P3323 Separation of variables - Spherical coordinates September 14-16, 2016

A spherical shell of radius R carries a uniform surface charge  $\sigma_0$  on the "northern" hemisphere and a uniform surface charge  $-\sigma_0$  on the "southern" hemisphere.

1. Write a general expression, as a sum of spherical harmonics, for the potential outside the sphere. What do we know about the potential at  $r \to \infty$ ?

2. Write a general expression for the potential inside the sphere. What do we know about the potential at  $r \rightarrow 0$ ?

3. What are the boundary conditions on the potential at the radius of the sphere? Is there a discontinuity in the potential? How are the expansion coefficients for the potential inside related to the coefficients for the potential outside?

4. How is the potential at r = R related to the surface charge density  $\sigma(\theta)$ ?

5. Write an integral expression for the expansion coefficients. Which coefficients are zero?

6. Calculate the l = 0 and l = 1 coefficients

7. Write an integral expression, for the dipole moment of the sphere.

8. Integrate to get the dipole moment.

9. Write an expression for the potential of a dipole and compare your result with the exact calculation of the potential of the split sphere.