

ELECTRIC DISPLACEMENT

If you put a dielectric in an external field \mathbf{E}_{ext} , it polarizes, adding a new field, $\mathbf{E}_{\text{induced}}$ (from the bound charges).

These superpose, making a total field \mathbf{E}_{tot} .

What is the vector equation relating these three fields?

A) $\vec{\mathbf{E}}_{\text{tot}} + \vec{\mathbf{E}}_{\text{ext}} + \vec{\mathbf{E}}_{\text{induced}} = 0$

B) $\vec{\mathbf{E}}_{\text{tot}} = \vec{\mathbf{E}}_{\text{ext}} - \vec{\mathbf{E}}_{\text{induced}}$

C) $\vec{\mathbf{E}}_{\text{tot}} = \vec{\mathbf{E}}_{\text{ext}} + \vec{\mathbf{E}}_{\text{induced}}$

D) $\vec{\mathbf{E}}_{\text{tot}} = -\vec{\mathbf{E}}_{\text{ext}} + \vec{\mathbf{E}}_{\text{induced}}$

E) Something else!

We define "Electric Displacement" or "D" field: $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$.

If you put a dielectric in an external field \mathbf{E}_{ext} , it polarizes, adding a new field, $\mathbf{E}_{\text{induced}}$ (from the bound charges).

These superpose, making a total field \mathbf{E}_{tot} .

Which of these three E fields is the "E" in the formula for D above?

A) \mathbf{E}_{ext}

B) $\mathbf{E}_{\text{induced}}$

C) \mathbf{E}_{tot}

Linear Dielectric:

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$$

χ_e is the “Electric Susceptibility

(Usually small, always positive)

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χ_e is the “Electric Susceptibility”

$$\begin{aligned} \mathbf{D} &= \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi_e \mathbf{E} \\ &= \varepsilon_0 (1 + \chi_e) \mathbf{E} \\ &\equiv \varepsilon_0 \varepsilon_r \mathbf{E} \end{aligned}$$

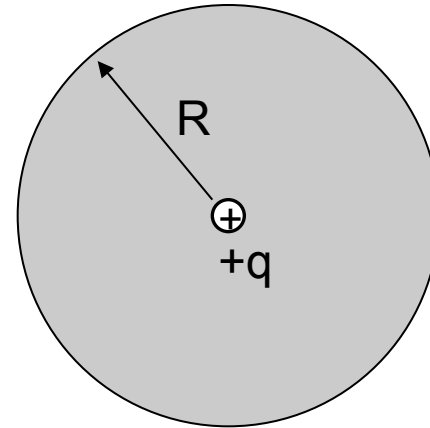
ε_r is the *dielectric constant*

$\varepsilon \equiv \varepsilon_0 \varepsilon_r$ is the *permittivity*

We define $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$, with

$$\oint \vec{\mathbf{D}} \cdot d\vec{\mathbf{a}} = Q_{\text{free, enclosed}}$$

A point charge $+q$ is placed at the center of a dielectric sphere (radius R). There are no other free charges anywhere. **What is $|\mathbf{D}(r)|$?**

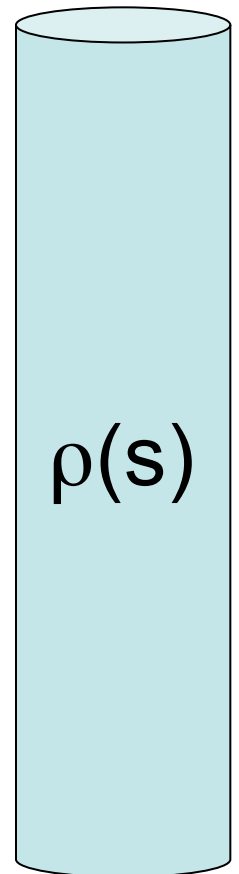


- A) $q/(4 \pi r^2)$ everywhere
- B) $q/(4 \pi \epsilon_0 r^2)$ everywhere
- C) $q/(4 \pi r^2)$ for $r < R$, but $q/(4 \pi \epsilon_0 r^2)$ for $r > R$
- D) None of the above, it's more complicated
- E) We need more info to answer!

