LDC question TR_7: Magnetic Field

What quality of the field do we need in the TPC, SIT, and other detectors?

How can we measure and monitor the field distortions at the required level of accuracy?

Can the large distortions in the large crossing angle be accounted for?

Can control samples be used to improve the knowledge of the field map?

Does it make sense to eliminate the plug, at the cost of a shorter magnet and thus a less homogeneous field?

Of course, this is all preliminary.
What quality of the field do we need in the TPC, SIT, and other detectors?

GOAL: TPC-only momentum resolution: $\delta(1/p) < 2 \times 10^{-4}$ /GeV
System momentum resolution: $\delta(1/p) < 5 \times 10^{-5}$ /GeV

An example of a solution (Dan’s simple MC)

100 $\mu$m TPC, IR=.32m, OR= 1.68m
$\delta(1/p) = 1.3 \times 10^{-4}$ /GeV

system including
10 $\mu$m VTX, 6 layers, IR= 2cm, OR= 6cm
10 $\mu$m SIT, 2 layers, 28cm, 30cm
$\delta(1/p) = 4.2 \times 10^{-5}$ /GeV

Several ways to spoil resolution

If a magnetic field non-uniformity results in degrading a detector resolution, the momentum resolution will be quickly spoiled.

160 $\mu$m TPC (TPC only) 2.1 $\times 10^{-4}$ /GeV

200 $\mu$m TPC (within system) 5.8 $\times 10^{-5}$ /GeV
40 $\mu$m VTX, 40 mm SIT 6.5
VTX and SIT, rotated 25 $\mu$m (10 $\mu$m resolution) 5.7
no SIT ( still have 10 $\mu$m VTX ) 5.3
What quality of the field do we need in the TPC, SIT, and other detectors?

There are 2 ways that the magnetic field distortions affect the track.

Distortions in the **path of the track** directly change the curvature of the track path.

    If the field non-uniformity can not be ignored, a transport fitting method must be used to take the local field into account.

Distortions in the **drift path** affect the point of ionization collection on the endplate.

    If the field non-uniformity can not be ignored, the drift trajectory can be corrected either with a mapping or using a transport method through the field.
What quality of the field do we need in the TPC, SIT, and other detectors?

From the figure to the right, the sagitta is,

\[ s = \frac{L^2}{8R}, \text{ where } R \text{ is the radius of curvature of the helix.} \]

The momentum of a track is related to \( R \);

\[ P_t = R B \times (0.3 \text{ GeV})/(m \text{ Tesla}) \]

Thus, substituting for \( 1/R \) in the first equation,

\[ P_t = (1/s) \times (L^2 B) \times (0.3/(8)(\text{GeV})/(m \text{ Tesla})) \]

Then the momentum resolution limit becomes

\[ \delta(1/P_t) = \frac{\delta s}{(L^2 B) (0.3/(8)(\text{GeV})/(m \text{ Tesla}))} < 5 \times 10^{-5} /\text{GeV} \]

With \( L=1.6 \text{m}, B=4 \text{ Tesla}, \)

\[ \delta s < 19 \times 10^{-6} \text{ m} \]

The above sagitta corresponds to the resolution limit.

The sagitta corresponding to a distortion that results in a 5% increase in the resolution when added in quadrature is

\[ \delta s < \left[ \left( (1.05)^2 - 1 \right)^{1/2} = 0.32 \right] \times (19 \times 10^{-6} \text{ m}) = 6 \times 10^{-6} \text{ m} \]
What quality of the field do we need in the TPC, SIT, and other detectors?

The effect of a radial field on the drift trajectory will be an effective increase in the length of the track, while maintaining constant sagitta. The resolution limit can be expressed in terms of $\delta L$.

