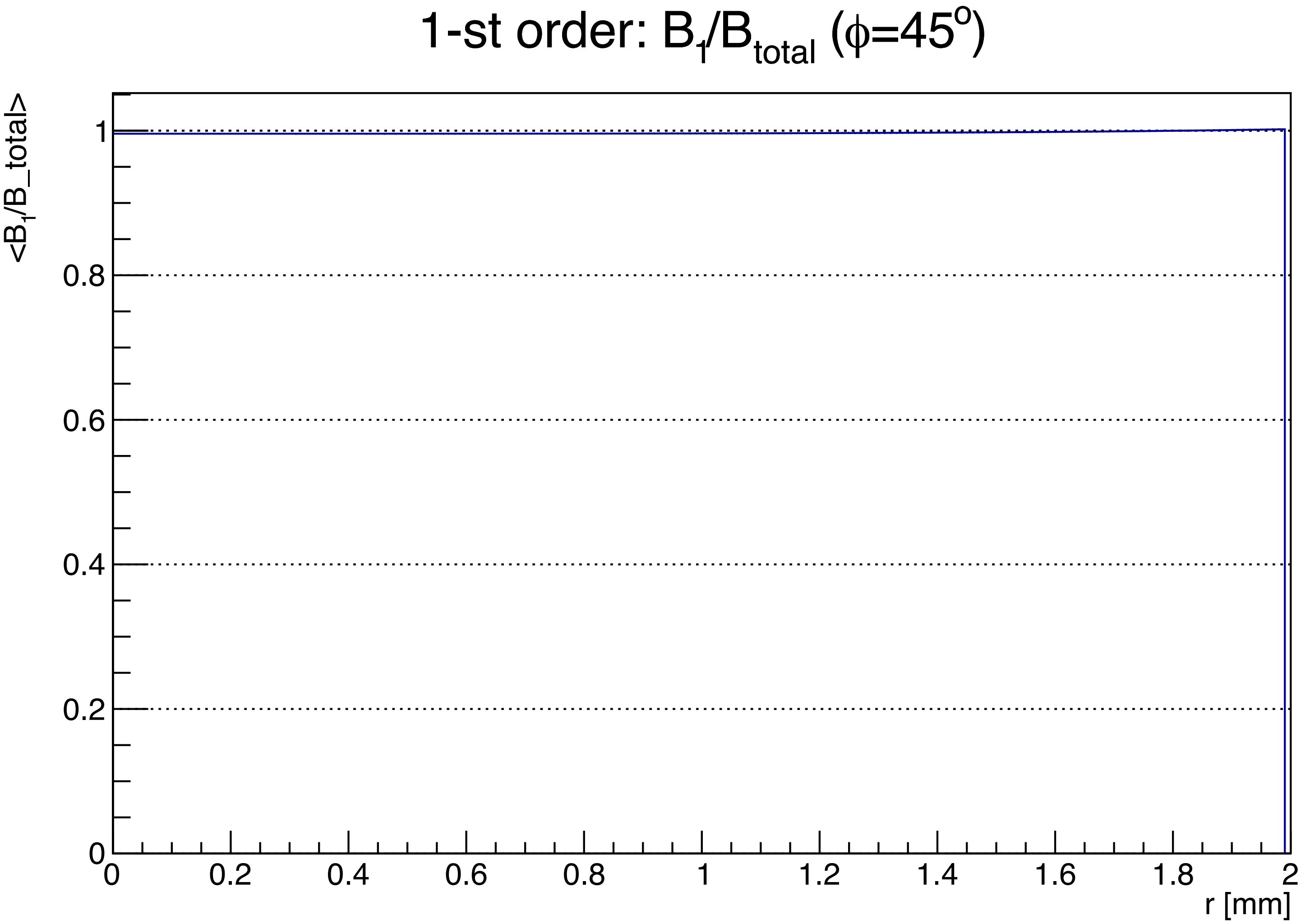
Control plots

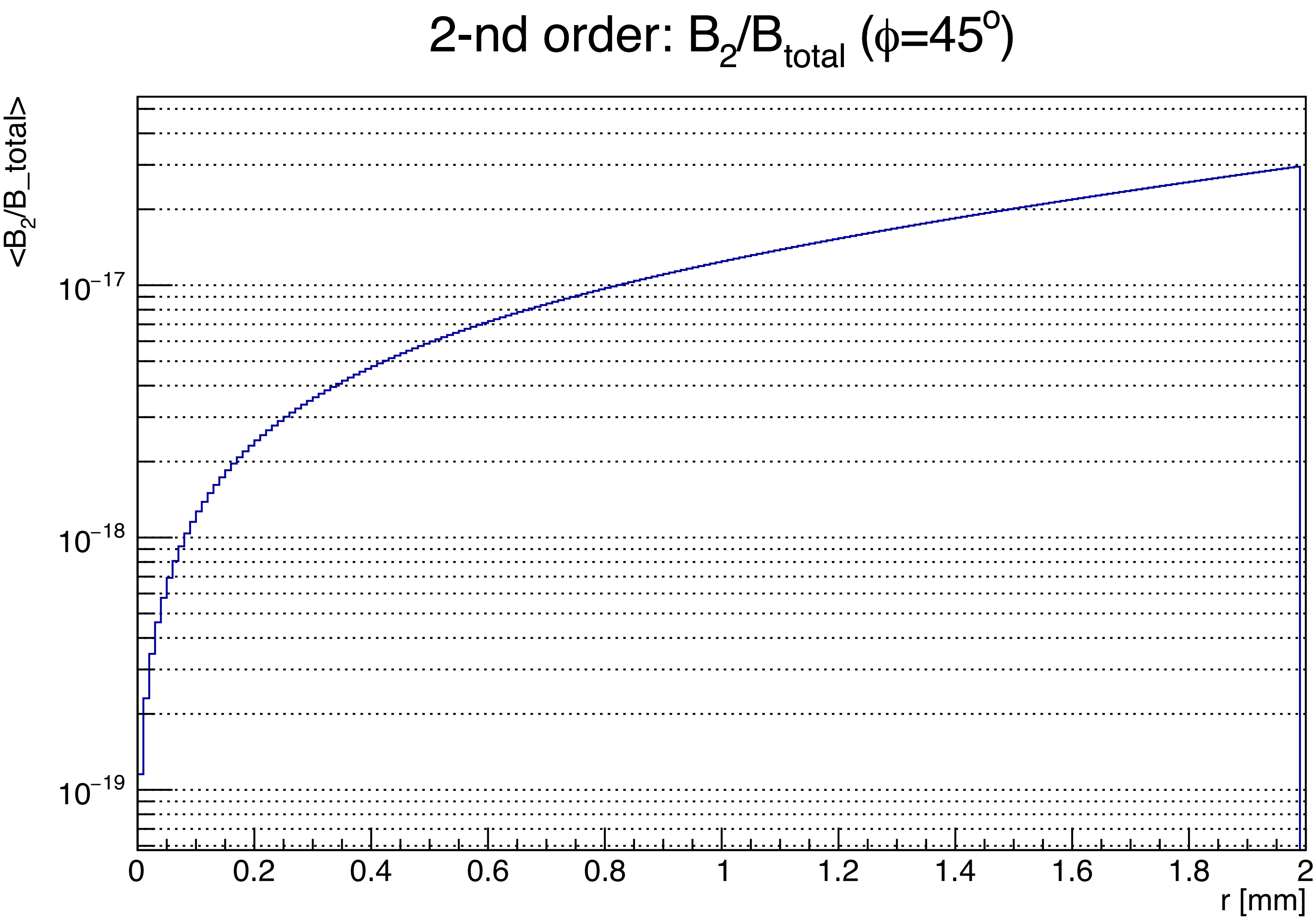


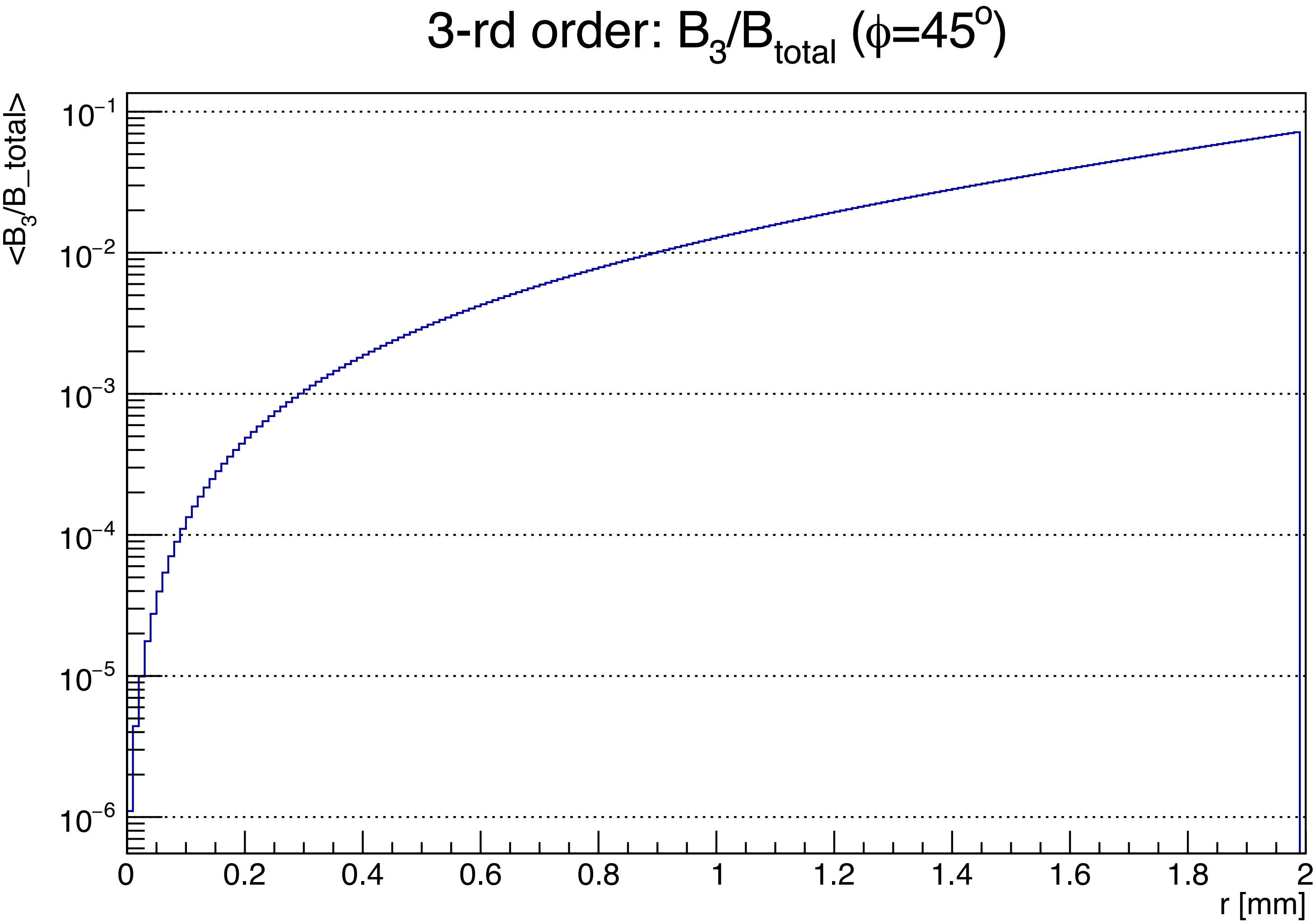


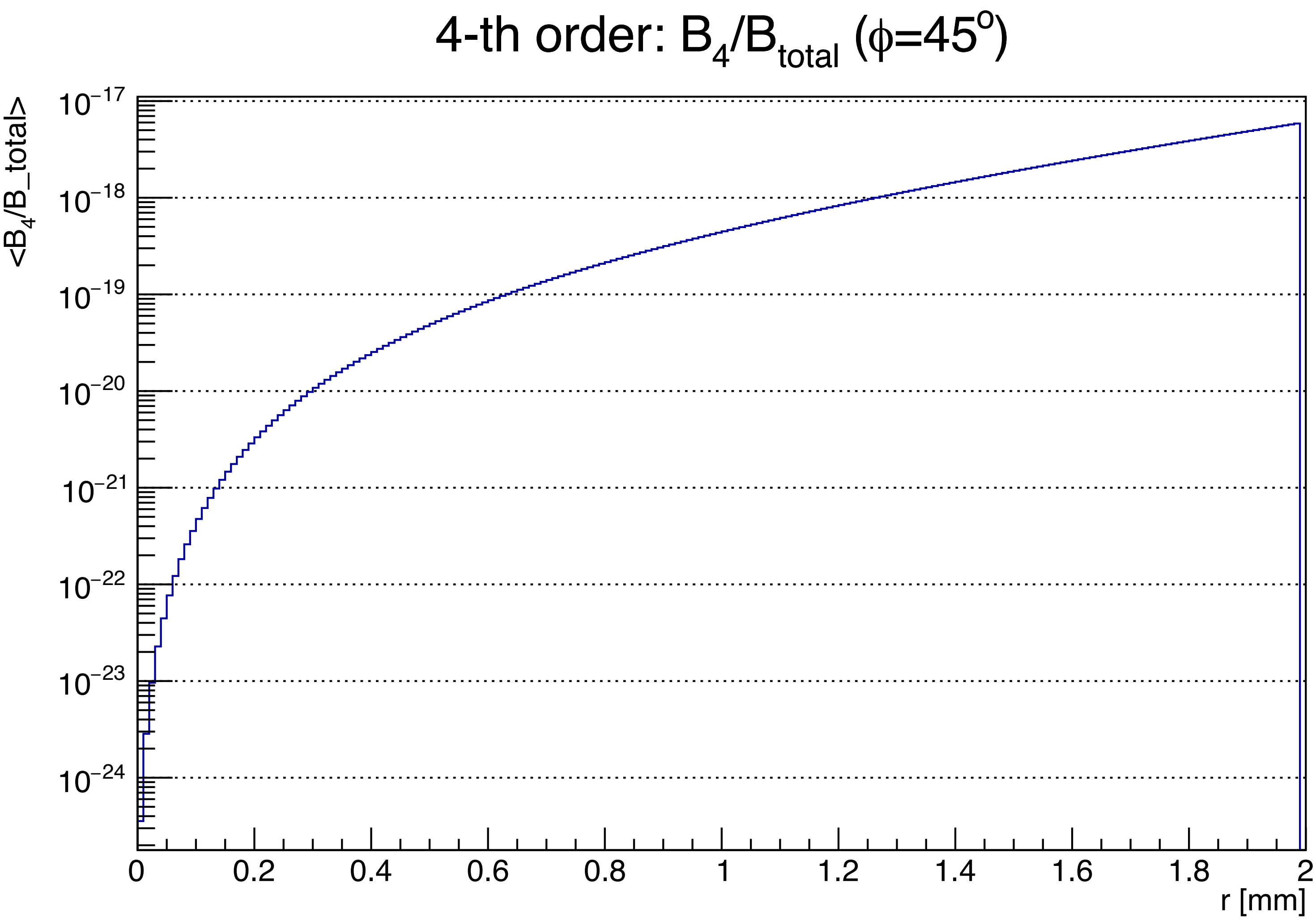