A solid non-conducting dielectric rod has been injected ("doped") with a fixed, known charge distribution $\rho(s)$.
(The material responds, polarizing internally)

When computing D in the rod, do you treat this $\rho(s)$ as the "free charges" or "bound charges" ?

- A) "free charge"
- B) "bound charge"
- C) Neither of these - $\rho(s)$ is some combination of free and bound
- D) Something else.



A very large (effectively infinite) capacitor has charge Q .
A neutral (homogeneous) dielectric is inserted into the gap
(and of course, it will polarize).

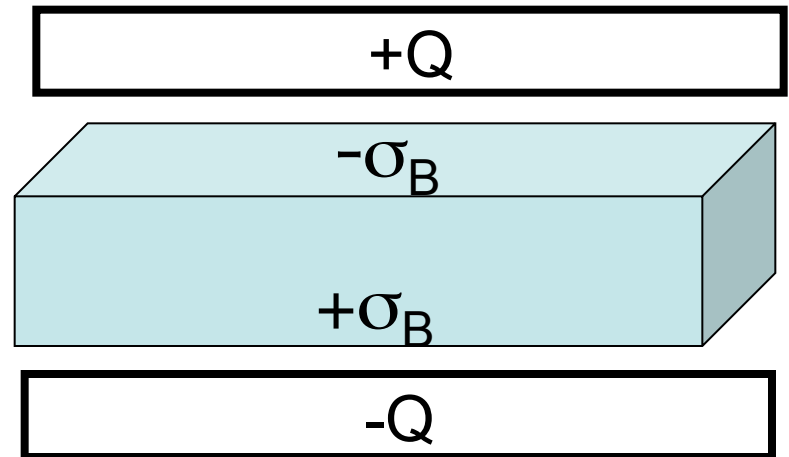
We want to find \mathbf{D} *everywhere*.

Which equation would *you* head to first?

i) $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

ii) $\oint \vec{D} \cdot d\vec{a} = Q_{\text{free}}$

iii) $\oint \vec{E} \cdot d\vec{a} = Q / \epsilon_0$



A) i B) ii C) iii

D) More than one of these would work OK.

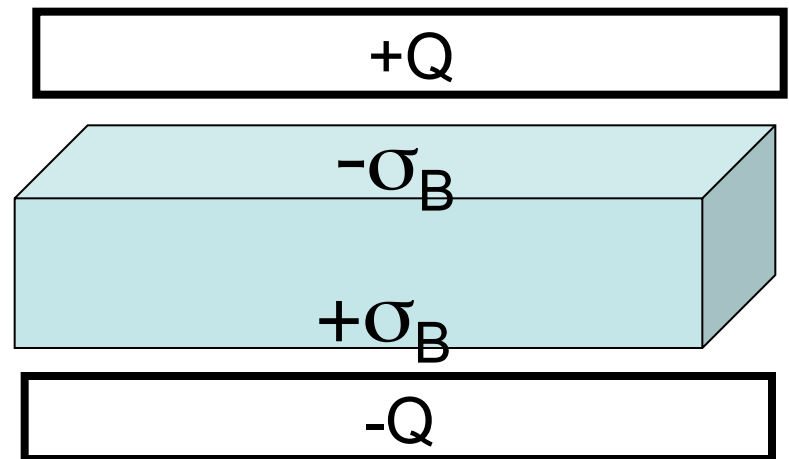
An ideal (large) capacitor has charge Q .
A neutral dielectric is inserted into the
gap (and of course, it will polarize)

We want to find \mathbf{E} *everywhere*

(i) $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$

(ii) $\oiint \vec{\mathbf{D}} \cdot d\vec{\mathbf{A}} = Q_{\text{free}}$

(iii) $\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = Q/\epsilon_0$



Which equation would *you* go to first?

A) i B) ii C) iii

D) Your call: *more* than 1 of these would work!

E) *Can't* solve, unless know the dielectric is linear!

