The exponential processes require a more complex explanation than is provided by the usual mechanism assumed for crystal luminescence. It is suggested that the idea of electron traps might be applied to these phenomena. Energy dissipation has proved to be a useful parameter in correlation of results in different sections of the work, but for its adequate exploitation requires a knowledge of voltage and current changes in phosphors which we do not possess.

The simple hypothesis of linear voltage change in absorption accounts for some of the experimental results: in view of the complicated nature of luminescence, even so far as it is understood, such apparently simple explanations are not to be accepted too readily. There is a need for more experimental work on the lines of that recorded in these papers, and a more fundamental study of the electron-absorption process in phosphors.

ACKNOWLEDGMENTS

Our thanks are due to Mr. I. Shoenberg, of E.M.I., Ltd., for permission to publish this paper; and to Dr. W. Ehrenberg, formerly of the E.M.I. Research Laboratories, for helpful discussions, especially with regard to § 2.

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THE INTEGRAL BREADTHS OF DEBYE–SCHERRER LINES PRODUCED BY DIVERGENT X RAYS

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MS. received 18 *December* 1945

ABSTRACT. Divergence of the incident x-ray beam produces appreciable phase differences between different parts of a crystal, even in the size range for which line broadening occurs. The broadening due to this phase difference is calculated. It is ordinarily negligible, as the increase in the width of the line is of the order of $1{\cdot}6\, t \cos\theta$, where t^3 is the volume of a crystal and θ is the Bragg angle. The special case of film and source equidistant from the crystal is investigated in greater detail.

§ 1. INTRODUCTION

In the theoretical treatment of the diffraction of x rays by crystals it is usual to consider a parallel incident beam. In practice, however, the rays diverge from a source a few centimetres from the specimen. The source may be real, the focus of the x-ray tube, or effective, part of the slit system of the camera. X-ray wave-lengths are sufficiently small for the divergence to introduce appreciable phase differences between different parts of a crystal small enough to produce line broadening. It seemed desirable, therefore, to investigate the diffraction of divergent x rays by crystals of this order of size, in case the divergence should have an appreciable effect on the line broadening.

It is found that the broadening on the film is of the order of $1{\cdot}6\,t\cos\theta$, where t^3 is the volume of a crystal and θ is the Bragg angle. To this approximation it is independent of the radius of the camera and the wave-length of the x rays. The broadening is thus negligible for crystals small enough to have their sizes measured by line broadening ($\sim 10^{-5}$ cm.), and it is not practically significant for the larger crystals used to give comparison lines. Debye-Scherrer lines would still have a finite breadth, due to the non-homogeneity of the incident x rays and imperfections in the experimental arrangement, even if diffraction broadening were entirely absent. Divergence broadening of the comparison lines would be indistinguishable from broadening due to the finite diameter of the specimen, and would disappear in the elimination of the broadening due to experimental conditions by methods such as that of Jones (1938).

The mathematical treatment is greatly simplified in the special case in which the source and the film are equidistant from the specimen. The calculation of the integral breadths of reflections from spherical crystals is carried through in detail for this case, so that the transition from small-particle (diffraction) broadening to large-particle (divergence) broadening can be followed. For any particular angle of reflection the integral breadth, β, is a minimum for a particle size given by $t = 1{\cdot}00(Q\lambda)^{\frac{1}{2}}/\cos\theta$, where Q is the camera radius and λ is the wave-length of the x rays. The broadening on the film, $Q\beta$, is $1{\cdot}40(Q\lambda)^{\frac{1}{2}}$. This is too small to be detected with normal technique.

§ 2. DIFFRACTION OF DIVERGENT X RAYS

In figure 1, the source of the x rays is at O, $\mathbf{P}$ is a vector joining the source to the centre of gravity of the crystal, $\mathbf{Q}$ is a vector from $\mathbf{P}$ to the point on the circumference of the camera at which it is desired to calculate the reflected intensity, $\mathbf{r}_j$ is a vector from $\mathbf{P}$ to the jth unit cell of the crystal, and 2θ is the angle of deviation.