The magnetic field of the helical undulator in BMAD is in good agreement with the B-field calculated by formulas (n=1).

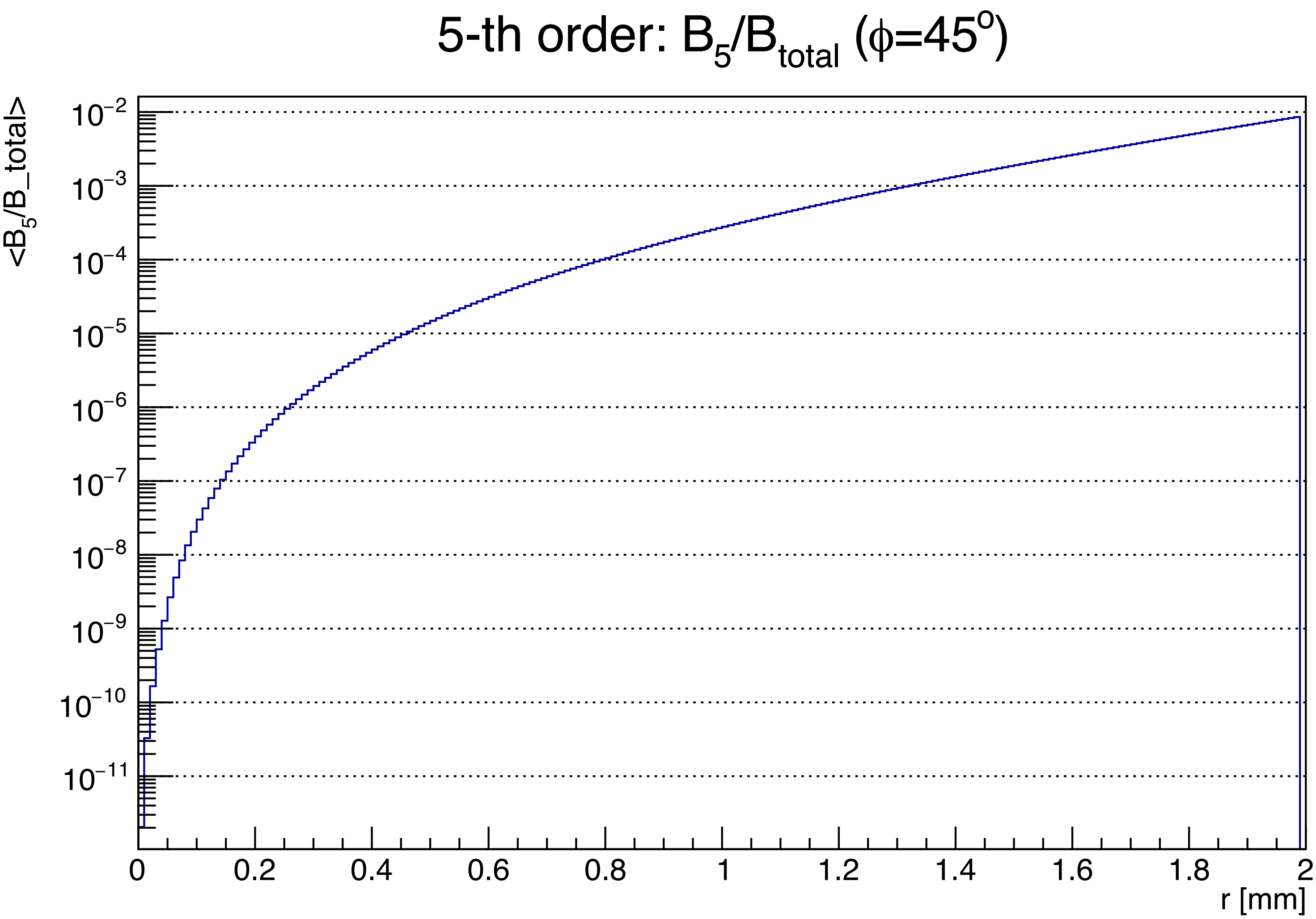
Next pages show the next term (n>1) contributions vs radius (averaged over z).