Linear Dielectric:

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$$

χ_e is the “Electric Susceptibility”

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$= \varepsilon_0 (1 + \chi_e) \mathbf{E}$$

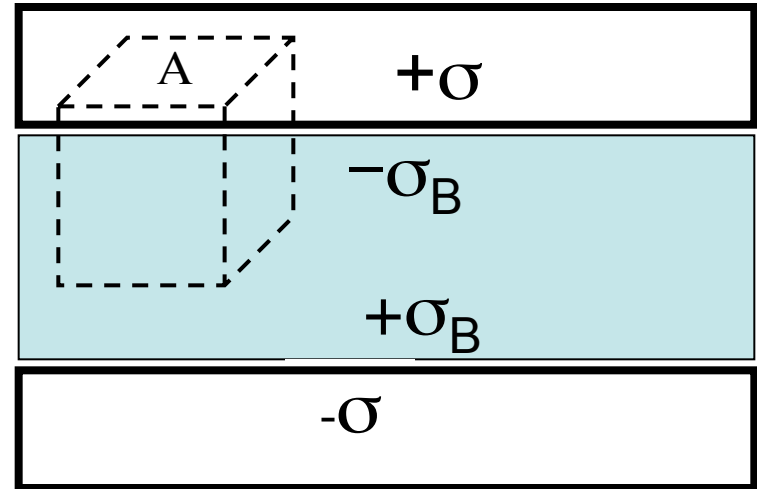
$$\equiv \varepsilon_0 \varepsilon_r \mathbf{E}$$

ε_r is the *dielectric constant*

An ideal (large) capacitor has charge Q .
A neutral linear dielectric is inserted into the gap.
We want to find \mathbf{D} in the dielectric.

$$\oiint \vec{D} \cdot d\vec{a} = Q_{free}$$

For the Gaussian pillbox shown,
what is $Q_{free,enclosed}$?

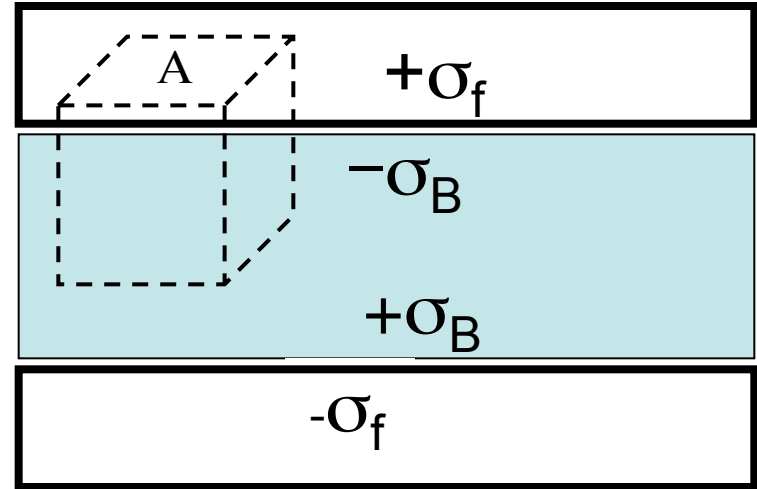


- A) σA B) $-\sigma_B A$ C) $(\sigma - \sigma_B)A$
D) $(\sigma + \sigma_B)A$ E) Something else

An ideal (large) capacitor has charge Q .
 A neutral linear dielectric is inserted into the gap.
 We want to find \mathbf{D} in the dielectric.

$$\oiint \vec{D} \cdot d\vec{a} = Q_{free}$$

For the Gaussian pillbox shown,
 what is $Q_{free,enclosed}$?



