P3323 Dielectric Sphere
October 3, 2016

A uniform linear dielectric sphere with relative permitivity ϵ_r and radius R is placed in an otherwise uniform electric field $\mathbf{E} = E_0\hat{\mathbf{z}}$.

1. The electric field $\mathbf{E} = E_0\hat{\mathbf{z}}$ is the gradient of what potential?

2. Write a general expression for the potential outside the sphere consistent with the boundary condition at $r \to \infty$.

3. Write a general expression for the potential inside the sphere consistent with the boundary condition at $r \to 0$.

4. What is the boundary condition at the surface of the sphere? Use it to solve for A_l (the expansion coefficients for V^{in}) in terms of B_l (the expansion coefficients for V^{out}) and E_0.

5. There is no free charge on the sphere and the surface free charge density $\sigma_f = 0$. The boundary condition for the electric displacement at the surface of the sphere is $\mathbf{D}_\perp{}^{above} - \mathbf{D}_\perp{}^{below} = \sigma_f$.

   What can you conclude about the relationship between

$$\frac{\partial V_{out}}{\partial r} \mid_R \text{ and } \frac{\partial V_{in}}{\partial r} \mid_R ?$$

6. Use the relationship between the derivative of the potentials at the surface of the sphere (that you found in part 5) to solve for B_l in terms of E_0, ϵ_r and R.

7. What is the polarization $\mathbf{P}$ inside the sphere? (Hint: Since it is a linear dielectric we know that $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$ and $\nabla \cdot \mathbf{P} = \epsilon_0 \chi \nabla \cdot \mathbf{E}$)

8. What is the surface bound charge density?