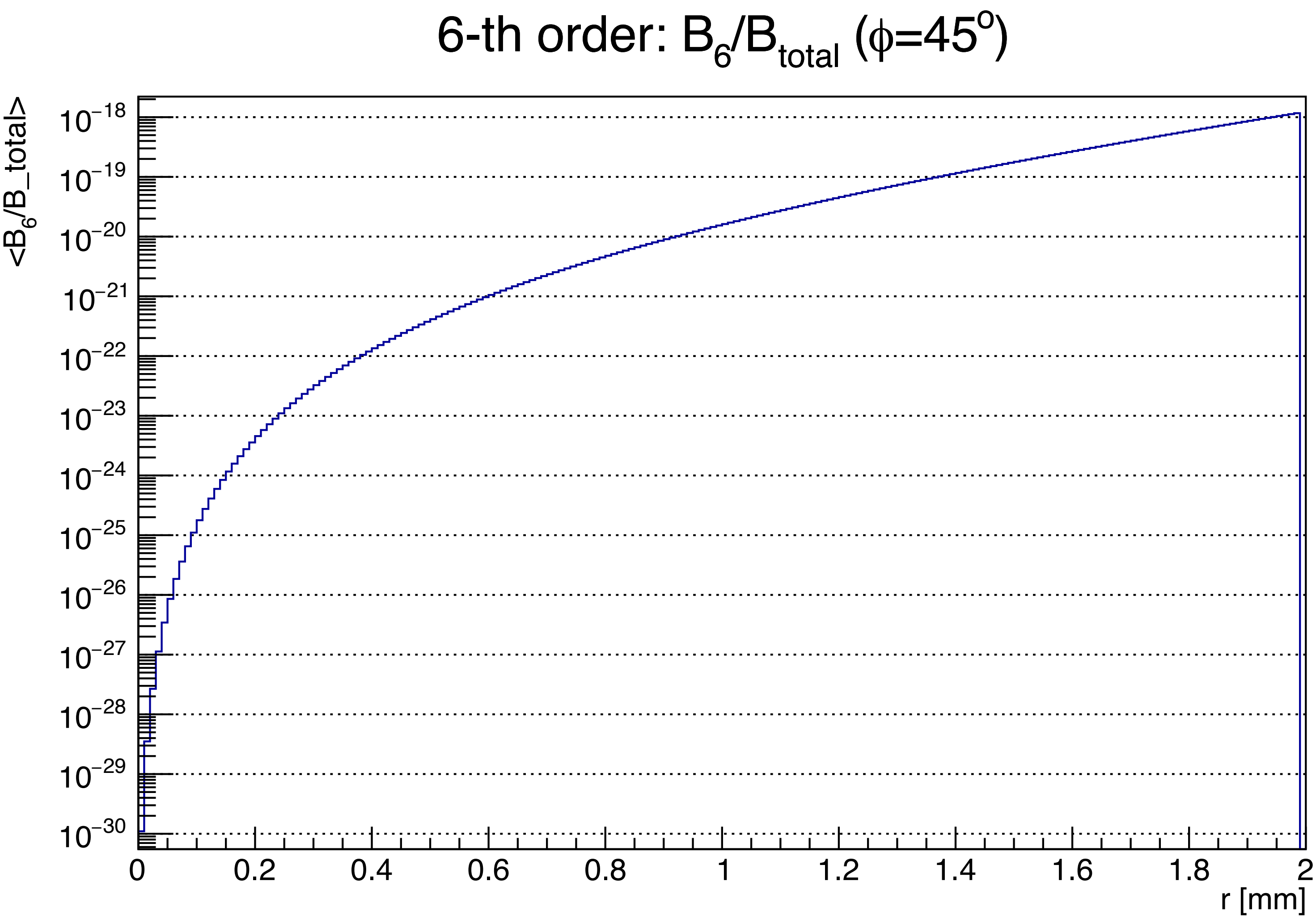

1-st order: B_1/B_{total} (ϕ=45^o)
<B_1/B_total>
0
0.2
0.4
0.6
0.8
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]


2-nd order: B2/Btotal (ϕ=45o)
<B2/B_total>
10−17
10−18
10−19
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

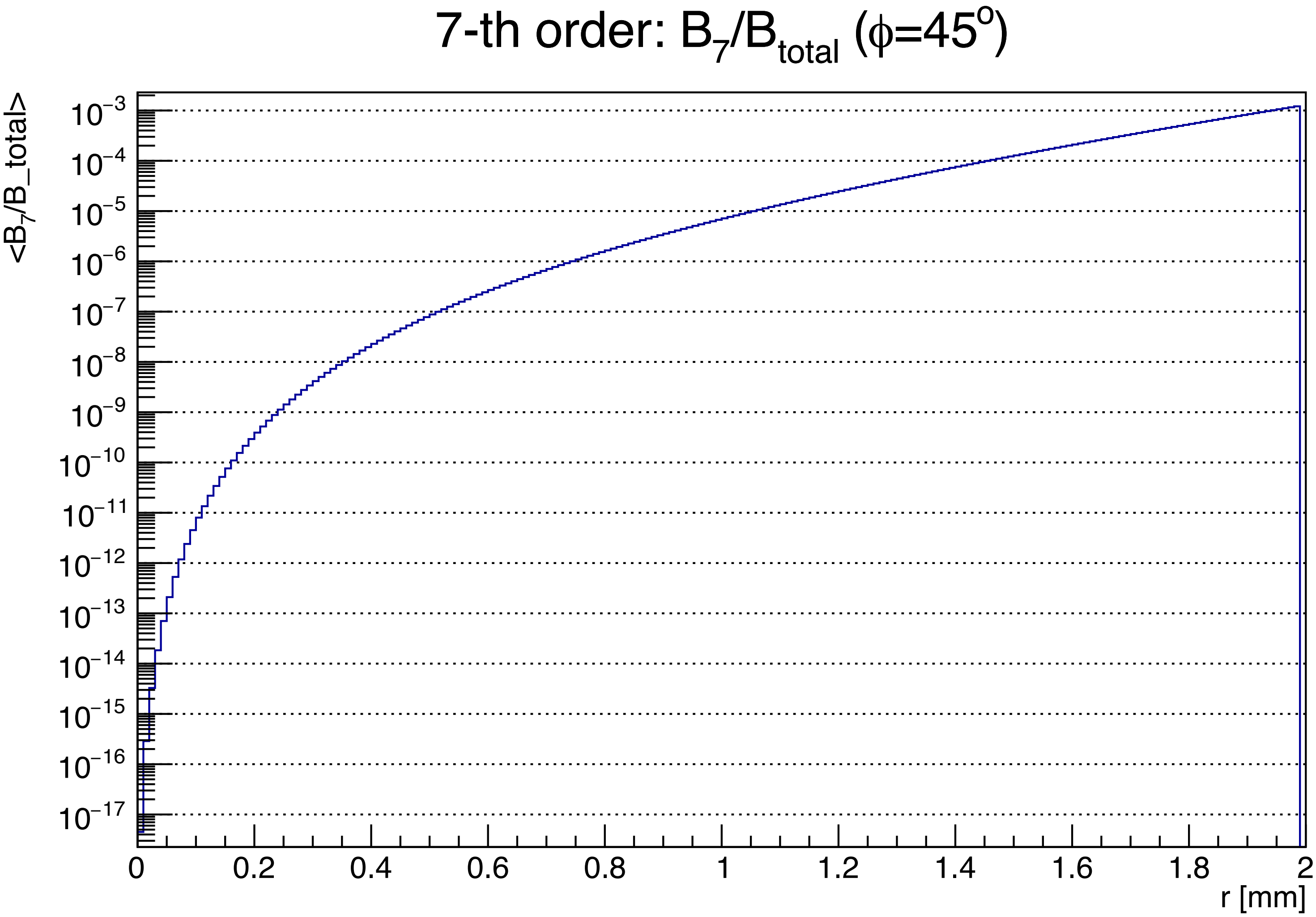

3-rd order: B3/Btotal (ϕ=45o)
<B3/B_total>
10−1
10−2
10−3
10−4
10−5
10−6
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

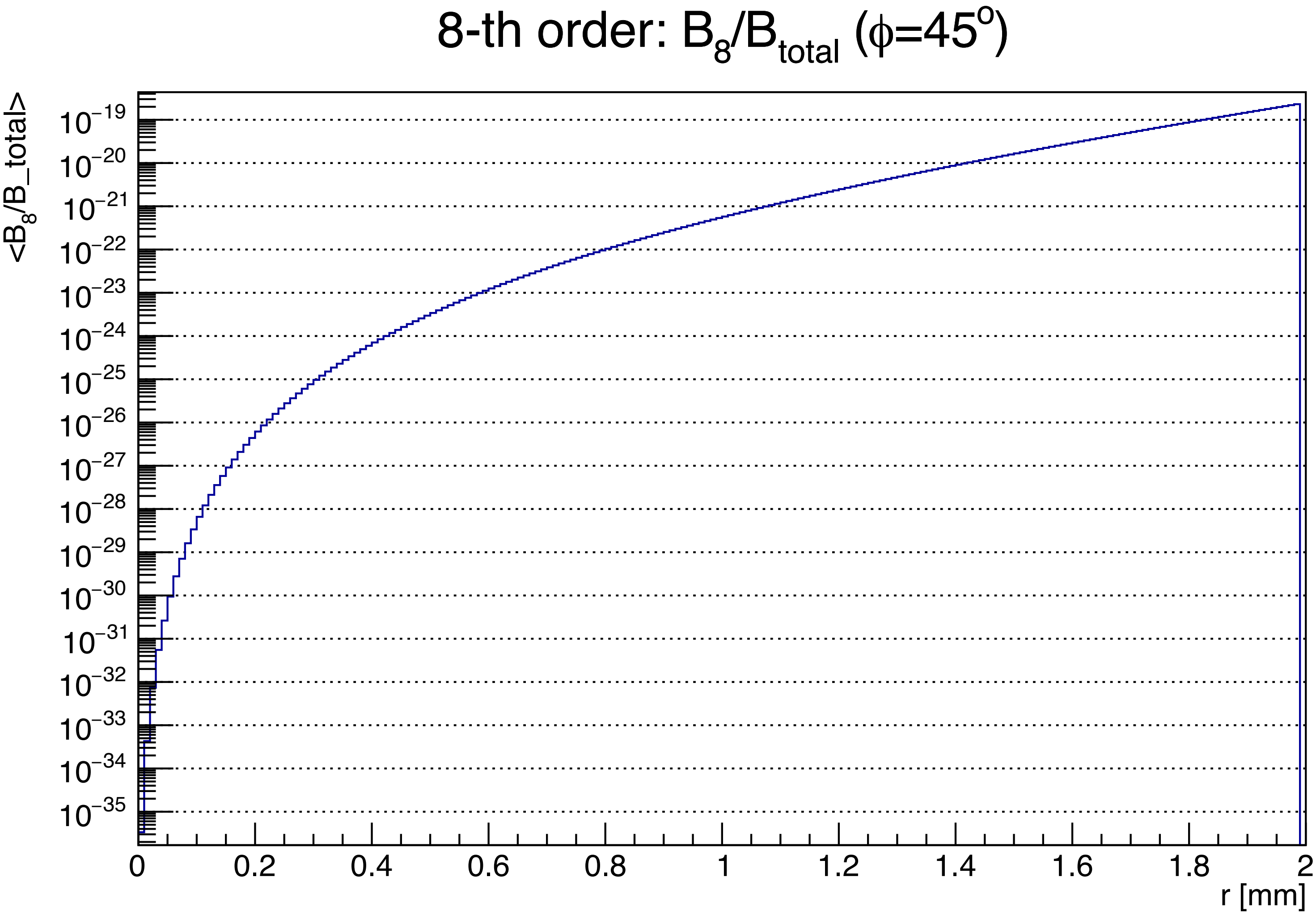

4-th order: B4/Btotal (φ=45o)
<B4/B_total>
10−17
10−18
10−19
10−20
10−21
10−22
10−23
10−24
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]


