1. **Griffiths 11.12.** Calculate the total cross-section for scattering from a Yukawa potential, in the Born Approximation. Express your answer as a function of $E$.

2. **Scattering from a square well.** Evaluate the Born approximation to scattering of particles by the spherical square-well potential.

![Figure 1: Square well potential](image)

Show that

$$
\left( \frac{d\sigma}{d\Omega} \right) = \left( \frac{2mV_0a^3}{\hbar^2} \right)^2 \left( \frac{\sin Ka - Ka \cos Ka}{K^3a^3} \right)^2
$$

and that the low energy limit of this is

$$
\left( \frac{d\sigma}{d\Omega} \right) = \left( \frac{2mV_0a^3}{3\hbar^2} \right)^2 \left( 1 - \frac{1}{5}K^2a^2 + \ldots \right)
$$

where

$$K = 2k \sin \frac{\theta}{2}
$$

Evaluate the total cross section in the limits of low and high energies; the answers are

$$
\sigma = \pi \left( \frac{4m}{3\hbar^2} \right)^2 (V_0a^3)^2 (1 - \frac{2}{5}k^2a^2 + \ldots) \text{ low energy}
$$

and

$$
\sigma = 2\pi \left( \frac{m}{\hbar^2} \right)^2 \left( \frac{V_0a^3}{ka} \right)^2 \text{ high energy}
$$

By the way:

$$
\int \frac{(\sin x - x \cos x)^2}{x^5} dx = - \frac{1}{4} \left[ \frac{(\sin x - x \cos x)^2}{x^4} + \frac{\sin^2 x}{x^2} \right]
$$
3. Dirac Equation

The Dirac equation is

\[ i\hbar \frac{\partial \psi}{\partial t} = [c\alpha \cdot p + \beta mc^2] \psi \]

where

\[ \alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \]

and

\[ \psi(r,t) = \begin{pmatrix} \psi_1(r,t) \\ \psi_2(r,t) \end{pmatrix} \]

and \( \psi_1 \) and \( \psi_2 \) are each two component spinors. If we assume a time dependence \( \psi(r,t) = \psi(r)e^{-iEt/\hbar} \), then the Dirac equation becomes

\[ E\psi = [c\alpha \cdot p + \beta mc^2] \psi \]  

(1)

In order to include a magnetic field we replace \( p \rightarrow p - qA \).

(a) Show that in the nonrelativistic limit

\[ E_S \psi_1 = \frac{[\sigma \cdot (p - qA)][\sigma \cdot (p - qA)]}{2m} \psi_1 \]  

(2)

where \( E_S = E - mc^2 \).

(b) Prove the identity

\[ \sigma \cdot A \sigma \cdot B = A \cdot B + i\sigma \cdot A \times B \]

for any vectors \( A \) and \( B \).

(c) Show that

\[ (p - qA) \times (p - qA) = iq\hbar B \]

(d) And that Equation 2 becomes

\[ \left[ \frac{(p - qA)^2}{2m} - \mu \cdot B \right] \psi_1 = E_S \psi_1 \]

where \( \mu = \frac{q}{m}s \) and \( s = \frac{\hbar}{2}\sigma \).