1. Spin 1/2 (S&N 3.10)

   (a) Consider a pure ensemble of identically prepared spin \( \frac{1}{2} \) systems. Suppose the expectation values \( \langle S_x \rangle, \langle S_z \rangle \) and the sign of \( \langle S_y \rangle \) are known. Show how we may determine the state vector. Why is it unnecessary to know the magnitude of \( \langle S_y \rangle \)?

   (b) Consider a mixed ensemble of spin \( \frac{1}{2} \) systems. Suppose the ensemble averages \([S_x], [S_y], \text{and} [S_z]\) are known. Show how we may construct the \(2 \times 2\) density matrix that characterizes the ensemble.

2. Time Evolution (S&N 3.11)

   (a) Prove that the time evolution of the density operator \( \rho \) (in the Schrodinger picture) is given by

   \[
   \rho(t) = U(t,t_0)\rho(t_0)U^\dagger(t,t_0).
   \]

   (b) Suppose we have a pure ensemble at \( t = 0 \). Prove that it cannot evolve into a mixed ensemble as long as the time evolution is governed by the Schrodinger equation.

3. Spin 1 (S&N 3.12)

   Consider an ensemble of spin 1 systems. The density matrix is now a \(3 \times 3\) matrix. How may independent (real) parameters are needed to characterize the density matrix? What must we know in addition to \([S_x], [S_y], \text{and} [S_z]\) to characterize the ensemble completely?