5-th order: B_5/B_{total} (ϕ=45^o)
<B_5/B_total>
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

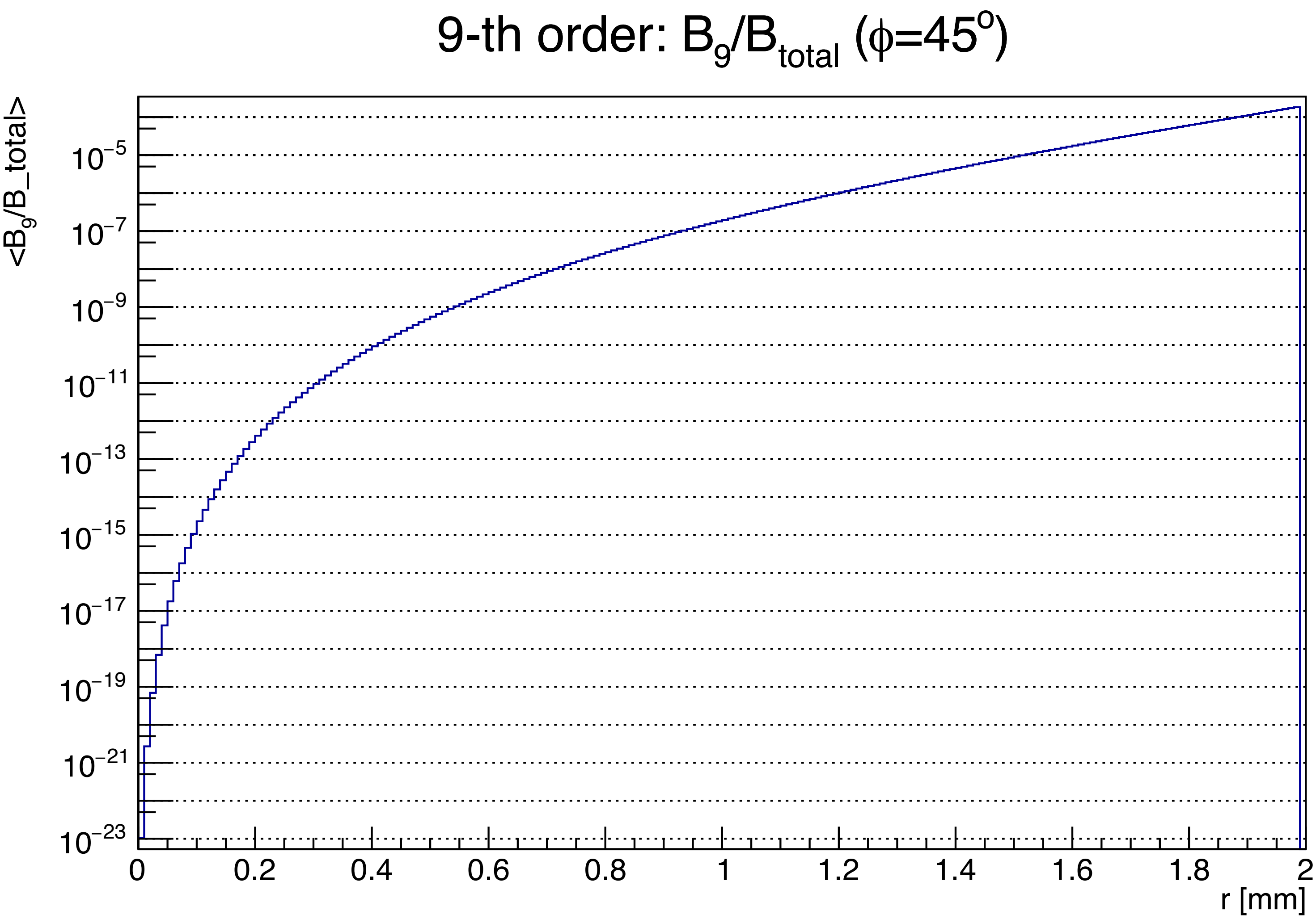


6-th order: B_6/B_{total} (ϕ=45^o)
<B_6/B_total>
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

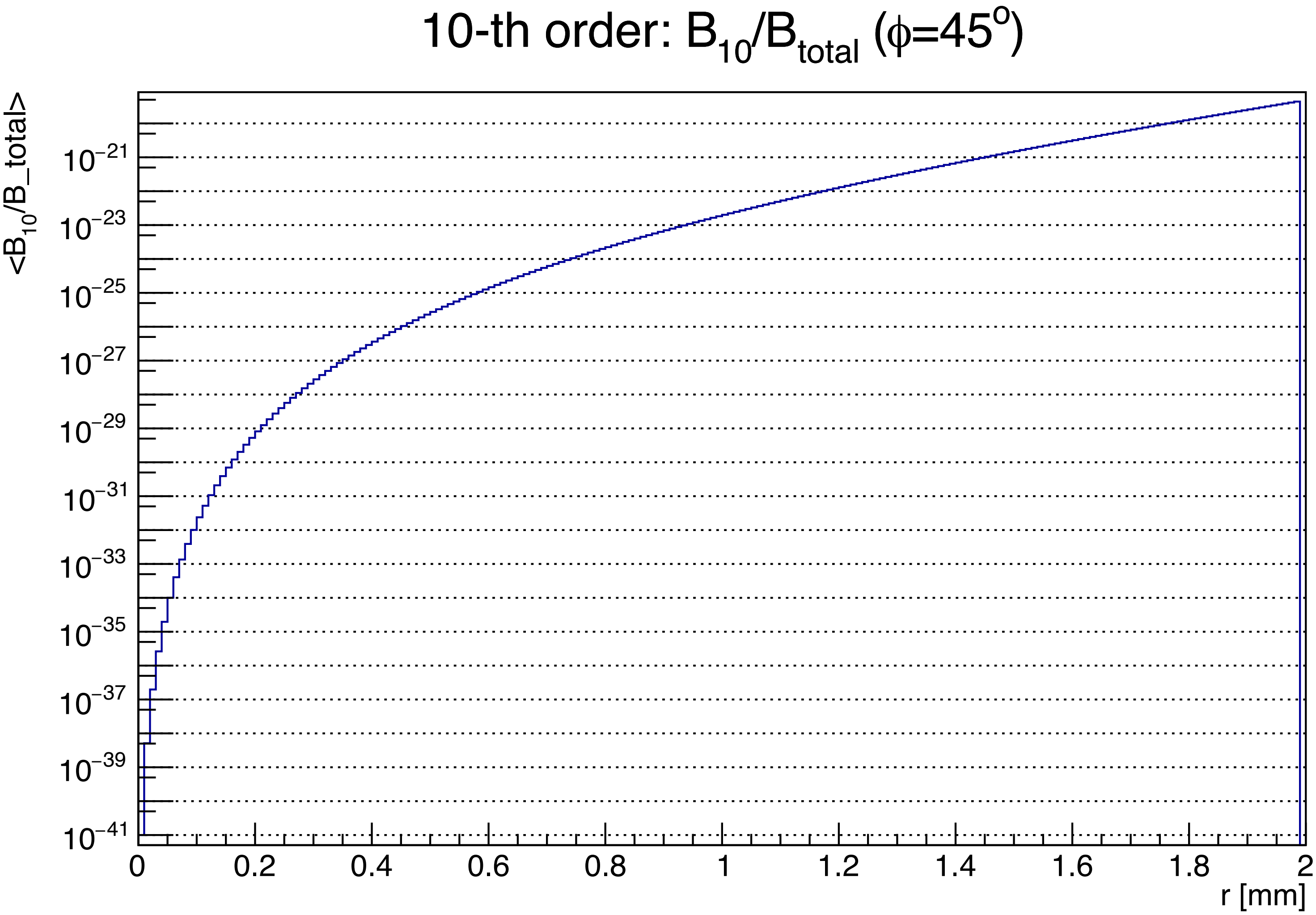

7-th order: B7/Btotal (φ=45°)
<B7/B_total>
10−3
10−4
10−5
10−6
10−7
10−8
10−9
10−10
10−11
10−12
10−13
10−14
10−15
10−16
10−17
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

