P3323 Transmission Line
November 4, 2016

A certain transmission line is constructed from two thin metal "ribbons", of width w, a very small distance $h \ll w$ apart. The current travels down one strip and back along the other. In each case, it spreads out uniformly over the surface of the ribbon. ($\epsilon_0 = 8.85 \times 10^{-12}\mathrm{C}^2/\mathrm{Nm}^2$,$\mu_0 = 4\pi \times 10^{-7}\mathrm{N}/\mathrm{A}^2$)

1. Find the capacitance per unit length, $\mathcal{C}$.

   [Solution: Capacitance $C = Q/V$. Suppose there is surface charge density $+\sigma$ on the top ribbon, and $-\sigma$ on the bottom. We use Gauss's law to determine the E-field in the gap.

$$\begin{aligned} \oint \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{enc}}{\epsilon_0} \\ \mathbf{E}A &= \frac{\sigma A}{\epsilon_0}\hat{\mathbf{z}} \rightarrow \mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{z}} \end{aligned}$$

   Then $V = Eh$, and

$$C = \frac{\sigma w l}{\sigma h/\epsilon_0}$$

   where the area $A = lw$. The capacitance per unit length

$$\mathcal{C} = \frac{\epsilon_0 w}{h}]$$

2. Find the inductance per unit length, $\mathcal{L}$

   [Solution: If the current is running in the z-direction, and the ribbons are parallel to the y-z plane, the magnetic field will be in the y-direction. According to Amperes law

$$\begin{aligned} \oint \mathbf{B} \cdot d\mathbf{l} &= \mu_0 I_{enc} \\ \mathbf{B} &= \frac{\mu_0 I}{w} \end{aligned}$$

   The magnetic flux in the gap is

$$\Phi = \int \mathbf{B} \cdot d\mathbf{a} = \frac{\mu_0 I}{w} lh$$

   The inductance

$$L = \Phi/I = \mu_0 \frac{lh}{w}$$

   and the induction per unit length

$$\mathcal{L} = \mu_0 \frac{h}{w}]$$

3. What are the dimensions of the product $\mathcal{LC}$ and what is the product numerically?

   [Solution: The product $\mathcal{LC} = \mu_0 \epsilon_0 = 1/c^2$ dimensions inverse velocity squared.]

4. If the strips are insulated from one another by a nonconducting material of permittivity ϵ and perameablity μ, what is the product $\mathcal{LC}$?

   [Solution: $\mathcal{LC} = \mu\epsilon$]