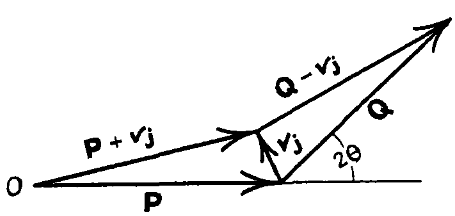

Figure 1.

The path difference between the rays diffracted from $\mathbf{P}$ and $\mathbf{P} + \mathbf{r}_j$ is

$$
\begin{aligned}
&|\mathbf{P}+\mathbf{r}_j| - |\mathbf{P}| + |\mathbf{Q}-\mathbf{r}_j| - |\mathbf{Q}| \\
&= \{(\mathbf{P}+\mathbf{r}_j)\cdot(\mathbf{P}+\mathbf{r}_j)\}^{\frac{1}{2}} - P + \{(\mathbf{Q}-\mathbf{r}_j)\cdot(\mathbf{Q}-\mathbf{r}_j)\}^{\frac{1}{2}} - Q \\
&= P\left\{\left[1+\frac{2\mathbf{P}\cdot\mathbf{r}_j}{P^2}+\frac{\mathbf{r}_j\cdot\mathbf{r}_j}{P^2}\right]^{\frac{1}{2}}-1\right\} + Q\left\{\left[1-\frac{2\mathbf{Q}\cdot\mathbf{r}_j}{Q^2}+\frac{\mathbf{r}_j\cdot\mathbf{r}_j}{Q^2}\right]^{\frac{1}{2}}-1\right\}
\end{aligned}
$$

$$=P\left\{\frac{\mathbf{P}\cdot\mathbf{r}_j}{P^2}+\frac{\mathbf{r}_j\cdot\mathbf{r}_j}{2P^2}-\frac{\mathbf{P}\cdot\mathbf{r}_j{}^2}{2P^4}+\dots\right\}+Q\left\{-\frac{\mathbf{Q}\cdot\mathbf{r}_j}{Q^2}+\frac{\mathbf{r}_j\cdot\mathbf{r}_j}{2Q^2}-\frac{\mathbf{Q}\cdot\mathbf{r}_j{}^2}{2Q^4}+\dots\right\}$$

$$=\mathbf{r}_j\cdot(\mathbf{p}-\mathbf{q})+\left(\frac{1}{2P}+\frac{1}{2Q}\right)\mathbf{r}_j\cdot\mathbf{r}_j-\mathbf{r}_j\cdot\left(\frac{\mathbf{pp}}{2P}+\frac{\mathbf{qq}}{2Q}\right)\cdot\mathbf{r}_j, \qquad \dots\dots(1)$$

where $\mathbf{p}$ and $\mathbf{q}$ are unit vectors in the direction of $\mathbf{P}$ and $\mathbf{Q}$. Let $\mathbf{p}-\mathbf{q}=\lambda(\mathbf{h}+\boldsymbol{\eta})$, where $\mathbf{h}$ is the vector of the reciprocal lattice which makes $|\boldsymbol{\eta}|$ smallest. Then $\mathbf{r}_j\cdot\mathbf{h}$ is an integer and $\mathbf{r}_j\cdot\mathbf{h}\lambda$ can be dropped from the expression for the path difference. Let

$$\sigma=\frac{1}{2P\lambda}+\frac{1}{2Q\lambda},\quad \delta=\frac{1}{2P\lambda}-\frac{1}{2Q\lambda},\quad \rho=\sigma/\delta,\ \text{and}\ \boldsymbol{\Psi}=\frac{\mathbf{pp}}{2P\lambda}+\frac{\mathbf{qq}}{2Q\lambda}, \qquad \dots\dots(2)$$

where $\boldsymbol{\Psi}$ is a dyadic. The amplitude of the reflected beam is then (the "structure factor" being omitted for brevity)