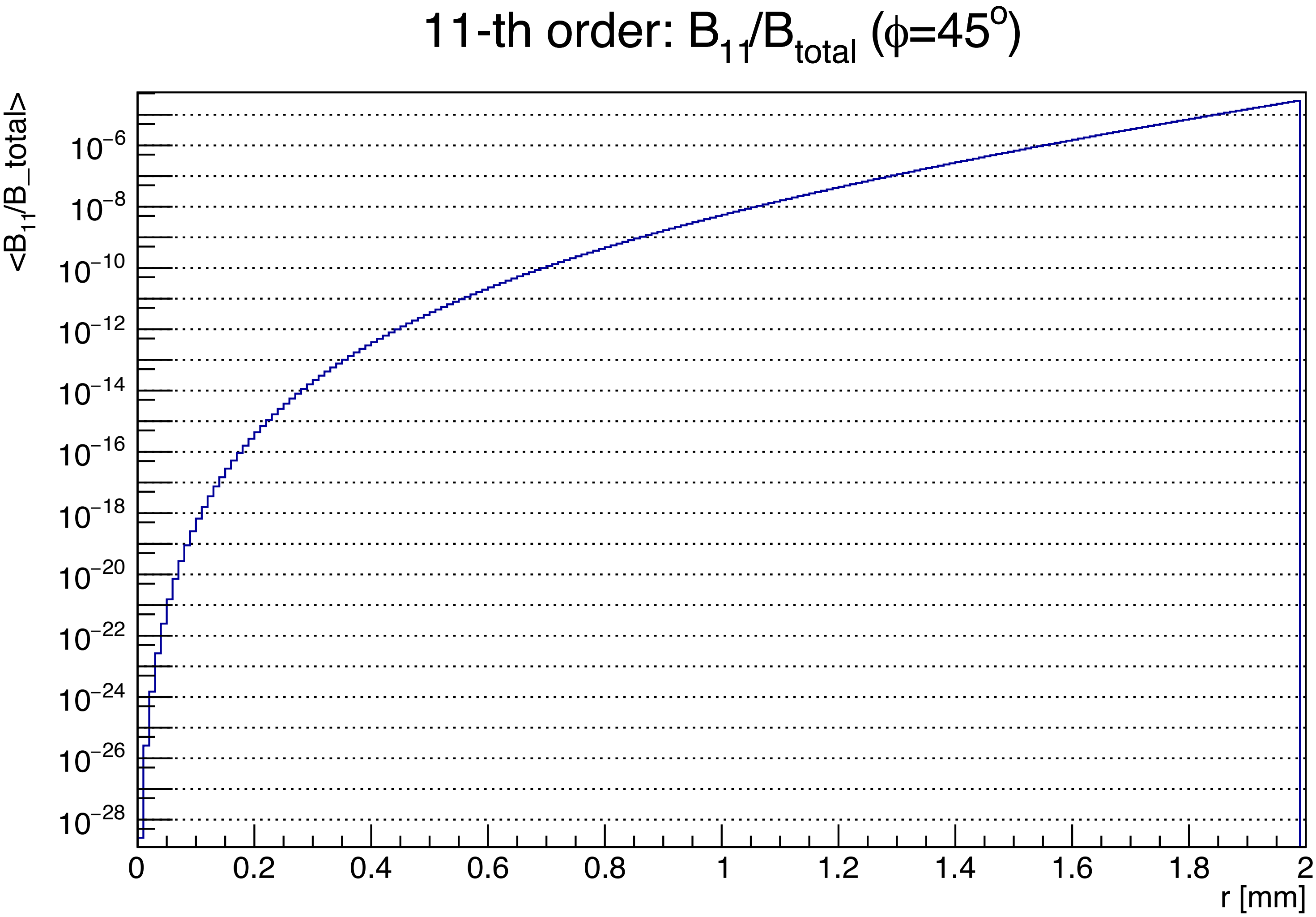


8-th order: B8/Btotal (ϕ=45o)
<B8/B_total>
10−19
10−20
10−21
10−22
10−23
10−24
10−25
10−26
10−27
10−28
10−29
10−30
10−31
10−32
10−33
10−34
10−35
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

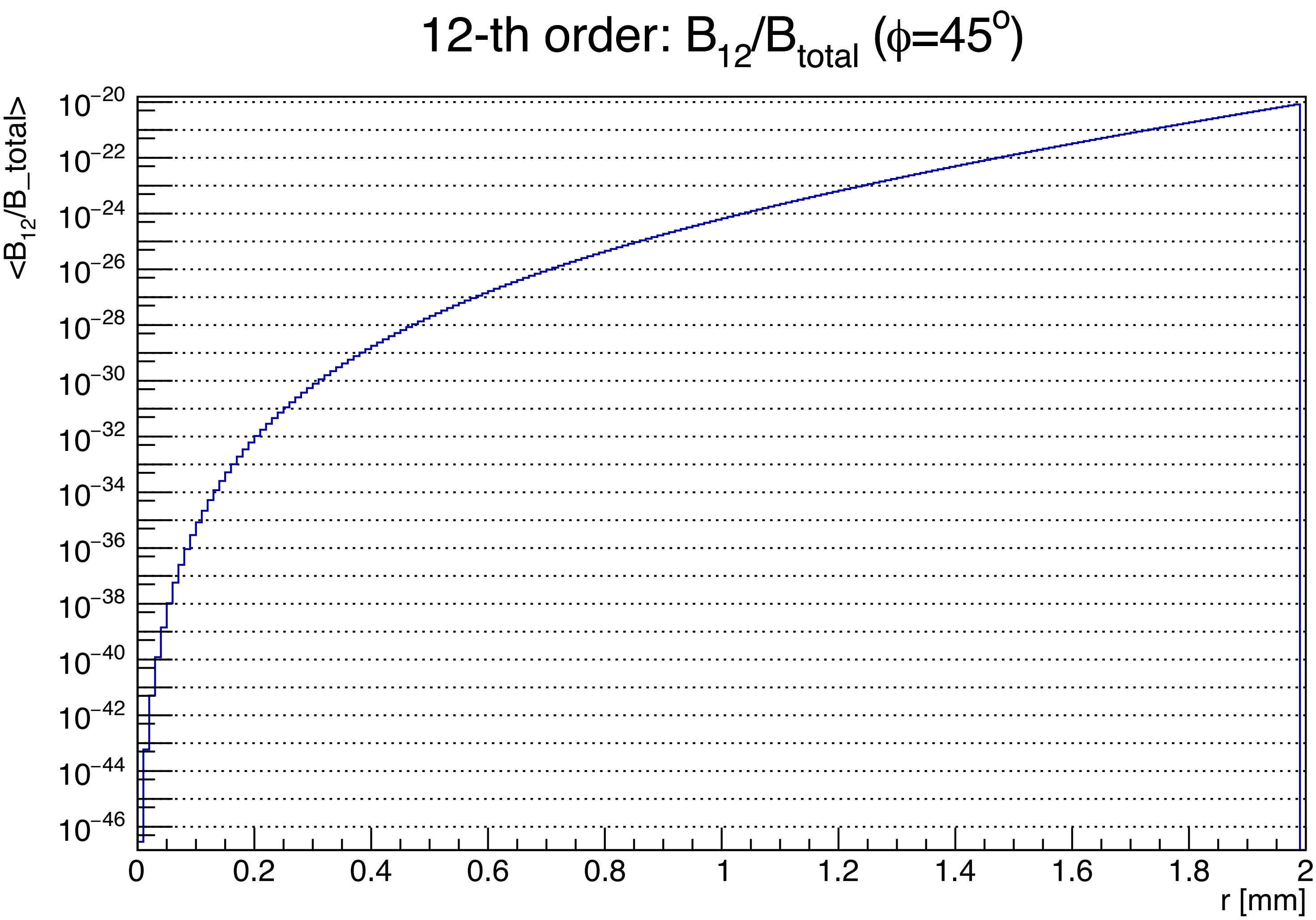


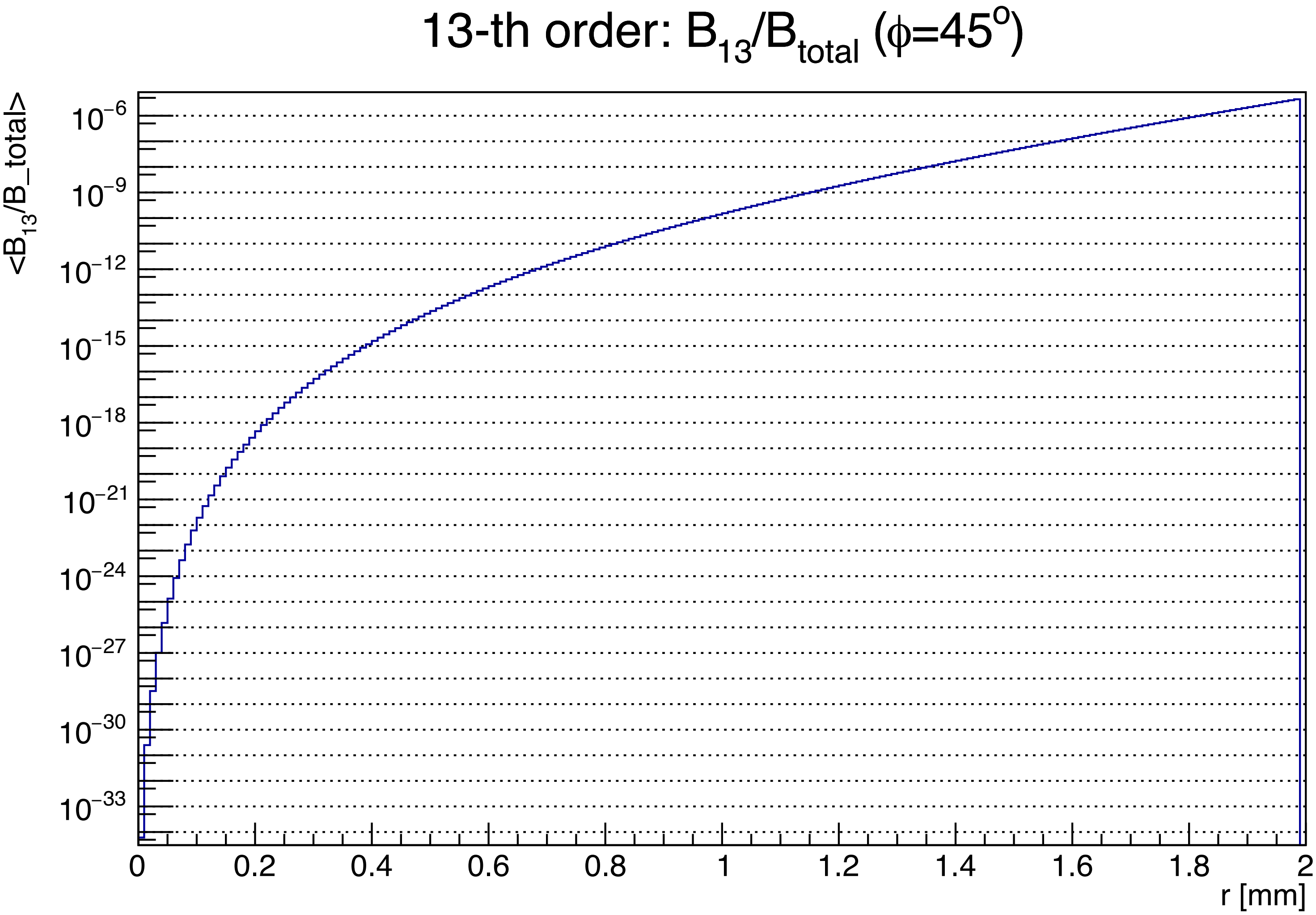
9-th order: B9/Btotal (φ=45o)
<B9/B_total>
10−5
10−7
10−9
10−11
10−13
10−15
10−17
10−19
10−21
10−23
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]


