Intermediate Electricity and Magnetism

Lecture 1
August 24, 2016

1 Electricity and magnetism

1. Coulombs Law. Two (and only) two types of charge, positive and negative. Like sign repel, unlike signs attract. Force between charges depends on the charge and the separation and also velocity and acceleration and the speed of light. Coulomb's law gets electrostatics right. → Coulomb's law. Measure the force with some kind of balance. Or measure the acceleration. Interesting to think about how Coulomb did this. By measuring he determined that

$$F = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^{\,2}}\hat{\boldsymbol{\mathscr{r}}}$$

$$\boldsymbol{\mathscr{r}}_{21} = \mathbf{r}_2 - \mathbf{r}_1 = \mathbf{r}_{21}$$

   Draw picture of two charges in coordinate system and vectors

$$\mathbf{F}(q_2) = \frac{1}{4\pi\epsilon_0}\frac{q_2 q_1}{(\mathbf{r}_2 - \mathbf{r}_1)^2}\frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

   Units and dimensions $\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9$ $\mathrm{Nm}^2/\mathrm{C}^2$ Charge is conserved, global and local
   Charge is quantized. Quarks 1/3.
   Clicker, What is the x-component of the force?
   Clicker, What is the correct form for $\mathscr{r}$ *?*
   And Inverse square law. Flux of a charge distribution is equal to the charge. inverse square law implies that photon is massless. But it didn't have to be this way. Charges are at rest for the time being.

2. Superposition

$$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2}$$

$$\mathbf{F} = \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{\mathscr{r}_1^{\,2}}\hat{\boldsymbol{\mathscr{r}}}_1 + \frac{q_2}{\mathscr{r}_2^{\,2}}\hat{\boldsymbol{\mathscr{r}}}_2 + \frac{q_3}{\mathscr{r}_3^{\,2}}\hat{\boldsymbol{\mathscr{r}}}_3 + \dots\right)$$

3. Electric field E-field is force on the test charge F/Q. E-fields can be added by linear superposition. Of course we have to pay atention to direction.

4. Superposition again
   Did not have to be this way. In fact.
   Four forces - gravity, E&M, Weak, Strong,

   - Strong holds quarks together to form protons and neutrons and protons and neutrons together as nuclei, energy from the sun
   - E&M holds atoms and molecules together, chemistry, energy from fossil fuels
   - Gravity holds the solar system together.
   - Weak interactions - only way we know about neutrinos, neutron decay.

   EM fields have no charge. Strong fields have "color". Gravitational fields have energy and therefore mass and therefore are source of gravitational field. Weak fields carry weak charge. So of the four forces only E&M obeys superposition principle. (Gravity is close because it is so weak.)

5. *Clicker, What is the direction of the field at P* opposite charges
   What is the force on the charge when P is very far away?

6. *Clicker, What is the direction of the field at P* same charge
   What is the force on the charge when P is very far away?

7. Conservation of charge, global and local

8. Magnetism deals with current or moving charge. Electricity and magnetism related by Maxwell equations. Historically discovered independently and then Maxwell related them neatly.

9. What if you start with Coulomb, and then try to make a theory that is relativistically invariant? $\rightarrow$ EM. Guides the development of more complciated field theories.

Lecture 2
August 26, 2016

2 Electric field

From Griffiths: Superposition and Coulomb's law implies the force on charge Q due to charges q_i is

$$\begin{aligned}\mathbf{F} &= \frac{1}{4\pi\epsilon_0}\left(\frac{Qq_1}{\mathscr{r}_1^2}\hat{\boldsymbol{\mathscr{r}}}_1 + \frac{Qq_2}{\mathscr{r}_2^2}\hat{\boldsymbol{\mathscr{r}}}_2 + \frac{Qq_3}{\mathscr{r}_3^2}\hat{\boldsymbol{\mathscr{r}}}_3 + \dots\right)\\ &= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{\mathscr{r}_1^2}\hat{\boldsymbol{\mathscr{r}}}_1 + \frac{q_2}{\mathscr{r}_2^2}\hat{\boldsymbol{\mathscr{r}}}_2 + \frac{q_3}{\mathscr{r}_3^2}\hat{\boldsymbol{\mathscr{r}}}_3 + \dots\right)\\ &= Q\mathbf{E}\end{aligned}$$

where

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{\mathscr{r}_1^2}\hat{\boldsymbol{\mathscr{r}}}_1 + \frac{q_2}{\mathscr{r}_2^2}\hat{\boldsymbol{\mathscr{r}}}_2 + \frac{q_3}{\mathscr{r}_3^2}\hat{\boldsymbol{\mathscr{r}}}_3 + \dots\right)$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\sum_i\left(\frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}_i\right)$$

Q is the "test" charge that we can use to measure the field.

3 Coulomb's law and symmetry

clicker pentagon

4 Continuous charge distribution

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\,dq$$

where dq is the charge element. Line charge $dq \to \lambda dl$.
Surface charge $dq \to \sigma da$
Volume charge $dq \to \rho dv$
and

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\,\rho(\mathbf{r}')dv$$

5 Field of an infinitely long wire

Wire is along the x-axis. Observation point is a distance z from the wire on the z-axis. There is clearly field only in the z-direction. The component of charge λdx along the wire contributes

$$d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{x^2+z^2}\frac{z}{\sqrt{x^2+z^2}}\hat{\mathbf{z}}$$

Then

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\lambda dx}{x^2+z^2}\frac{z}{\sqrt{x^2+z^2}}\hat{\mathbf{z}} \\
&= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{zx\hat{\mathbf{z}}}{z^2\sqrt{z^2+x^2}}\right]_{-L}^{L} \\
&= \frac{2\lambda}{4\pi\epsilon_0}\frac{zL\hat{\mathbf{z}}}{z^2\sqrt{z^2+L^2}}
\end{aligned}$$

Far from the line $z \gg L$,

$$\frac{2\lambda}{4\pi\epsilon_0}\frac{L\hat{\mathbf{z}}}{z^2}$$

As $L \to \infty$,

$$E \to \frac{2\lambda}{4\pi\epsilon_0}\frac{\hat{\mathbf{z}}}{z}$$

6 Field of a spherical shell

$$\begin{aligned}
\mathbf{E} &= \frac{\sigma}{4\pi\epsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{(z-R\cos\theta)\hat{\mathbf{z}}}{[R^2+z^2-2Rz\cos\theta]^{3/2}}R^2\sin\theta d\theta d\phi \\
\mathbf{E} &= \frac{2\pi\sigma}{4\pi\epsilon_0}\int_{-1}^{1}\frac{(z-Rx)\hat{\mathbf{z}}}{[R^2+z^2-2Rzx]^{3/2}}R^2 dx \\
\mathbf{E} &= \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{(zx-R)\hat{\mathbf{z}}}{z^2[R^2+z^2-2Rzx]^{1/2}}R^2\ \Big|_{-1}^{1}
\end{aligned}$$