$$G=\Sigma_j\exp\{2\pi i(\mathbf{r}_j\cdot\boldsymbol{\eta}+\sigma\mathbf{r}_j\cdot\mathbf{r}_j-\mathbf{r}_j\cdot\boldsymbol{\Psi}\cdot\mathbf{r}_j)\} \qquad \dots\dots(3)$$

$$\doteq V^{-1}\int\exp\{2\pi i(\mathbf{r}\cdot\boldsymbol{\eta}+\sigma\mathbf{r}\cdot\mathbf{r}-\mathbf{r}\cdot\boldsymbol{\Psi}\cdot\mathbf{r})\}\,dv, \qquad \dots\dots(4)$$

where V is the volume of one unit cell and the integration is over the volume of the crystal. This integral may be simplified somewhat by a proper choice of axes. Let

$$\mathbf{a}=(\mathbf{p}-\mathbf{q})/2\sin\theta,\quad \mathbf{b}=(\mathbf{p}+\mathbf{q})/2\cos\theta,\quad \boldsymbol{\gamma}=\mathbf{p}\times\mathbf{q}/\sin 2\theta. \qquad \dots\dots(5)$$

It will readily be seen that $\mathbf{a}$, $\mathbf{b}$, $\boldsymbol{\gamma}$ are unit and orthogonal. Then

$$\boldsymbol{\Psi}=\sigma\sin^2\theta\mathbf{aa}+\delta\sin\theta\cos\theta\mathbf{ab}$$
$$+\delta\sin\theta\cos\theta\mathbf{ba}+\sigma\cos^2\theta\mathbf{bb}. \qquad \dots\dots(6)$$

It is desired to find unit and orthogonal axes $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ such that

$$\boldsymbol{\Psi}\cdot\boldsymbol{\alpha}=A\boldsymbol{\alpha},\quad \boldsymbol{\Psi}\cdot\boldsymbol{\beta}=B\boldsymbol{\beta}, \qquad \dots\dots(7)$$

where A and B are constants. It can be verified that

$$\left.\begin{aligned}\boldsymbol{\alpha}&=\frac{\mathbf{a}+\{|\rho\cot 2\theta|-|(\rho^2\cot^2 2\theta+1)^{\frac12}|\}\mathbf{b}}{\sqrt{2}\,|\{1-|\rho\cot 2\theta(\rho^2\cot^2 2\theta+1)^{\frac12}|+\rho^2\cot^2 2\theta\}^{\frac12}|},\\ \boldsymbol{\beta}&=\frac{\mathbf{a}+\{|\rho\cot 2\theta|+|(\rho^2\cot^2 2\theta+1)^{\frac12}|\}\mathbf{b}}{\sqrt{2}\,|\{1+|\rho\cot 2\theta(\rho^2\cot^2 2\theta+1)^{\frac12}|+\rho^2\cot^2 2\theta\}^{\frac12}|},\end{aligned}\right\} \qquad \dots\dots(8)$$

$$\left.\begin{aligned}A&=\tfrac12\{\sigma-|(\sigma^2\cos^2 2\theta+\delta^2\sin^2 2\theta)^{\frac12}|\},\\ B&=\tfrac12\{\sigma+|(\sigma^2\cos^2 2\theta+\delta^2\sin^2 2\theta)^{\frac12}|\},\end{aligned}\right\} \qquad \dots\dots(9)$$

and that $\boldsymbol{\alpha}\to\mathbf{a}$ and $\boldsymbol{\beta}\to\mathbf{b}$ as $\delta\to 0$. With axes in the directions $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$,

$$\mathbf{r}=x\boldsymbol{\alpha}+y\boldsymbol{\beta}+z\boldsymbol{\gamma},\quad \boldsymbol{\eta}=\xi\boldsymbol{\alpha}+\eta\boldsymbol{\beta}+\zeta\boldsymbol{\gamma},$$

$$G=V^{-1}\int\exp\{2\pi i[x\xi+y\eta+z\zeta+\sigma(x^2+y^2+z^2)-Ax^2-By^2]\}\,dv$$

$$=V^{-1}\int\exp\{2\pi i(x\xi+y\eta+z\zeta+Bx^2+Ay^2+\sigma z^2)\}dv \qquad \dots\dots(10)$$

$$=V^{-1}\int\exp\{2\pi i[(B^{\frac12}x+\xi/2B^{\frac12})^2-\xi^2/4B]\}dx\int\exp\{2\pi i[(A^{\frac12}y+\eta/2A^{\frac12})^2$$
$$-\eta^2/4A]\}dy\int\exp\{2\pi i[(\sigma^{\frac12}z+\zeta/2\sigma^{\frac12})^2-\zeta^2/4\sigma]\}dz. \qquad \dots\dots(11)$$

