P3323 energy November 7, 2016

A fast wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor.



1. Find the electric and magnetic fields in the gap, as functions of distance s from the axis and the time t. (Assume the charge is zero at t = 0.)

2. Find the energy density u_{em} and the Poynting vector **S** in the gap. Note especially the *direction* of **S**.

3. Check that the continuity equation for energy is satisfied

$$\frac{\partial u}{\partial t} = -\nabla \cdot \mathbf{S}$$

4. Determine the total energy in the gap, as a function of time.

5. Calculate the total power flowing into the gap, by integrating the Poynting vector over the appropriate surface.

6. Check that the power input is equal to the rate of increase of energy in the gap.