- A) $\sigma_f A$ B) $-\sigma_B A$ C) $(\sigma_f - \sigma_B)A$
 D) $(\sigma_f + \sigma_B)A$ E) Something else

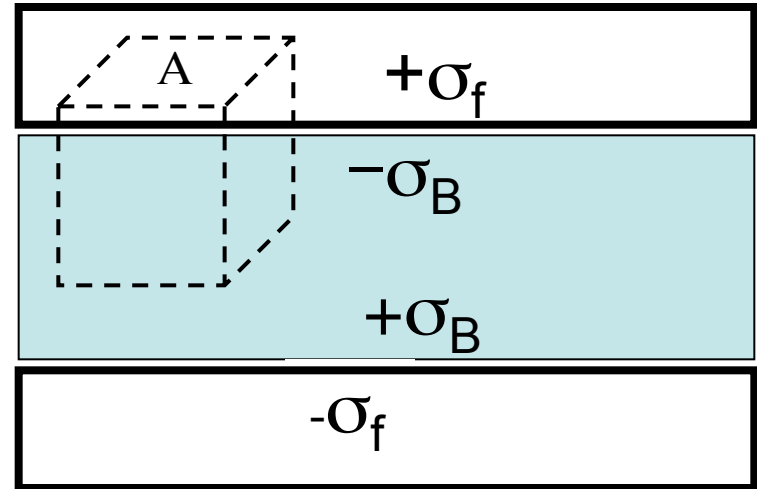
An ideal (large) capacitor has charge Q .

A neutral linear dielectric is inserted into the gap.

We want to find \mathbf{D} in the dielectric.

$$\oiint \vec{D} \cdot d\vec{a} = Q_{free}$$

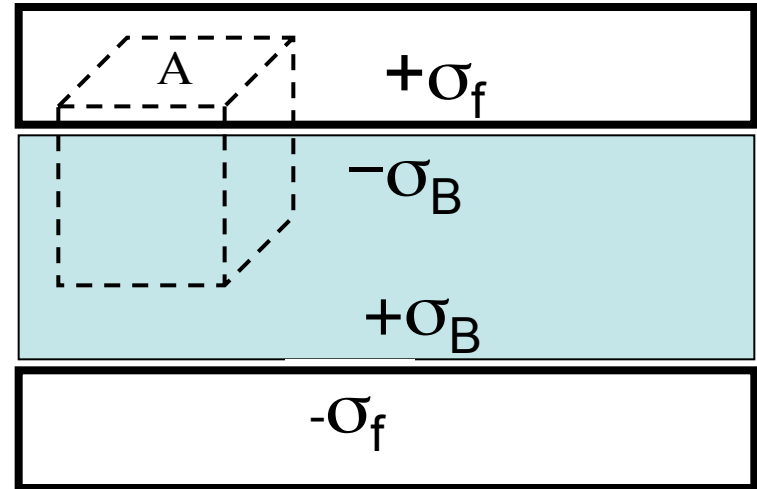
Is D zero INSIDE the metal?
(i.e. on the top face of our
cubical Gaussian surface)



- A) It must be zero in there
- B) It depends
- C) It is definitely NOT zero in there...

An ideal (large) capacitor has charge Q .
A neutral linear dielectric is inserted into the gap.
We want to find \mathbf{D} in the dielectric.

$$\oiint \vec{D} \cdot d\vec{a} = Q_{free}$$



What is $|\mathbf{D}|$ in the dielectric?

- A) σ_f B) $2\sigma_f$ C) $\sigma_f / 2$
D) $\sigma_f + \sigma_b$ E) Something else

An ideal (large) capacitor has charge Q .
A neutral linear dielectric is inserted into the gap.
Now that we have \mathbf{D} in the dielectric,
what is \mathbf{E} inside the dielectric ?

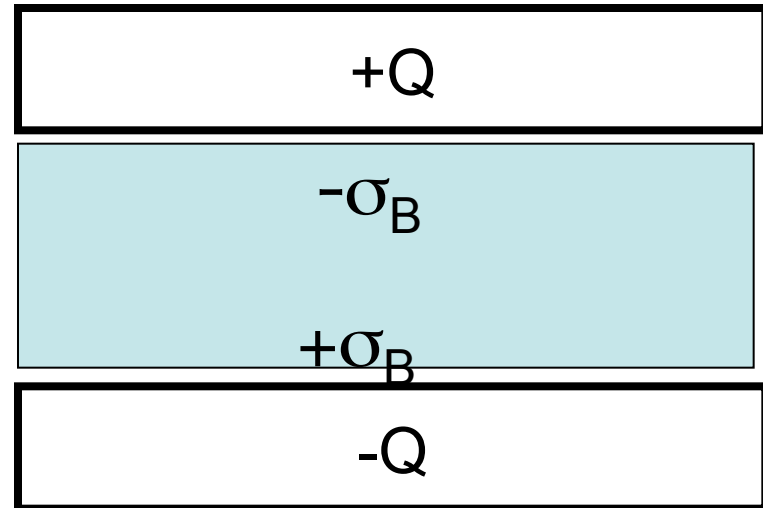
A) $\mathbf{E} = \mathbf{D} \epsilon_0 \epsilon_r$