10-th order: B10/Btotal (ϕ=45o)
<B10/B_total>
10−21
10−23
10−25
10−27
10−29
10−31
10−33
10−35
10−37
10−39
10−41
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

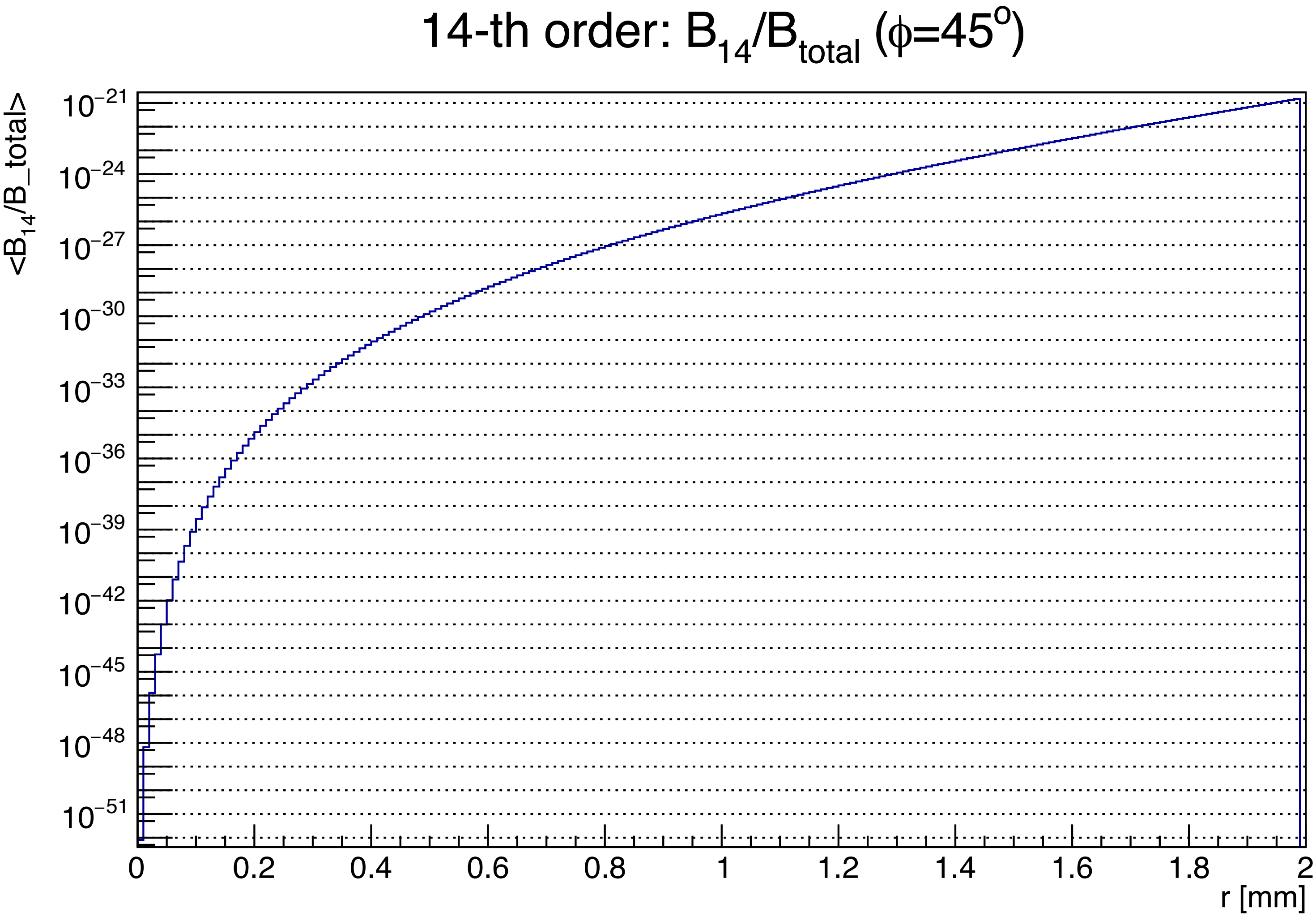


11-th order: B11/Btotal (φ=45o)
<B11/B_total>
10−6
10−8
10−10
10−12
10−14
10−16
10−18
10−20
10−22
10−24
10−26
10−28
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

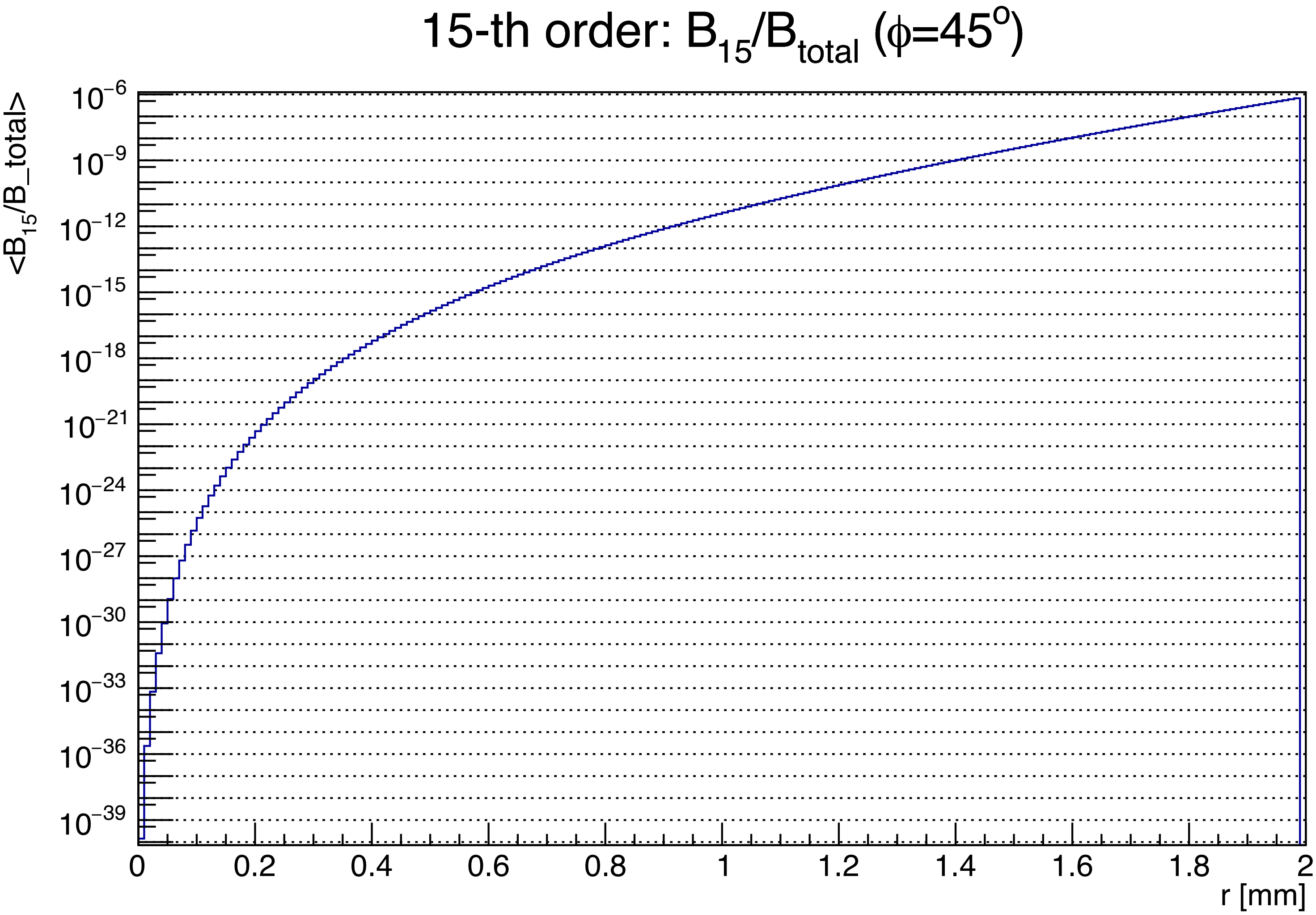

12-th order: B_{12}/B_{total} (ϕ=45^o)
<B_{12}/B_total>
10^{-20}
10^{-22}
10^{-24}
10^{-26}
10^{-28}
10^{-30}
10^{-32}
10^{-34}
10^{-36}
10^{-38}
10^{-40}
10^{-42}
10^{-44}
10^{-46}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]



