Emittance growth from undulators in the OSC insertion

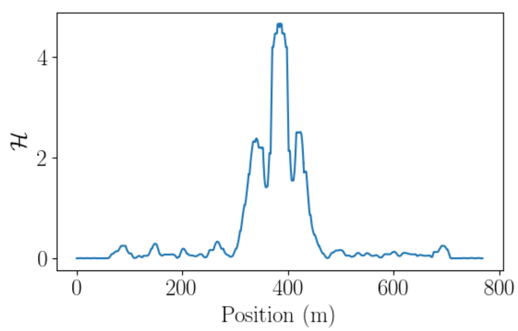
M. Andorf

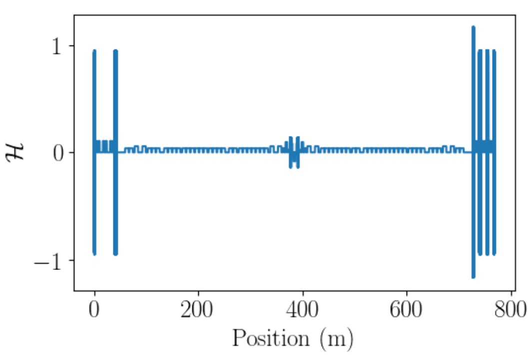
Standard analysis of equilibrium emittance yields expressions for the natural emittance and energy spread in a ring.

$$\epsilon = \frac{\tau_x}{4LU_s^2} \oint \mathcal{H}(s) N_\gamma \langle u^2 \rangle ds \qquad \frac{\sigma_u^2}{U_s^2} = \frac{\tau_u}{4LU_s^2} \oint N_\gamma \langle u^2 \rangle ds$$

If we only consider the local bend radius along (so that locally everything is treated like a dipole) we get

$$N_\gamma = \frac{5}{\sqrt{3}} \frac{\alpha_f \gamma c}{\rho} \qquad \langle u^2 \rangle = \frac{1}{N_\gamma} \int_0^\infty u^2 n_\gamma(u) du \approx 0.41 u_c^2 \qquad u_c = \frac{3c}{2\rho}\gamma^3$$





Taking the on-axis magnetic field to compute the local photon-energy distribution, and the lattice functions to find curly H we can numerically compute the above integrals to get the natural emittance and energy spread.

Doing this gave

emittance_x=3.94 and energy spread 4.03*1e-3

Using bmad I get

emittance_x=3.78 and energy spread 4.06*1e-3

In a paper by E.L. Saldin et al they present a formula for ‘rate of energy diffusion’ in an undulator

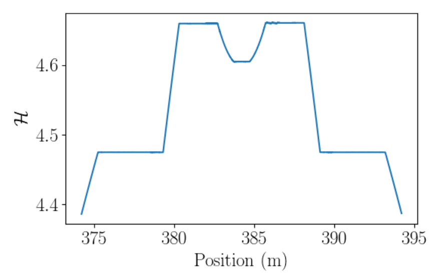
$$\frac{d\langle\gamma^2\rangle}{dt} = \frac{14}{15} c\lambda_c \gamma^4 k_u^3 K^2 F_K \longrightarrow \frac{d\langle U^2\rangle}{dt} = \frac{14}{15} c\lambda_c \gamma^4 k_u^3 K^2 F_K U_{rest}^2$$

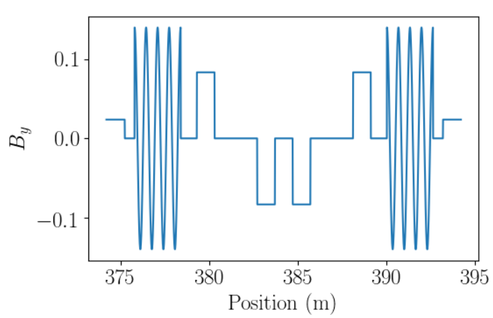
So now the integrals are broken into two parts. One giving emittance contribution from dipoles, the other from the undulators

$$\frac{\sigma_u^2}{U_s^2} = \frac{\tau_u}{4LU_s^2}\left(\int_{dip} N_\gamma \langle u^2\rangle ds + 2\int_0^{L_u} \frac{d\langle U^2\rangle}{dt} ds\right)$$

$$= \frac{\tau_u}{4LU_s^2}\left(\int_{dip} N_\gamma \langle u^2\rangle ds + 2L_u \frac{d\langle U^2\rangle}{dt}\right)$$