From the previous page, the momentum of a track is related to $R$;  
\[
P_t = R B \times (.3 \text{ GeV})/(\text{m Tesla}) \quad \text{or} \quad (1/P_t) = 1/R \times 1/(B \times (.3 \text{ GeV})/(\text{m Tesla}))
\]

Also, \(1/R = 8s/L^2\)

Then, 
\[
\delta(1/P_t) = \delta(8s/L^2) \times \frac{1}{(B \times (.3 \text{ GeV})/(\text{m Tesla}))}
\]

\[
= 2 (8s/L^2) \frac{\delta L/L}{1/(B \times (.3 \text{ GeV})/(\text{m Tesla}))}
\]

\[
\delta(1/P_t) = 2/P_t \frac{\delta L/L}{< 5 \times 10^{-5} /\text{GeV}}
\]

Again, the limit must be reduced by \([( (1.05)^2 -1 )^{1/2} = .32 ]\) to correspond to a distortion that results in a 5% increase in the resolution when added in quadrature.

\[
\delta(1/P_t) = 2/P_t \frac{\delta L/L}{< 1.6 \times 10^{-5} /\text{GeV}} \quad \text{or}
\]

\[
\delta L < L P_t \times (0.8 \times 10^{-5} /\text{GeV})
\]
What quality of the field do we need in the TPC, SIT, and other detectors?

The limit, $\delta s < 6 \times 10^{-6} \text{ m}$ (or the limit on $\delta L$) is derived from the system resolution and corresponds to a distortion that would result in a 5% increase in the resolution.

The $6 \, \mu\text{m}$ sagitta limit is relevant for magnetic distortions in the VTX and SIT.

In the TPC, it may be more relevant to use the TPC-only resolution requirement. Thus, requiring $\delta(1/p) < 2 \times 10^{-4} /\text{GeV}$ would result a sagitta limit, $\delta s < 24 \, \mu\text{m}$.

However, to allow aligning the silicon to the TPC, using TPC-only tracks, without introducing systematic errors, the sagitta due to magnetic distortions must be limited to $6 \, \mu\text{m}$. 

What quality of the field do we need in the TPC, SIT, and other detectors?

Consider the case of magnetic field distortions in the drift path, in particular, the effect of a $B_\phi$ with the following characteristics.
- $B_\phi = 0$ at $z = 0$,
- and increases linearly with $z$.
- $B_\phi$ is maximum at $R = 0$,
- and decreases linearly to zero at mid-radius.

There will be a kink observed in the track.

Ionization follows the magnetic field lines resulting in a displacement of $B_{\phi,\text{max}}/B_z \times l_{\text{drift}}$. The sagitta is ½ of the motion. The arm is ½ of the TPC radius. The fitted sagitta will be about ½ of that shown. If $B_{\phi,\text{max}} = 1\%$, the systematic error in the fitted sagitta is $(1/8) (1\%) (2 \text{ m}) = 2.8 \text{mm}$ which is huge!
What quality of the field do we need in the TPC, SIT, and other detectors?

The field uniformity requirement: a $B_\phi$ distortion

A magnetic field distortion in the drift path, $\delta B_\phi$, with magnitude 1% and particular $z$ and $R$ dependence, creates an error in sagitta of $\Delta s = 2.8 \times 10^{-3} \text{ m}$. The sagitta limit is $\delta s < 6 \times 10^{-6} \text{ m}$.

Thus the field uniformity requirement is

$$\frac{\delta B}{B} < 1\% \left( \frac{6 \times 10^{-6}}{2.8 \times 10^{-3}} \right) = 2 \times 10^{-5}.$$
What quality of the field do we need in the TPC, SIT, and other detectors?

Consider another case of magnetic field distortions in the drift path, $B_r$ with the characteristics,

- $B_r = 0$ at $z=0$,
- and increases linearly with $z$.
- $B_r = 0$ at $R=0$
- and increases linearly with $R$
- with a maximum $B_{r,max}$ at $z=l_{TPC}$ and $R=R_{TPC}$.

In this case, the sagitta is constant; the cord, $L$, increases by $\delta L = B_{r,max}/B_z \times l_{drift}$.

For $B_{r,max}/B_z = .01$, $l_{drift} = 2m$, $\delta L = 2$ cm

I will calculate $\delta(1/P_t) = 2 \ (1/P_t) \ \delta L/L$

with $P_t = 10 \text{ GeV}$;

at lower momenta multiple scattering dominates.