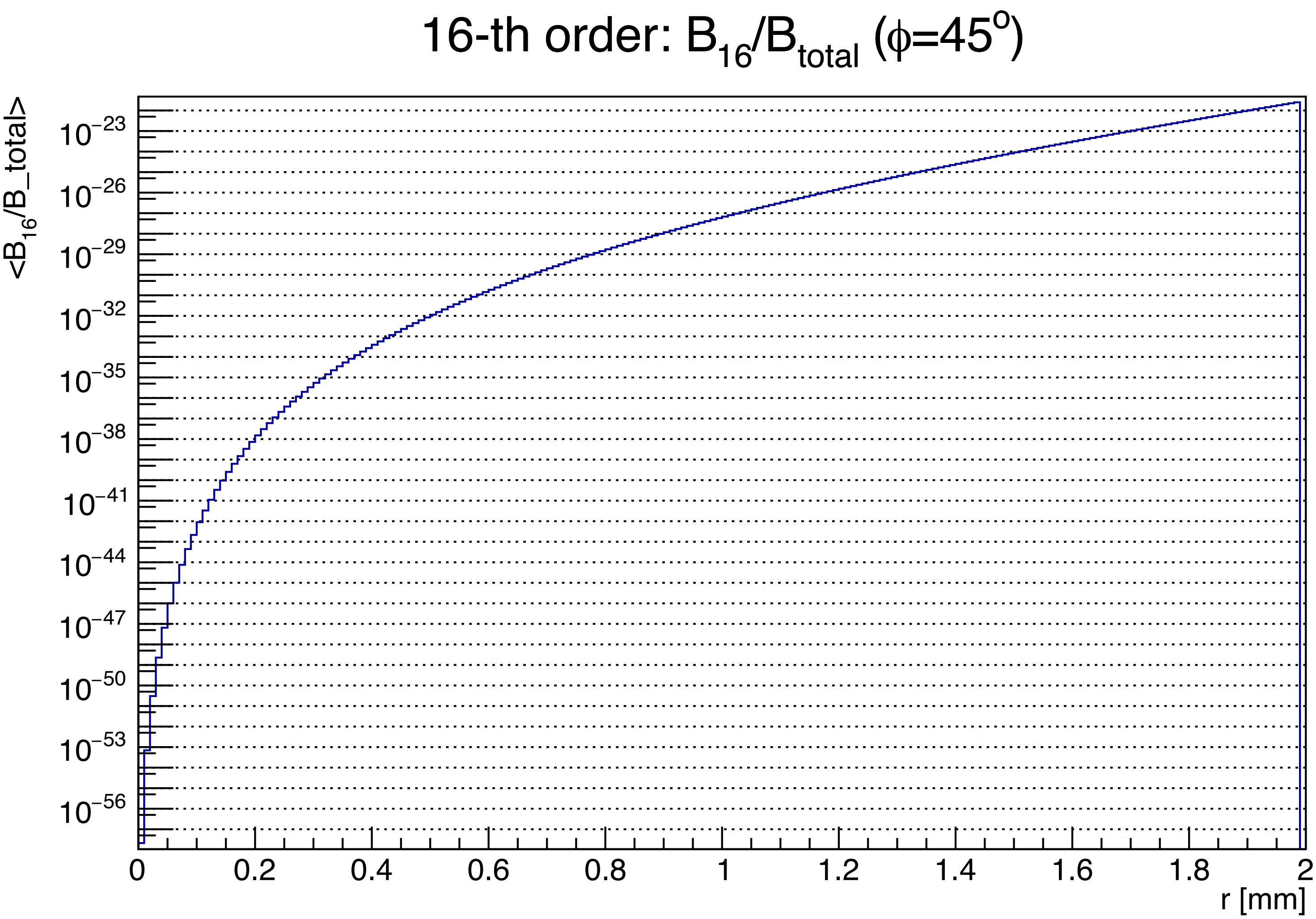
13-th order: B13/Btotal (φ=45°)
<B13/B_total>
10−6
10−9
10−12
10−15
10−18
10−21
10−24
10−27
10−30
10−33
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

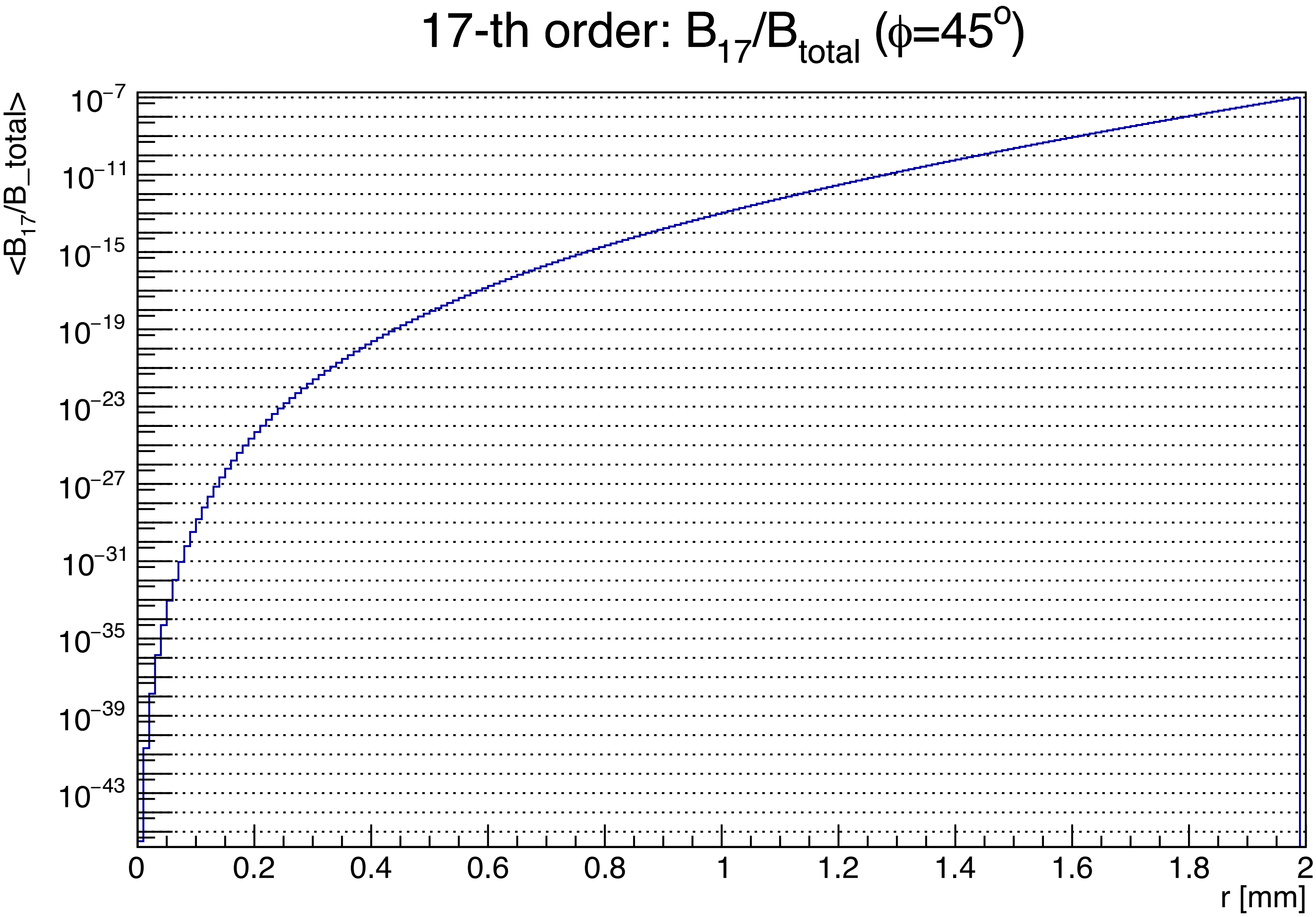


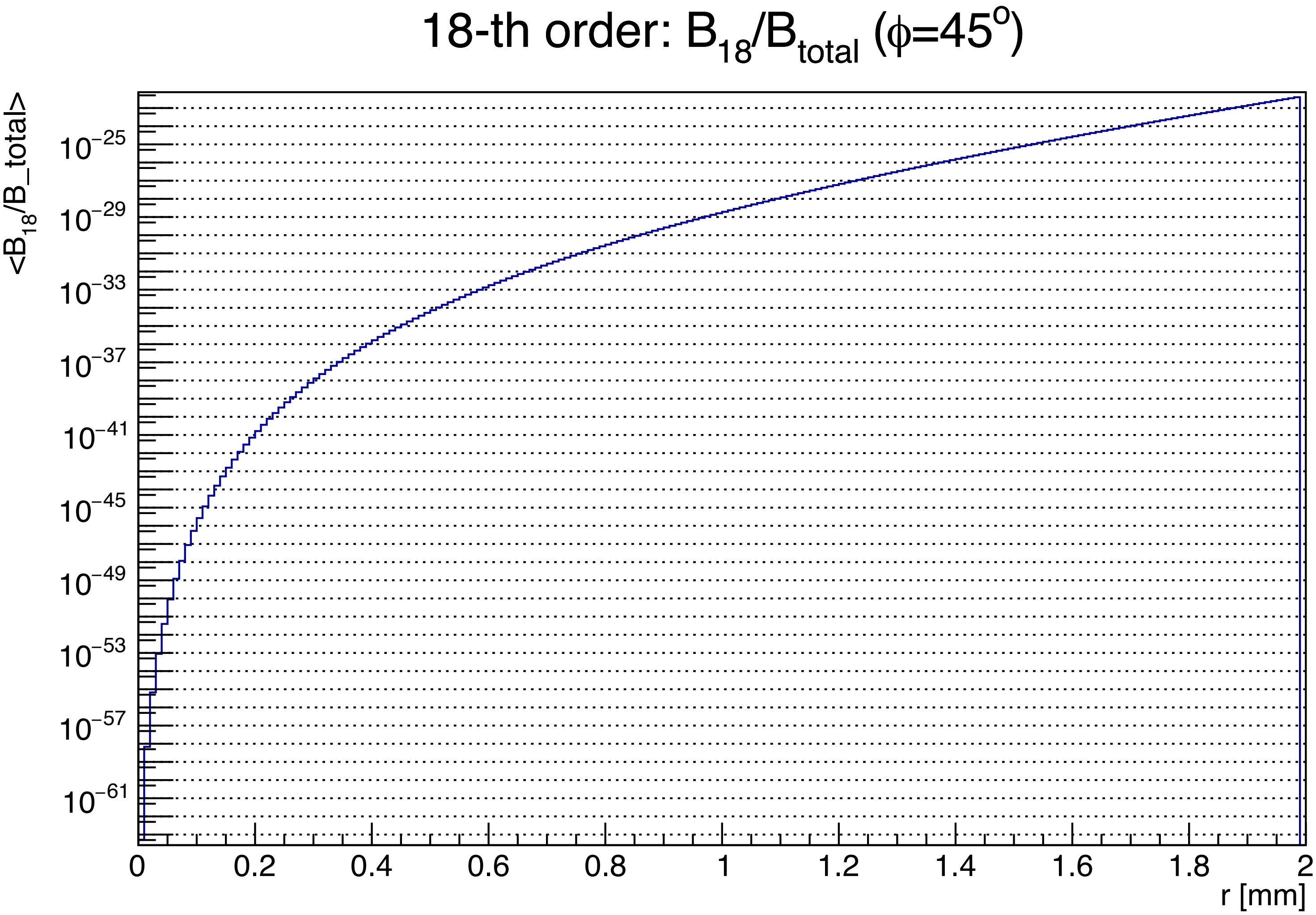
14-th order: B_{14}/B_{total} (ϕ=45^o)
10^{-21}
10^{-24}
10^{-27}
10^{-30}
10^{-33}
10^{-36}
10^{-39}
10^{-42}
10^{-45}
10^{-48}
10^{-51}
<B_{14}/B_total>
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]