B) $\mathbf{E} = \mathbf{D} / \epsilon_0 \epsilon_r$

C) $\mathbf{E} = \mathbf{D} \epsilon_0$

D) $\mathbf{E} = \mathbf{D} / \epsilon_0$

E) Not so simple! Need another method



Linear Dielectric:

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E}$$

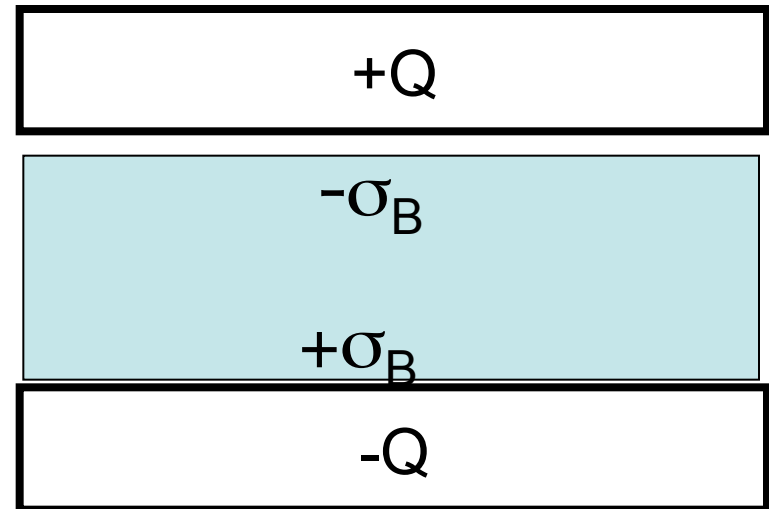
ε_r is the *dielectric constant*

An ideal (large) capacitor has charge Q .

A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)

Now that we have \mathbf{D} in the dielectric,

what is \mathbf{E} in that small gap above the dielectric ?



A) $\mathbf{E} = \mathbf{D} \epsilon_0 \epsilon_r$

B) $\mathbf{E} = \mathbf{D} / \epsilon_0 \epsilon_r$

C) $\mathbf{E} = \mathbf{D} \epsilon_0$

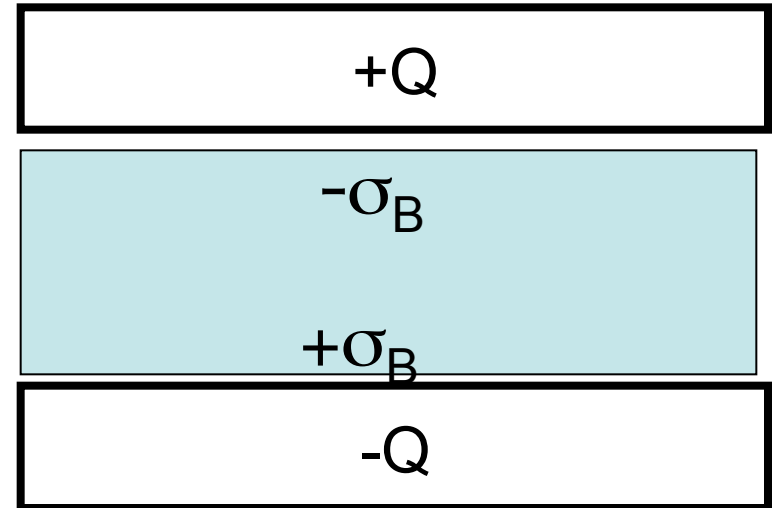
D) $\mathbf{E} = \mathbf{D} / \epsilon_0$

E) Not so simple! Need another method

An ideal (large) capacitor has charge Q .
A neutral linear dielectric is inserted into the gap
(with given dielectric constant)

Where is E discontinuous?

