P3323 LC circuit
November 28, 2016

In a conductor the wave equations for E and B become

$$\nabla^{\mathbf{2}}\mathbf{E} = \mu\epsilon\frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu\sigma\frac{\partial \mathbf{E}}{\partial t}, \quad \nabla^{\mathbf{2}}\mathbf{E} = \mu\epsilon\frac{\partial^2 \mathbf{B}}{\partial t^2} + \mu\sigma\frac{\partial \mathbf{B}}{\partial t}$$

with solutions

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(\tilde{k}z-\omega t)}, \quad \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(\tilde{k}z-\omega t)}$$

Substitution of the plane wave for E or B into the appropriate wave equation gives

$$\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$$

and

$$\tilde{k} = k + i\kappa$$

1. Compute k and κ, the real and imaginary parts of $\tilde{k}$ in terms of μ, ϵ, σ, and ω.

2. Determine κ, and the skin depth d in the limit where $\sigma/\epsilon\omega \gg 1$

3. Determine κ, and the skin depth d in the limit where $\sigma/\epsilon\omega \ll 1$

4. Determine the propagation speed $v = \omega/k$ in the limit where $\sigma/\epsilon\omega \ll 1$

5. Now we can write

$$\begin{aligned}\tilde{\mathbf{E}}(z,t) &= \tilde{E}_0 e^{-\kappa z} e^{i(kz-\omega t)} \hat{\mathbf{x}} \\ \tilde{\mathbf{B}}(z,t) &= \frac{\tilde{k}}{\omega} \tilde{E}_0 e^{-\kappa z} e^{i(kz-\omega t)} \hat{\mathbf{y}}\end{aligned}$$

In view of the above two equations and your knowledge of k and κ what is the relative phase of E and B fields in a good conductor ($\sigma/\epsilon\omega \gg 1$)?

6. Compute the intensity of an electromagnetic plane wave in a conducting medium. $I = \langle \mathbf{S} \rangle$.