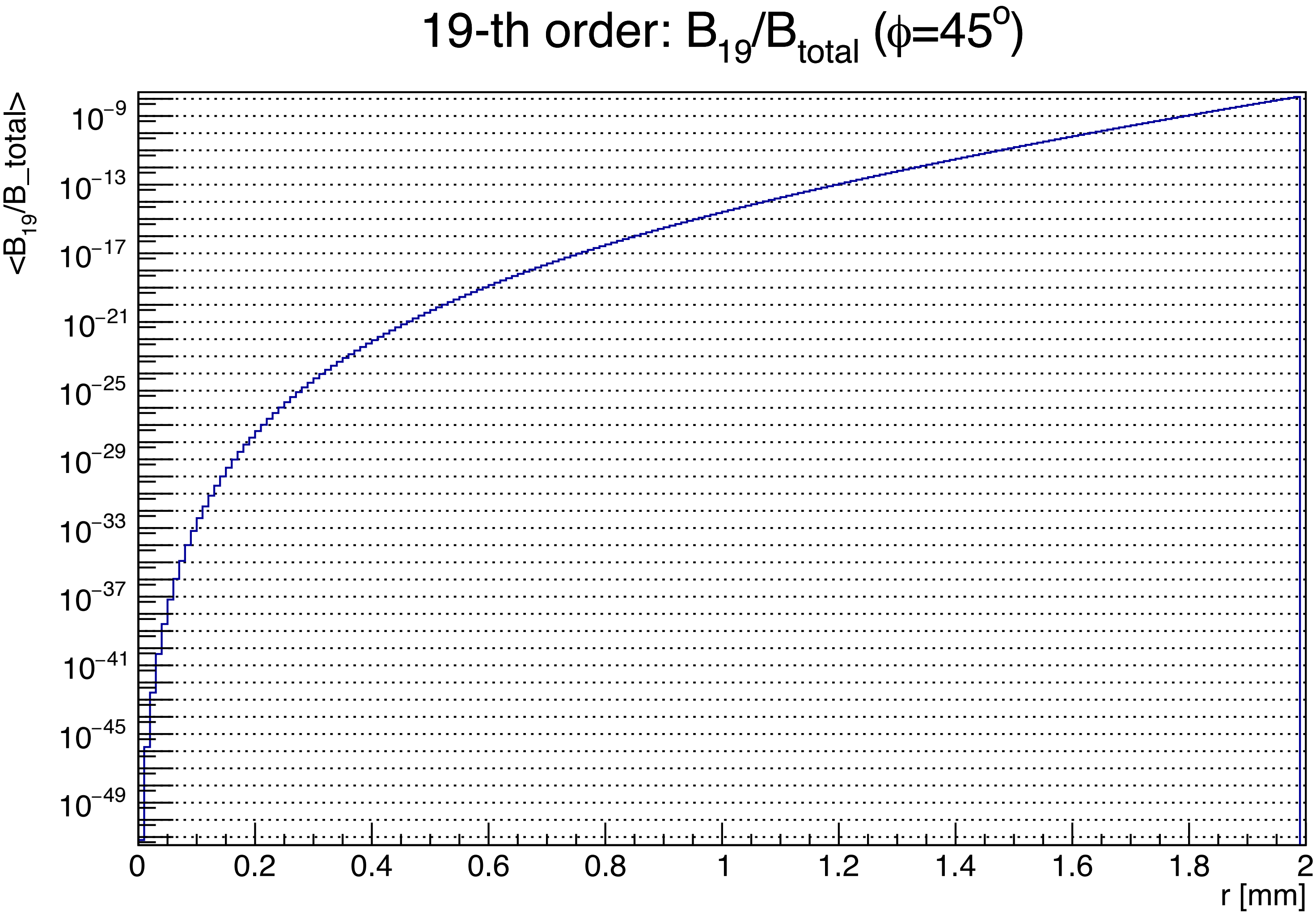
15-th order: B_{15}/B_{total} (ϕ=45^o)
<B_{15}/B_total>
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-24}
10^{-27}
10^{-30}
10^{-33}
10^{-36}
10^{-39}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]

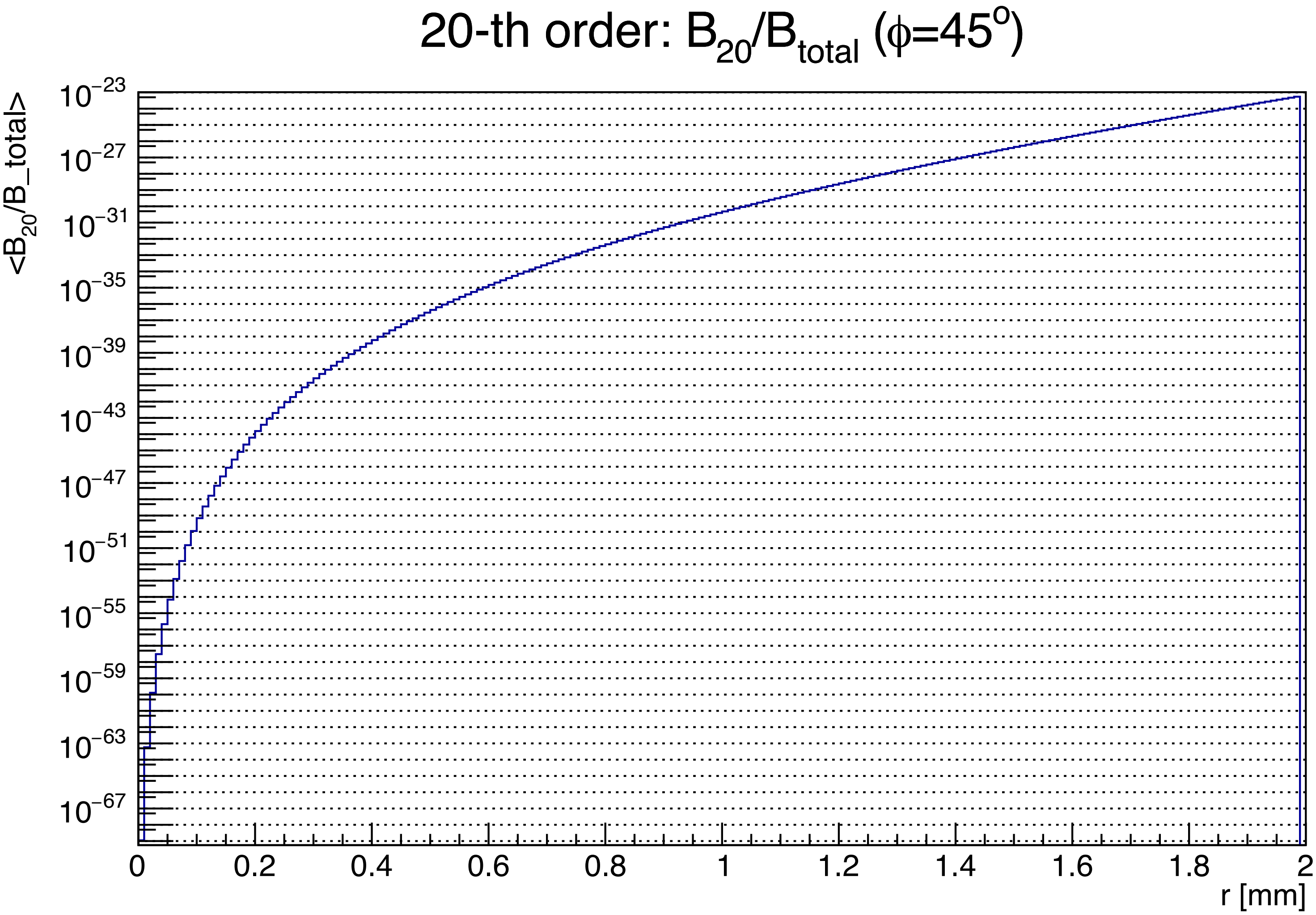

16-th order: B_{16}/B_{total} (ϕ=45^o)
<B_{16}/B_total>
10^{-23}
10^{-26}
10^{-29}
10^{-32}
10^{-35}
10^{-38}
10^{-41}
10^{-44}
10^{-47}
10^{-50}
10^{-53}
10^{-56}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]


17-th order: B17/Btotal (φ=45o)
<B17/B_total>
10−7
10−11
10−15
10−19
10−23
10−27
10−31
10−35
10−39
10−43
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]



18-th order: B18/Btotal (φ=45o)
<B18/B_total>
10−25
10−29
10−33
10−37
10−41
10−45
10−49
10−53
10−57
10−61
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]


19-th order: B_{19}/B_{total} (ϕ=45^o)
$\langle B_{19}/B\_total \rangle$
10^{-9}
10^{-13}
10^{-17}
10^{-21}
10^{-25}
10^{-29}
10^{-33}
10^{-37}
10^{-41}
10^{-45}
10^{-49}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [mm]



20-th order: B20/Btotal (φ=45°)
<B20/B_total>
r [mm]