This integral is difficult tc evaluate for an arbitrary crystal shape, but for a sphere whose radius a is large compared with $B^{-\frac{1}{2}}$, $A^{-\frac{1}{2}}$, $\sigma^{-\frac{1}{2}}$, it may be evaluated approximately by the use of properties of Fresnel integrals. It is well known that

$$\left.\begin{aligned}\left|\int_{x_1}^{x_2}\exp\{2\pi i c x^2\}dx\right|^2 &\sim (2c)^{-1} \text{ for } x_1 \text{ and } x_2 \text{ of opposite sign and } x_1^2,\ x_2^2 \gg c^{-1},\\ &\sim 0 \quad \text{ for } x_1 \text{ and } x_2 \text{ of the same sign.}\end{aligned}\right\} \quad \ldots\ldots(12)$$

For a sphere, the limits of integration of the three integrals in equation (11) are $\pm(a^2-y^2-z^2)^{\frac{1}{2}}$, $\pm(a^2-z^2)^{\frac{1}{2}}$, $\pm a$ respectively. The modulus of the value of the first integral in equation (11) is, therefore, approximately $(2B)^{-\frac{1}{2}}$ for $\xi^2<4B^2\{a^2-y^2-z^2\}$ (i.e. for $y^2<\{a^2-z^2-\xi^2/4B^2\}$), 0 otherwise. Similarly the modulus of the product of the first two integrals is $(4AB)^{-\frac{1}{2}}$ for $\eta^2<4A^2\{a^2-z^2-\xi^2/4B^2\}$ (i.e. for $z^2<\{a^2-\xi^2/4B^2-\eta^2/4A^2\}$) and zero otherwise, and the modulus of the triple integral is $(8AB\sigma)^{-\frac{1}{2}}$ for $\zeta^2<4\sigma^2\{a^2-\xi^2/4B^2-\eta^2/4A^2\}$ (i.e. for $\xi^2/4B^2+\eta^2/4A^2+\zeta^2/4\sigma^2<a^2$) and zero otherwise. Then the intensity of reflection as a function of $\boldsymbol{\eta}$ is

$$\left.\begin{aligned}H=VGG^*&=\frac{1}{8AB\sigma V}, \quad \frac{\xi^2}{4B^2}+\frac{\eta^2}{4A^2}+\frac{\zeta^2}{4\sigma^2}<a^2,\\ &=0, \text{ otherwise.}\end{aligned}\right\} \quad \ldots\ldots(13)$$

In other words, the intensity of reflection is approximately constant and equal to $(8AB\sigma V)^{-1}$ within an ellipsoid of semi-axes of lengths $2Ba$, $2Aa$, $2\sigma a$ and directions $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, approximately constant and equal to zero outside. The total intensity I is equal to $(8AB\sigma V)^{-1}$ times the volume of the ellipsoid:

$$I=\tfrac{4}{3}\pi\,.\,2Ba\,.\,2Aa\,.\,2\sigma a\,.\,(8AB\sigma V)^{-1}=\tfrac{4}{3}\pi a^3V^{-1},$$

which is the total number of unit cells, as it should be. The intensity with θ between θ and $\theta+d\theta$ is proportional to the volume of the ellipsoid contained between two planes perpendicular to $\mathbf{a}$ and a distance $2\cos\theta d\theta/\lambda$ apart. The maximum intensity is, therefore (figure 2),