For $L = 1.7m$, $\delta(1/P_t) = 2 \ (1/P_t) \ \delta L/L = .0024/\text{GeV}$.
What quality of the field do we need in the TPC, SIT, and other detectors?

The field uniformity requirement: a radial distortion

A magnetic field distortion in the drift path, $\delta B_r$, with magnitude 1% and maximum magnitude at maximum $z$ and $R$, creates a shift in $P_t$, $\Delta P_t = 2.4 \times 10^{-3} /\text{GeV}$.

The limit is $\delta(1/P_t) < 1.6 \times 10^{-5} /\text{GeV}$.

(The contribution that results in a 5% increase in $\delta(1/P_t)$)

Thus the field uniformity requirement is then

$$\delta B/B < 1\% \quad (1.6 \times 10^{-5}) / (2.4 \times 10^{-3}) = 7 \times 10^{-5}.$$
What quality of the field do we need in the TPC, SIT, and other detectors?

**The field uniformity requirement, Ron Settles**

\( \sigma_0 \) is defined as the residual added fluctuation of the \( r\phi \) measurements on the endplate and limited such that the resolution increase is 5%. The value can be simply derived as

\[
\left( \frac{1.05}{1} \right)^{1/2} \times 100 \mu m = 30 \mu m.
\]

\( \sigma_0 \) is related to the residual magnetic uncertainty by

\[ \sigma_0 = \delta B_{r,\phi} / B_z \times l_{drift}. \]

Thus, for \( l_{drift} = 2m \), the magnetic field uncertainty is limited by

\[ \delta B_{r,\phi} / B_z < \sigma_0 / l_{drift} = 1.5 \times 10^{-5}. \]

The above limit is based on the tpc-only resolution; the limit based on the system resolution is

\[ \delta B_{r,\phi} / B_z < \sigma_0 / l_{drift} = 0.4 \times 10^{-5}. \]
What quality of the field do we need in the TPC, SIT, and other detectors?

The field uniformity requirement

from a study of a particular systematic $B_\phi$, $\delta B/B_z < 2 \times 10^{-5}$

from a study of a particular systematic $B_r$, $\delta B/B_z < 7 \times 10^{-5}$

from residual measurement fluctuations, $\delta B/B_z < .4 \times 10^{-5}$ (system resolution)

The $B_r$ result indicates that the tracking system is less sensitive to this model of a distortion.

The results from the $B_\phi$ and from the measurement fluctuations are different conditions.

A field uniformity of $1 \times 10^{-5}$ would probably be sufficient (without mapping).

A field uniformity of $1 \times 10^{-5}$ probably can not be achieved. Systematic uncertainties must be limited to this value.

(And, that brings us to the next question.)

All track trajectories must be corrected by a transport fitter.

All drift trajectories must be corrected with a map.
How can we measure and monitor the field distortions at the required level of accuracy?

Mapping the magnetic field to an accuracy of $\delta B/B < 1 \times 10^{-5}$ is a difficult measurement.

The Aleph field map was internally self consistent to $40 \times 10^{-5}$.

The map measurements are fit to conform with Maxwell’s equations.
Differences of the corrected map, with respect to a model of the magnet, are within $40 \times 10^{-5}$.

However, the “consistency” is not a direct measure of the accuracy.

The observed Aleph momentum resolution implies that the field map has an accuracy of $\sigma_0=70\mu$m.
Thus, a magnetic field uncertainty achieved was

$$\delta B/B = 70\text{mm}/30\text{mm} \times (1.5 \times 10^{-5}) = 3.5 \times 10^{-5}$$

We must measure the field map to the best possible accuracy, probably $3.5 \times 10^{-5}$.
We will require an independent measurement of the field distortions to achieve the required accuracy, $1 \times 10^{-5}$. 
How can we measure and monitor the field distortions at the required level of accuracy?

It would be convenient to measure the magnetic field distortions with the tracks.
It would also be convenient to align the endplate sub-panels with the tracks.
Due to correlations, we probably can not do both.