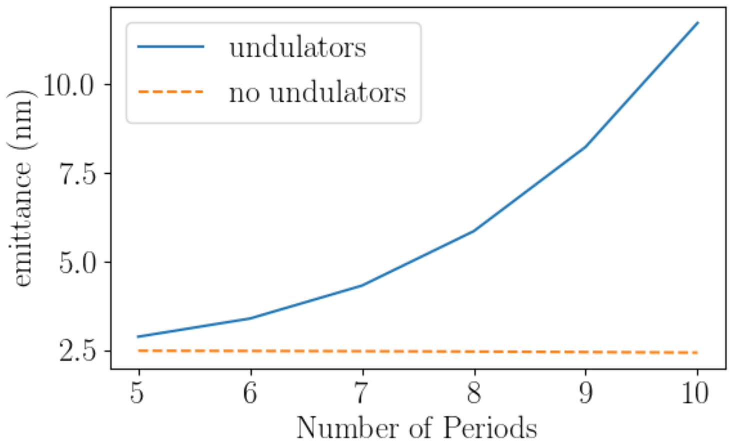
Factor of 2 accounts pickup+kicker

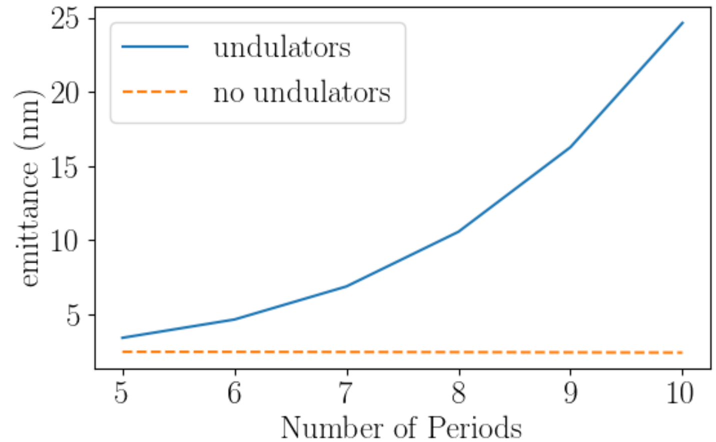


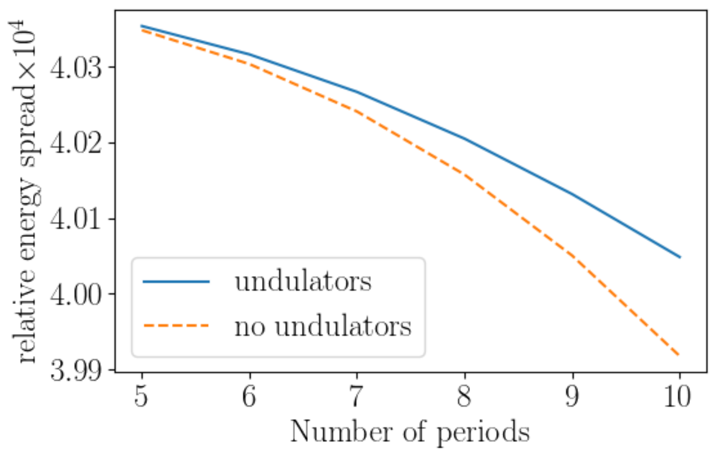


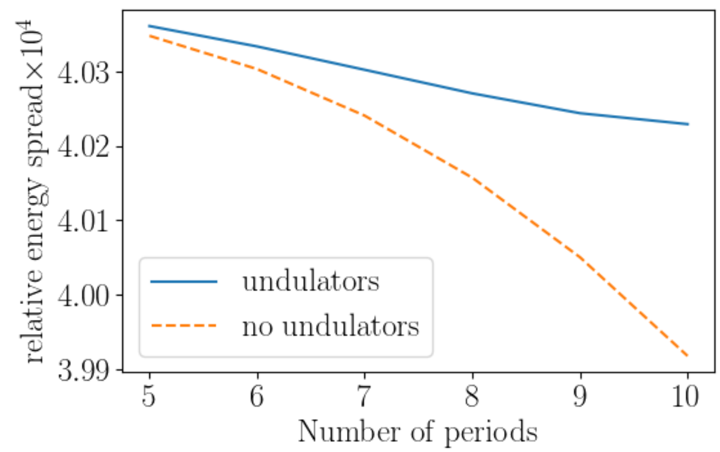
$$\epsilon = \frac{\tau_x}{4LU_s^2}\left(\int_{dip} \mathcal{H}(s) N_\gamma \langle u^2\rangle ds + 2\int_0^{L_u} \mathcal{H}(s) \frac{d\langle U^2\rangle}{dt} ds\right)$$

In our case curly H is constant in the undulator so the 2[nd] integral in the emittance expression can be exactly evaluated.