$$\begin{aligned}\left(\frac{dI}{d\theta}\right)_0&=\frac{2\cos\theta}{\lambda}\,.\,H\,.\,\pi\,.\,2\sigma a\,.\,T\\ &=\frac{\cos\theta}{2\lambda}\cdot\frac{\pi a T}{ABV},\end{aligned} \quad \ldots\ldots(14)$$

where T is the semi-diameter of the ellipsoid in the direction of $\mathbf{b}$. The integral breadth is therefore

$$\beta=\frac{2I}{(dI/d\theta)_0}=\frac{16\lambda a^2AB}{3T\cos\theta}. \quad \ldots\ldots(15)$$

The explicit expression for T is rather complex; it may be shown to be

$$T=a\left\{\frac{\boldsymbol{\beta}\cdot\mathbf{a}^2}{\dfrac{\boldsymbol{\alpha}\cdot\mathbf{a}^2}{4B^2}+\dfrac{\boldsymbol{\beta}\cdot\mathbf{a}^2}{4A^2}}+\frac{\boldsymbol{\alpha}\cdot\mathbf{a}^2}{\dfrac{\boldsymbol{\beta}\cdot\mathbf{a}^2}{4B^2}+\dfrac{\boldsymbol{\alpha}\cdot\mathbf{a}^2}{4A^2}}\right\}^{\frac{1}{2}}. \quad \ldots\ldots(16)$$

This simplifies considerably for $P=Q$. For most cameras this is approximately true, and the simplified expression will give some estimate of the integral breadth due to the divergence of the x-ray beam. The simplified expression is

$$T=2Aa, \quad \ldots\ldots(17)$$

and the integral breadth becomes

$$\beta=\frac{8\lambda aB}{3\cos\theta}=\frac{8a}{3}\cdot\frac{\cos\theta}{Q}=1\cdot 65t\frac{\cos\theta}{Q}. \qquad \ldots\ldots(18)$$

The actual broadening on the film is equal to the camera radius times the integral breadth, i.e. to $1\cdot 65\,t\cos\theta$. This becomes appreciable when a is a few hundredths of a millimetre.

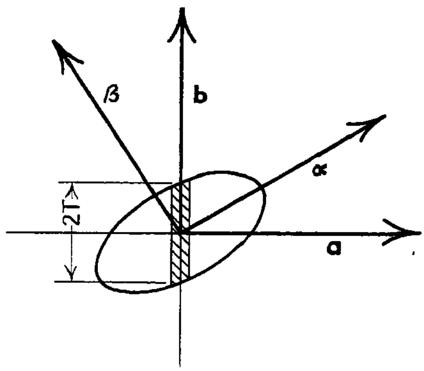

Figure 2.

The actual value of the numerical factor in equation (18) will depend on the shape of the crystal, and for non-spherical crystals on the indices of the reflection. Equation (11) can be evaluated similarly for other simple shapes; reflection from a face of a cubic crystal leads to a factor 2 instead of 1·65. Equation (18) should, however, give the order of magnitude of the effect.

§ 3. DETAILED CALCULATION FOR EQUIDISTANT SOURCE AND FILM

The whole calculation simplifies considerably for $P=Q$, so it is perhaps worth while to examine in more detail the transition from small-particle (diffraction) broadening to large-particle (divergence) broadening for a spherical particle in this special case. Equation (10) becomes

$$G=V^{-1}\int\exp\{2\pi i[x\xi+y\eta+z\zeta+\sigma(x^2\cos^2\theta+y^2\sin^2\theta+z^2)]\}dv, \qquad \ldots\ldots(19)$$

where both x and ξ are measured in the direction of $\mathbf{h}$. The intensity of reflection as a function of η is

$$H=VGG^*=V^{-1}\int_v\int_{v'}\exp\{2\pi i L\}\,dv\,dv' \text{ if}$$

$$L\equiv[\xi(x-x')+\eta(y-y')+\zeta(z-z')+(\{x^2-x'^2\}\cos^2\theta+\{y^2-y'^2\}\sin^2\theta+z^2-z'^2)] \qquad \ldots\ldots(20)$$

so that

$$\frac{dI}{d\theta}=\frac{2\cos\theta}{\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}H\,d\eta\,d\zeta$$

$$=\frac{2\cos\theta}{\lambda V}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_v\int_{v'}\exp\{2\pi i[L]\}dv\,dv'\,d\eta\,d\zeta. \qquad \ldots\ldots(21)$$