- i) near the free charges on the plates
- ii) near the bound charges on the dielectric surface

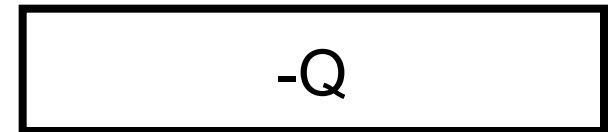
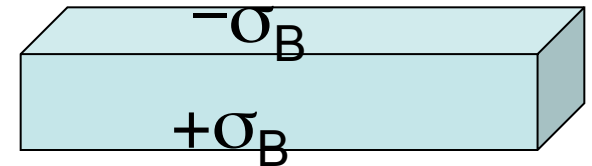
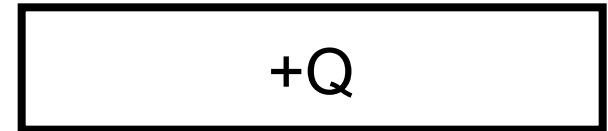


- A) i only
- B) ii only
- C) both i and ii (but nowhere else)
- D) both i and ii but also other places
- E) none of these/other/???

An ideal (large) capacitor has charge Q .
A neutral *linear* dielectric is inserted into
the gap (with given dielectric constant)

Where is E discontinuous?

- i) near the free charges
on the plates
- ii) near the bound charges
on the dielectric surface

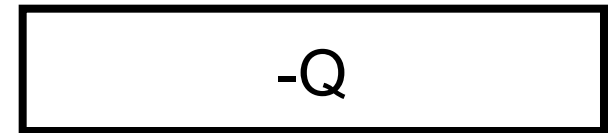
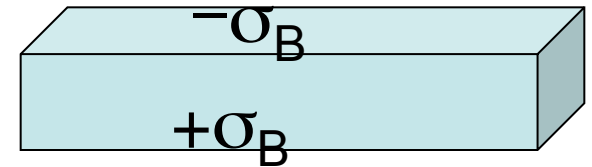
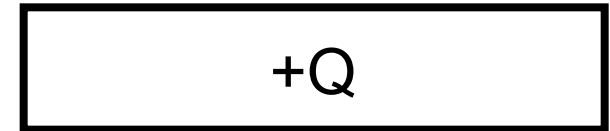


- A) i only
- B) ii only
- C) both i and ii (but nowhere else)
- D) both i and ii but also other places
- E) none of these/other/???

An ideal (large) capacitor has charge Q .
A neutral *linear* dielectric is inserted into
the gap (with given dielectric constant)

Where is D discontinuous?

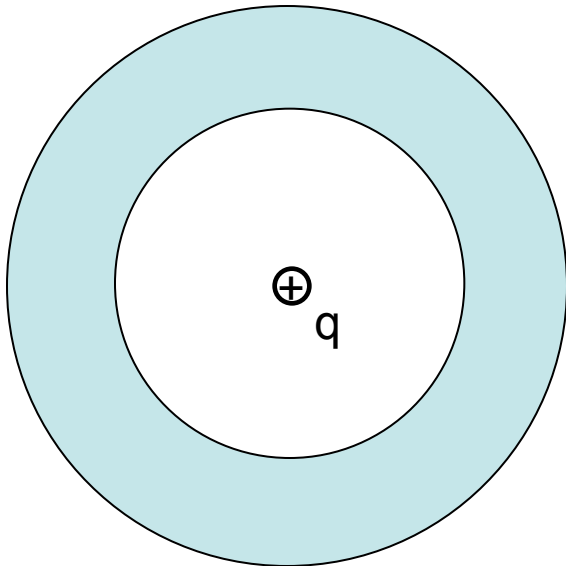
- i) near the free charges
on the plates
- ii) near the bound charges
on the dielectric surface



- A) i only
- B) ii only
- C) both i and ii (but nowhere else)
- D) both i and ii but also other places
- E) none of these/other/???

