Intermediate Electricity and Magnetism

September 7-9, 2016

Lecture 6
September 7, 2016

1 Boundary conditions

$$\oint_S \mathbf{E}\cdot\hat{\mathbf{n}}da = \frac{Q_{end}}{\epsilon_0} = \frac{1}{\epsilon_0}\sigma A \Rightarrow E_{\perp}^{above} - E_{\perp}^{below} = \frac{\sigma}{\epsilon_0}$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = 0 \Rightarrow E_{\parallel}^{above} = E_{\parallel}^{below}$$

Summary

$$\mathbf{E}_{above} - \mathbf{E}_{below} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$$

Potential continuous at boundary. since $\int_a^b \mathbf{E}\cdot d\mathbf{l}$ across boundary $\to 0$. Gradient consistent with BC for E.

2 Conductors

Electri field is zero inside conductors. Conductors havefree charge, and if E is not zero the force on the charges will cause them to rearrange themselves until there are no remaining forces. Free charge is restricted to the surfaces.

- Free charge resides on surfaces
- The conductor is an equipotential, (since there is no internal E-field)
- The field at the surface is perpendicular to the surface. Again any parallel component would rearrange charges.
- Inside a cavity embedded in a conductor, in which there is no free charge, the electric field is zero. Field lines begin and end on positive and negative charges. Since E=0 in the conductor the contribution to the line integral in the cavity must be zero.

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3 Force on Conductor

Surface charge generates E-field. Force = qE. Consider thick conductor. Above the surface $E = \sigma/\epsilon_0$. Below $E = 0$. $\langle E \rangle = \frac{\sigma}{2\epsilon_0}$ and

$$F/A = \langle E \rangle \sigma = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2_{outside}$$

The Efield at the surface has a contribution from external field and the surface charge density. Use a nonconductor or very thin conudctor to explain.

Consider a sheet of charge in otherwise empty space. What is the force? None because efield on opposite sides is in opposite directions. Average field at the survace is zero.

What about the force on the plate of a capacitor. Better take the average field, namely

$$\langle E \rangle = \frac{1}{2}(\frac{\sigma}{\epsilon_0} + 0) = \frac{\sigma}{2\epsilon_0}$$

Can check by computing $\frac{dW}{dz}$ where z is the separation of the plates and the stored energy is

$$\frac{1}{2}\epsilon_0 E^2 Az$$

4 Capacitance

Any two conductors with total charge $\pm Q$ are equipotentials with voltage difference $V \propto Q$. The constant of proportionality is the capacitance. $C = Q/V$. Double the charge, double the voltage.

It takes some work to take positive charges from one conductor and move them to the other, namely

$$dW = dqV = dq \int \mathbf{E} \cdot d\mathbf{l}$$

Since $V = q/C$

$$dW = \frac{q}{C} dq$$

and

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}$$

The same amount that is stored in the electric field that results from the rearrangement of the charge. For a parallel plate capacitor for example.

$$E = \frac{\sigma}{\epsilon_0}$$

$$V = Ed$$

where d is the spacing between the plates Then

$$C = Q/V = \frac{A\sigma}{V} = \frac{A\epsilon_0}{d}$$