The integrals with respect to η and ζ are singular, being zero if $y' \neq y$ or $z' \neq z$, and infinite for $y' = y$ and $z' = z$, but "by an appropriate limiting process" (Patterson, 1939a, p. 973) it may be shown that the double integrals with respect to η and y', ζ and z' each have the value unity. Equation (21) becomes

$$\frac{dI}{d\theta} = \frac{2\cos\theta}{\lambda V}\iiiint \exp\{2\pi i[\xi(x-x') + \sigma(x^2 - x'^2)\cos^2\theta]\}\,dx\,dy\,dz\,dx', \quad \ldots\ldots(22)$$

$$\left(\frac{dI}{d\theta}\right)_0 = \frac{2\cos\theta}{\lambda V}\iiiint \exp\{2\pi i\sigma\cos^2\theta(x^2 - x'^2)\}dx\,dy\,dz\,dx'. \quad \ldots\ldots(23)$$

This may be expressed in terms of Fresnel integrals and cylindrical coordinates (axis in the direction of $\mathbf{h}$) as

$$\left(\frac{dI}{d\theta}\right)_0 = \frac{2\cos\theta}{\lambda V}\int_0^{2\pi}\int_0^a \left[\int_{-(a^2-r^2)^{\frac{1}{2}}}^{(a^2-r^2)^{\frac{1}{2}}} \exp\{2\pi i\sigma\cos^2\theta x^2\}dx\right]$$
$$\left[\int_{-(a^2-r^2)^{\frac{1}{2}}}^{(a^2-r^2)^{\frac{1}{2}}} \exp\{-2\pi i\sigma\cos^2\theta x'^2\}dx'\right] r\,dr\,d\phi$$
$$= \frac{4\pi}{\lambda V\sigma\cos\theta}\int_0^a [C^2(2\sigma^{\frac{1}{2}}\cos\theta\sqrt{a^2-r^2}) + S^2(2\sigma^{\frac{1}{2}}\cos\theta\sqrt{a^2-r^2})]r\,dr$$
$$= \frac{\pi}{\lambda V\sigma^2\cos^3\theta}\int_0^{2\sigma^{\frac{1}{2}}a\cos\theta} [C^2(u) + S^2(u)]u\,du, \quad \ldots\ldots(24)$$

where $$C(u) + iS(u) = \int_0^u \exp\{\pi i u^2/2\}du.$$

The integral breadth is, therefore,

$$\beta = \frac{2I}{(dI/d\theta)_0} = 2 \cdot \frac{4\pi a^3}{3V} \cdot \frac{\lambda V\sigma^2\cos^3\theta}{\pi} \cdot \left[\int_0^{2\sigma^{\frac{1}{2}}a\cos\theta} [C^2(u) + S^2(u)]u\,du\right]^{-1}$$
$$= \frac{2\lambda}{3a\cos\theta} \cdot \frac{1}{D(2\sigma^{\frac{1}{2}}a\cos\theta)} \quad \ldots\ldots(25)$$

where $$D(u) = 4u^{-4}\int_0^u [C^2(u) + S^2(u)]u\,du. \quad \ldots\ldots(26)$$

To progress further it is necessary to evaluate the function $D(u)$. Since $C^2(u) + S^2(u) \to u^2$ as $u \to 0$, its value for small values of u is 1. For large values of u, $C^2(u) + S^2(u) \to \frac{1}{2}$, so that the asymptotic value of $D(u)$ is u^{-2}. A series convenient for small values of u may be obtained as follows. It is known (Preston, 1895, p. 276) that

$$C^2(u) + S^2(u) = M^2 + N^2, \quad \ldots\ldots(27)$$

where M and N are known series satisfying the differential equations

$$\frac{dM}{du} = 1 - \pi u N, \quad \frac{dN}{du} = \pi u M \quad \ldots\ldots(28)$$

and, therefore,

$$\frac{d(M^2 + N^2)}{du} = 2M, \quad \ldots\ldots(29)$$

and

$$D(u) = 2u^{-4}\int_0^{u^2}\left\{\int_0^{u^2} Mu^{-1}d(u^2)\right\}d(u^2). \quad \ldots\ldots(30)$$