A point charge $+q$ is placed at the center of a neutral, linear, homogeneous, dielectric teflon shell. Can D be computed from its divergence?

$$\oint \vec{D} \cdot d\vec{a} = Q_{\text{free}}$$



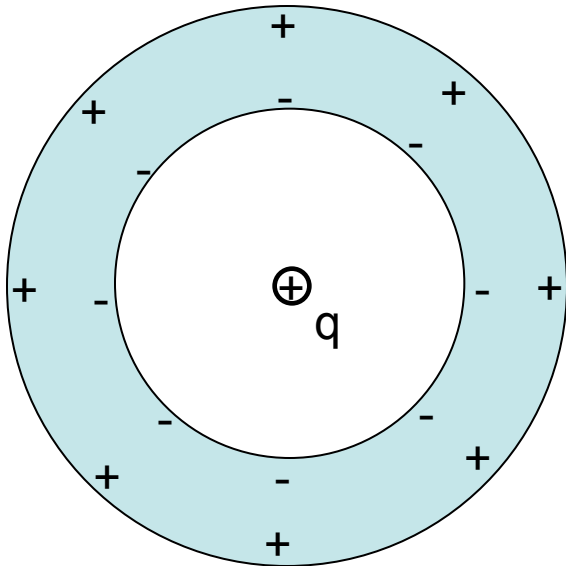
- A) Yes
- B) No
- C) Depends on other things not given

A point charge $+q$ is placed at the center of a neutral, linear, homogeneous dielectric teflon shell.

The shell polarizes due to the point charge.

Is the curl of the polarization \mathbf{P} zero everywhere?

$$\oint \vec{P} \cdot d\vec{l} = 0 \quad \text{for every possible loop?}$$



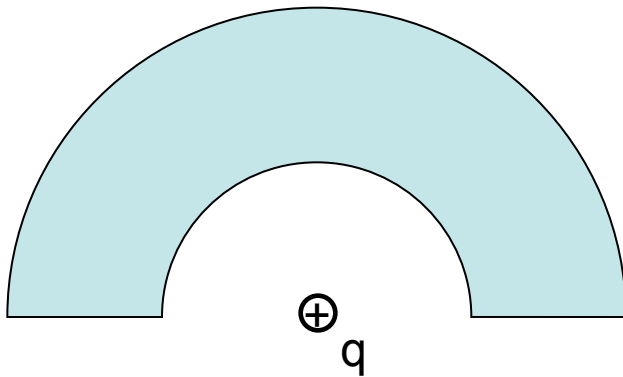
A) Yes

B) No

C) Depends on other things not given

A point charge $+q$ is placed at the center of a neutral, linear, dielectric **hemispherical** shell. Can D be computed from its divergence?

$$\oint \vec{D} \cdot d\vec{a} = Q_{\text{free}}$$

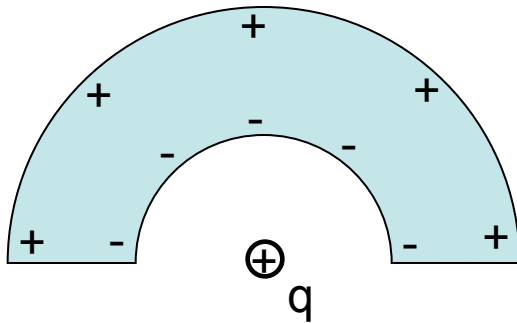


- A) Yes
- B) No
- C) Depends on the inner radius of the dielectric.

A point charge $+q$ is placed at the center of a neutral, linear, dielectric shell. The shell polarizes due to the point charge.