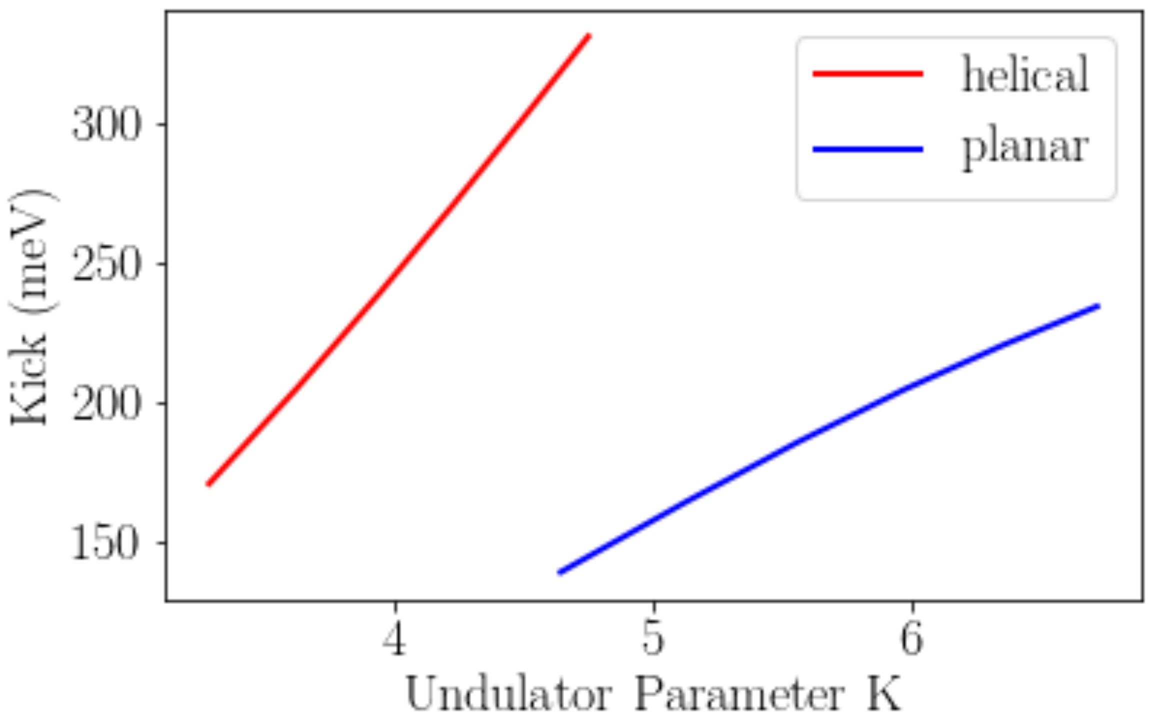






This model is predicting 5.85 nm emittance for helical and 10.6 nm using a planar undulator.

Reminder of what kick amplitude looks like as a function of K….



Cooling Range

We have $\lambda_x / \lambda_s \approx \Phi D^* h / \left(2\Delta s - \Phi D^* h\right)$ $\lambda x/\lambda s \approx 20 = R$

$$n_{\sigma s} \approx \frac{\mu_{01}}{\left(2\Delta s - \Phi D^* h\right) k_0 \sigma_p} \qquad n_{\sigma x} \approx \frac{\mu_{01}}{2k_0 h \Phi \sqrt{\varepsilon \beta^*}}$$

Cooling range in horizontal plane becomes

$$\eta_{\sigma_x} = \frac{\mu_{0,1}(R+1)}{4k\Delta s}\sqrt{\frac{\mathcal{D}^{*2}}{\epsilon\beta^*}} = \frac{\mu_{0,1}(R+1)}{4k\Delta s}\sqrt{\frac{\mathcal{H}^*}{\epsilon}}$$

Assuming 5.85 nm emittance gives a cooling range of 22.5.

We can reduce this number to ~10. Doing this by reducing H* will reduce emittance. Doing this by increasing beta* will decrease nonlinear path lengthening (decrease sextupole strength, or even eliminate the need for them.)
Should not reduce H* by too so that R stays fairly large.