There are competing distortions that must be eliminated, or solved simultaneously:

- magnetic field distortions affecting the track trajectory,
- magnetic field distortions affecting the drift trajectory,
- alignment of the TPC to the silicon detectors,
- alignment of the sub-panels of the readout of the TPC.

(In a drift chamber, the problem of magnetic field distortions of the drift trajectory is replaced by the problem of simultaneously solving the details of the drift function and $t_0$.)

Some competing distortions must be measured, with $12\mu$m (.0005 inch) certainty, independent of the tracks.
How can we measure and monitor the field distortions at the required level of accuracy?

Some competing distortions must be measured, with 12µm certainty, independent of the tracks.

The sub-panels must be aligned to 12µm (roughly half of the sagitta uncertainty limit) to separate distortion of the readout from distortion of the drift path in the magnetic field.

To provide alignment of the sub-panels of the TPC readout, the frame must have reference edges defined to 12µm.

Coordinate-Measuring-Machines allow (.0005 inch) 12.7µm over 1m, (.002 inch) 50µm over 4m.

We will be required to find a way to achieve the better resolution over the larger object.
How can we measure and monitor the field distortions at the required level of accuracy?

Extra slide showing CLEO measurements
How can we measure and monitor the field distortions at the required level of accuracy?

**Magnetic field distortions affecting the drift trajectory**

With proper quality control and measurement during manufacture, the TPC readout will be accurate enough to measure the magnetic field distortions. It will require a set of laser beams. (Not all experience shows they are useful.)

Figures show a possible configuration. Laser beams cross the chamber in a way to provide radial and azimuthal distortion measurements at several longitudinal positions.

**Magnetic field distortions affecting the track trajectory**

The field map can be derived from the drift map. Consistency can be made with tracks.

**Magnetic field measurements**

The field should be mapped with an accuracy of $1 \times 10^{-4}$, with all iron and compensation magnets in place, for a consistency check of the laser measurements.
Can control samples be used to improve the knowledge of the field map

The CLEO tracking group uses $e^+e^-$ and $\mu^+\mu^-$ events, $e^+e^-$ when we need higher statistics, $\mu^+\mu^-$ when we are concerned about the charge asymmetry.

CLEO uses a constrained fit of the two tracks to decouple the tracks from sub-detector mis-alignment. Two tracks, when not constrained, are described by 10 parameters. We require:
- meet at a point (2 constraints),
- back-to-back in the center-of-mass (2 constraints),
- equal momentum in the center-of-mass (1 constraint).

The two-track fit has a moment arm of 3.2m, compared to 1.3m in the TPC alone.

Hypothetically, the resolution of a two-track fit, without the VTX or SIT, is $\delta(1/p_t) = 4 \times 10^{-5}/\text{GeV}$, and competitive with single track system resolution.

At LDC, we can also use the tracks to measure magnetic field correction in the drift trajectory if the track trajectory is in a region of high-uniformity magnetic field. (It may be necessary to use only track trajectories near $z=0$.)

The two-track fits can also be used to align the VTX and SIT.

See slide 23 for added comments on the field homogeneity requirements near $z=0$. 
Can the large distortions in the large crossing angle be accounted for?

This is a 2% distortion.

The magnetic field of the “DID” must be understood at the level of

\[ 1 \times 10^{-5} / 0.02 = 5 \times 10^{-4} \] (of itself)

The “DID” violates the requirement for high field uniformity within \( |z| < 50 \text{cm} \).

The “DID” field is 0.02 Tesla, \( 5 \times 10^{-3} \times B_z \).

See slide 23 for added comments on the homogeneity requirements near \( z=0 \).

What about the anti-solenoid? Beyond 4m?
Can the large distortions in the large crossing angle be accounted for?

The main magnetic field must be measured. One can not rely on finite-element-analysis as the only measurement when there is saturated iron.

\[ \sim 0.5 \times 10^{-4} \text{ variation} \]
Does it make sense to eliminate the plug, at the cost of a shorter magnet and thus a less homogeneous field?

This changes $B_z$ by only 5%, at the maximum $Z$. The $B_r$ component of $< 1/4 \%$. It is all azimuthal independent.