Is the curl of the polarization \mathbf{P} zero everywhere?

$$\oint \vec{P} \cdot d\vec{l} = 0 \quad \text{for every possible loop?}$$



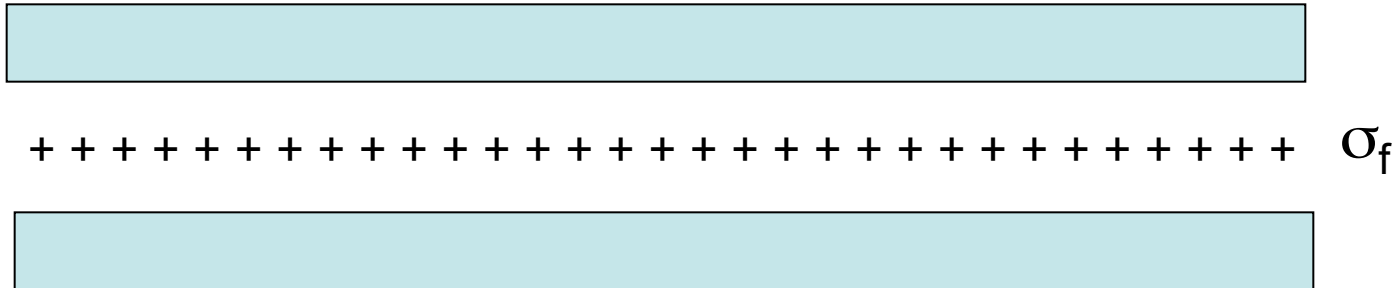
A) Yes

B) No

C) Depends on the inner radius of the dielectric.

An infinite plane of charge with surface charge density σ_f is between two infinite slabs of neutral linear dielectric (of dielectric constant ϵ_r), as shown. The "bare" E-field, due only to the plane of free charge, has magnitude $E_0 = \sigma_f / 2\epsilon_0$

$$x \quad E = ?$$



What is the magnitude of the E-field in the space above the top dielectric at the point x ?

- A) $E = E_0$ B) $E > E_0$ C) $E < E_0$

4.11

We argued that C goes UP by a factor of ϵ_r if you fill a capacitor with dielectric.

What happens to the stored energy of a capacitor if it's filled with a dielectric?

A) It goes up

B) It goes down

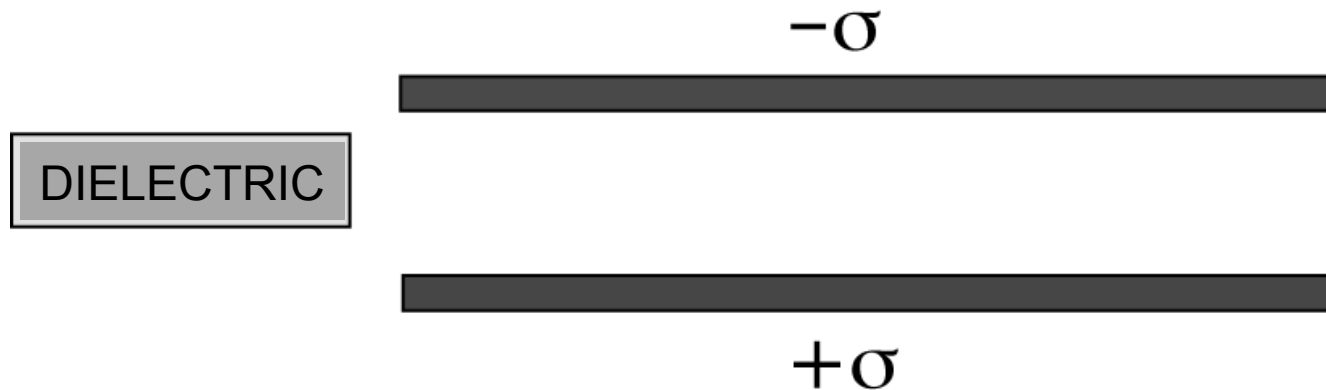
C) It is unchanged

D) The answer depends on what else is "held fixed" (V? Q?)

4.12

b

If we push this dielectric inside the *isolated* capacitor, will it be drawn into the capacitor or repelled?



- A. It gets sucked into the capacitor
- B. It gets pushed out from the capacitor
- C. I just don't know.