1. Griffiths 7.13. Find the lowest bound on the ground state of hydrogen you can get using a gaussian trial wave function

\[ \psi(r) = Ae^{-br^2}, \]

where \( A \) is determined by normalization and \( b \) is an adjustable parameter. Answer: -11.5eV.

2. Griffiths 7.14. If the photon had a nonzero mas (\( m_\gamma \neq 0 \)), the Coulomb potential would be replaced by the Yukawa potential,

\[ V(r) = -\frac{e^2}{4\pi \varepsilon_0} \frac{e^{-\mu r}}{r}, \]

where \( \mu = m_\gamma c/\hbar \). With a trial wave function of your own devising, estimate the binding energy of a "hydrogen" atom with this potential. Assume \( \mu a \ll 1 \), and give your answer correct to order \( (\mu a)^2 \).

3. Griffiths 7.15. Suppose you are given a quantum system whose Hamiltonian \( H_0 \) admits just two eigenstates, \( \psi_a \) (with energy \( E_a \)), and \( \psi_b \) (with energy \( E_b \)). They are orthogonal, normalized, and nondegenerate (assume \( E_a \) is the smaller of the two energies). Now we turn on a perturbation \( H' \), with the following matrix elements:

\[ \langle \psi_a | H' | \psi_a \rangle = \langle \psi_b | H' | \psi_b \rangle; \quad \langle \psi_a | H' | \psi_b \rangle = \langle \psi_b | H' | \psi_a \rangle = h, \]

where \( h \) is some specified constant.

(a) Find the exact eigenvalues of the perturbed Hamiltonian.

(b) Estimate the energies of the perturbed system using second-order perturbation theory.

(c) Estimate the ground state energy of the perturbed system using the variational principle, with a trial function of the form

\[ \psi = (\cos \phi)\psi_a + (\sin \phi)\psi_b, \]

where \( \phi \) is an adjustable parameter. Note: Writing the linear combination in this way is just a neat way to guarantee that \( \psi \) is normalized.
(d) Compare your answers to (a), (b), and (c). Why is the variational principle so accurate, in this case?