P3323 Reflection and Transmission
November 21, 2016

Consider a plane wave traveling through a medium with permittivity $\epsilon_1$ and permeability $\mu_0$ normally incident on a medium with permittivity $\epsilon_2$ and permeability $\mu_0$. We want to determine the fraction of the incident energy that is transmitted through the boundary of the materials, and the fraction of radiation reflected. The waves are propagating in the z-direction and the boundary is the x-y plane at $z = 0$.

**Boundary Conditions**

$$\begin{aligned} \epsilon_1 E_1^{\perp} &= \epsilon_2 E_2^{\perp}, \quad E_1^{\parallel} = E_2^{\parallel} \\ B_1^{\perp} &= B_2^{\perp}, \quad \frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel} \end{aligned}$$

The incident E and B-fields can be written

$$\begin{aligned} \tilde{\mathbf{E}}_I(z,t) &= \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \\ \tilde{\mathbf{B}}_I(z,t) &= \frac{1}{v_1} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}} \end{aligned}$$

1. How is $k_1$ related to $v_1$?

2. The reflected E-field is written

$$\tilde{\mathbf{E}}_R(z,t) = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{\mathbf{x}}$$

   Write an expression for the reflected magnetic field in terms of $\tilde{E}_{0R}, k_1, \omega,$ and $v_1$.

3. The transmitted E-field is

$$\tilde{\mathbf{E}}_T(z,t) \quad = \quad \tilde{E}_{0T} e^{i(k_2 z - \omega t)} \hat{\mathbf{x}}$$

   Write an expression for the transmitted magnetic field in terms of $\tilde{E}_{0T}, k_2, \omega$, and $v_2$.

4. Use the boundary conditions (above) to relate $\tilde{E}_{I0}, \tilde{E}_{R0}$ and $\tilde{E}_{T0}$.

5. Use the boundary conditions (above) to relate $\tilde{B}_{I0}, \tilde{B}_{R0}$ and $\tilde{B}_{T0}$.

6. Solve for $\tilde{E}_{0R}$ in terms of $\tilde{E}_{0I}$ and compute the reflection coefficient $R = \frac{I_R}{I_I}$. Write $R$ in terms of the indices of refraction $n_1$ and $n_2$. (Intensity is the time average of the Poynting vector. $I = \frac{1}{2}\epsilon_0 v E_0^2$.)

7. Solve for $\tilde{E}_{0T}$ in terms of $\tilde{E}_{0I}$ and compute the transmission coefficient $T = \frac{I_T}{I_I}$. Write $T$ in terms of the indices of refraction $n_1$ and $n_2$.

8. Show that $R + T = 1$.