Integrating the series for M (Preston, p. 275) term by term gives

$$D(u)=1-\frac{2\pi^2u^4}{3.4.1.3.5}+\frac{2\pi^4u^8}{5.6.1.3.5.7.9}-\frac{2\pi^6u^{12}}{7.8.1.3.5.7.9.11.13}+\dots \qquad \dots\dots(31)$$

The ratio of successive terms in this series is

$$-\frac{\pi^2u^4(2n-1)(2n)}{(2n+1)(2n+2)(4n-1)(4n+1)},$$

which approaches 0 for sufficiently large n for any value of u. It is therefore absolutely convergent for all values of u. It is, however, inconvenient for u greater than 2. Table 1 gives some numerical values of $D(u)$. Those for $u<2$

Table 1. Values of $D(u)$

	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0	1·0000	0·99999	0·99982	0·99911	0·99720	0·99317	0·98590	0·97405	0·95624	0·93101
1	0·8969	0·8534	0·8000	0·7373	0·6670	0·5917	0·5151	0·4413	0·3749	0·3190
2	0·2753	0·2440	0·2226	0·2077	0·1951	0·1821	0·1669	0·1508	0·1371	0·1257
3	0·1181	0·1118	0·1062	0·0992	0·0921	0·0856	0·0811	0·0774	0·0738	0·0695
4	0·0653	0·0628	0·0595	0·0569	0·0540	0·0512	0·0491	0·0472	0·0452	0·0430
5	0·0413									

were calculated from the series, those for $u>2$ by numerical integration of four-place tables of $C(u)$ and $S(u)$. In the range 0·5–2·0 the greatest difference between the values calculated by the two methods is 0·0003; the mean difference is about 0·0001.

With these values of $D(u)$, the integral breadth for any particular case can be calculated by equation (25). There is a minimum value of β which occurs for $D+uD'=0$, i.e. for $u \doteqdot 1{\cdot}24$. The corresponding value of a is

$$1{\cdot}24/2\sigma^{\frac{1}{2}}\cos\theta \sim 40{,}000\ \text{A. for } Q=10 \text{ cm.},\quad \lambda=2\ \text{A.},\quad \theta=45^\circ.$$

The minimum value of β is $1{\cdot}40\sqrt{\lambda/Q}$; the actual broadening on the film, $Q\beta$, is $1{\cdot}40\sqrt{Q\lambda}=0{\cdot}006$ mm., an amount undetectable with normal technique.

Equation (25) and the discussion in the previous paragraph have been given in terms of the radius of the particle, a. In terms of the cube root of its volume, t, the expressions become

$$\beta=\frac{1{\cdot}0747\,\lambda}{t\cos\theta}\cdot\frac{1}{D(1{\cdot}24\sigma t^{\frac{1}{2}}\cos\theta)}, \qquad \dots\dots(32)$$

$$t=1{\cdot}00/\sigma^{\frac{1}{2}}\cos\theta \text{ for minimum } \beta.$$

The minimum values of β and $Q\beta$ are unchanged. For $t/\sqrt{Q\lambda}$ small, equation (32) becomes $\beta=1{\cdot}0747\lambda/t\cos\theta$, in agreement with the results of Patterson (1939 b) and Stokes and Wilson (1942).

ACKNOWLEDGMENTS

The writer desires to thank Dr. H. Lipson, M.A., who suggested the desirability of investigating the effect of divergence on line breadths, for his encouragement and interest throughout the work, and Dr. P. P. Ewald for helpful criticisms.

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