Does it violate the uniformity of $< 10^{-4}$ close to $z=0$, as will be needed for the control sample tracks?

See slide 23 for added comments on homogeneity requirements near $z=0$.

Field at 3.8m, length of TPC plus a distance equal to the length of the plug.
Conclusions

Field quality, or uniformity, of $1 \times 10^{-5}$ cannot be achieved.

Systematic uncertainties must be limited to this precision to avoid introducing systematic error into the VTX and SIT alignment.

All track trajectories and drift trajectories must be corrected by mapping or a transport fit.

The magnetic field should be measured to an accuracy of $1 \times 10^{-5}$ with all compensation magnets and iron in place.

One cannot rely on finite element analysis for the measurement of the solenoid field.

To provide an independent measure of the drift trajectory distortion, locate the readout sub-panels to $12 \mu m$. Use lasers to measure the distortion of the drift trajectory.

Hypothetically, the resolution of a two-track fit, without the VTX or SIT, is $\delta(1/p_t) = 4 \times 10^{-5}/\text{GeV}$, and is competitive with single track system resolution.

Two-track fits can be used for consistency checks of the drift trajectory distortion and for aligning the VRX and SIT.

The magnetic field of the “DID” must be understood at the level of $5 \times 10^{-4}$ (of itself). However, the “DID” contributes significant non-homogeneity near $z=0$.

Removing the plug appears to be a smaller perturbation than the “DID”. But a field uniformity of $10^{-4}$ is required near $z=0$ for the control sample measurements.

See slide 23 for added comments on homogeneity requirements near $z=0$. 
Can control samples be used to improve the knowledge of the field map

Field homogeneity required near $z = 0$, $|z| < 50\text{cm}$

Field homogeneity, not only knowledge, is required near $|z|=0$
to decouple the distortions of the track trajectory from the distortions of the drift trajectory.

The requirements are less severe because the track trajectory is bent by the field while the drift trajectory follows the field lines.

With a momentum resolution goal: $\delta(1/p_t) < 5 \times 10^{-5}/\text{GeV}$, the magnetic field distortion which would yield a contribution equal to the resolution limit is

$$\delta B/B = \delta p_t/p_t = 5 \times 10^{-5} / \text{GeV} \times p_t.$$ 

This can be evaluated at the Z pole, $p_t = E_{\text{beam}} = 45 \text{ GeV}$.

We have specified the limit of the uncertainty of the magnetic field to correspond to a distortion that results in a 5% increase in the resolution when added in quadrature. The resolution increase due to the field homogeneity will add to this and should be limited 2%, when added in quadrature; $\sqrt{(1.02)^2 - 1} = 0.2$.

Thus, the required field homogeneity is:

$$\delta B/B = 5 \times 10^{-5} / \text{GeV} \times 45 \text{ GeV} \times 0.2 = 4.5 \times 10^{-4}.$$ 

The required field homogeneity is nearly equal to the stated Aleph field map internal self consistency, $4 \times 10^{-4}$. While the required field homogeneity is an order of magnitude more than the stated Aleph magnetic field uncertainty, $3.5 \times 10^{-5}$, the latter is derived from tracks and cannot be used as input to a track-based field measurement.
Can the large distortions in the large crossing angle be accounted for?

This slide was removed from the 22-Aug-2005 presentation, for the conclusion, 25-Aug-2005.

CLEO III (c) has a 1.5 (1.0) Tesla solenoid field with superconducting (and permanent) quadrupole magnets intruding into the tracking volume.

edge of larger green area is 95% contours are 0.5 % changes
Can the large distortions in the large crossing angle be accounted for?

CLEO uses a calculated magnetic field map of the contribution from the quadrupole to correct for the quadrupole distortions.

The Kalman fit is a transport method and, therefore, inherently allows application of a magnetic field map.

The magnetic field is distorted by the fringe field of the final focus quadrupoles causing a 2-cycle momentum dependence.

This is corrected in the Kalman fit.

(The residual 1-cycle momentum dependence is due to the 2.5 mr crossing angle)