LECTURE 18

Beam loss and beam emittance growth

Mechanisms for emittance growth and beam loss

Beam lifetime:

from residual gas interactions; Touschek effect; quantum lifetimes in electron machines; Beam lifetime due to beam-beam collisions

Emittance growth:

from residual gas interactions; intrabeam scattering; random noise sources

Mechanisms for beam loss

1. Scattering from atoms of the residual gas in the beam vacuum chamber

Large angle Coulomb scattering-can cause beam loss if scattered particle hits an aperture

Bremsstrahlung: (electrons only) Large radiative energy losses

Inelastic nuclear scattering (protons only): Beam loss through nuclear reactions

2. Scattering of one particle by another particle in the bunch:

Coulomb scattering of one particle by another particle in the bunch is called *intrabeam scattering*. Both the angle and the energy of both particles can change in this process. If the energy change is

large enough, the particles may find themselves outside the energy aperture and be lost. This type of loss is called the *Touschek effect*.

3. (Electron machines only): The quantum fluctuations due to photon radiation can cause a particle to exceed the energy aperture or physical aperture of the machine. The resulting lifetime is called the “quantum lifetime”.

4. Beam loss at the interaction point in colliders

Electron-positron colliders: beam loss occurs through radiative Bhabha scattering ($e^+ + e^- \to e^+ + e^- + \gamma$), in which the energy of one of the electrons falls outside the energy aperture.

Hadron colliders: beam loss occurs through inelastic reactions

$$p + p(\bar{p}) \; -> \; hadrons$$

Mechanisms for emittance growth

1. Scattering from atoms of the residual gas in the beam vacuum chamber

Elastic Coulomb scattering: Random, small angle scattering (multiple Coulomb scattering) causes transverse emittance growth

2. Small angle intrabeam scattering of one particle by another particle in the bunch:

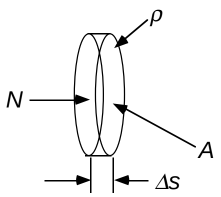
Small angle scattering equalizes the beam temperature in all dimensions: it causes a transfer of emittance from one dimension to another. Above transition, the emittance in all three degrees of freedom can grow.

3. Emittance growth from random noise sources:

Random power supply noise, and ground motion, can cause transverse emittance growth

Beam loss from residual gas interactions

We start by reviewing the concept of "cross section". The cross section for a particular reaction between two particles is the effective area which one particle presents to the other. Consider the volume element shown below, of infinitesimal length Δs, area *A*, containing a gas with atomic number density *n*. In this volume, there are $\Delta N_0 = nA\Delta s$ atoms, which have a reaction cross section σ with the incident beam particles.



The probability of a reaction is

$$\Delta P = \Delta N_0 \frac{\sigma}{A} = n\sigma\Delta s$$

so the change in the number of beam particles is

$$\Delta N = -N\Delta P = -Nn\sigma\Delta s \Rightarrow \frac{dN}{ds} = -Nn\sigma$$

If the beam is relativistic, then $ds = cdt$ and the change in N with time is

$$\frac{dN}{dt} = -Nnc\sigma = -\frac{N}{\tau}$$

This gives the equation for the beam lifetime $\tau = \frac{1}{nc\sigma}$ associated with beam loss when passing through the gas. The physics of the interaction which causes the loss of beam is contained in the cross section σ. We now consider the cross sections for the important beam loss mechanisms. The equations for these cross sections, and for many of the other formulae quoted in this lecture, have been taken from "Handbook of Accelerator Physics and Engineering", A.Chao and M. Tigner, eds, World Scientific (1999). The course web page has a link to the Handbook web page.

Coulomb scattering

The differential cross section for Coulomb scattering of a relativistic charge *e*, from a material whose nuclei have charge *Ze*, for small angles, is

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{2\pi\theta d\theta} = \frac{4Z^2 r_0^2}{\left(\theta^2 + \theta_1^2\right)^2}\left(\frac{m_e c}{p}\right)^2$$

in which θ is the polar angle. ("Handbook", p. 213) This is just the Rutherford scattering formula. The angle $\theta_1 = \alpha Z^{1/3}\left(\frac{m_e c}{p}\right)$, α=1/137, accounts for electron screening at small angles.

If a particle is scattered into a polar angle θ, at a point where the lattice function is β, then the maximum excursion of the resulting betatron oscillation is $z_{\max} = \theta\sqrt{\beta\beta_{\max}}/\sqrt{2}$ (for either plane). The particle will be lost if $z_{\max} > b$, where b is the radius of the vacuum chamber at β_{max}. So for all angles $\theta > \theta_{\min} = \frac{\sqrt{2}b}{\sqrt{\beta\beta_{\max}}}$, the particle will be lost. The cross section for loss of a particle due to a large angle Coulomb scatter, averaged around the ring, is

$$\sigma_{Coulomb} = 2\pi \int_{\theta_{\min}}^{\infty} \frac{d\sigma}{d\Omega}\theta d\theta = \frac{4\pi Z^2 r_0^2}{\theta_{\min}^2}\left(\frac{m_e c}{p}\right)^2 = \frac{\beta_{\max}\langle\beta\rangle}{b^2} 2\pi Z^2 r_0^2 \left(\frac{m_e c}{p}\right)^2$$

for the typical case of $\theta_1 << \theta_{\min}$.

Example: Z=7 (nitrogen gas-not a typical gas in an accelerator vacuum system, but we'll use it in this example). Take a vacuum chamber with radius b=30 mm, and a machine with β_{max}=30 m, $\langle\beta\rangle = 15$ m, and p=5 GeV/c. Then we find $\sigma_{Coulomb} = 0.13$ barn, where 1 barn=10^{-24} cm^2.

Bremsstrahlung (electrons only)

The differential cross section for bremsstrahlung in the nuclear field of a charge Z is

$$\frac{d\sigma}{du} \approx \frac{16\alpha r_0^2}{3} Z(Z+1)\ln\left(\frac{184}{Z^{1/3}}\right)\frac{1-u+0.75u^2}{u}$$

in which $u = \frac{\Delta E}{E}$ is the relative energy lost to radiation. ("Handbook", p 213). If the energy aperture of the machine is ΔE_a, then the cross section for particle loss due to bremsstrahlung is

$$\sigma_{Brem} = \int_{u_a}^{1} \frac{d\sigma}{du} du \approx \frac{16\alpha r_0^2}{3} Z(Z+1)\ln\left(\frac{184}{Z^{1/3}}\right)\left[\ln\frac{1}{u_a} - \frac{5}{8}\right]$$

for $u_a = \frac{\Delta E_a}{E}$ <<1.

Example: Z=7 and u_a=0.003. We find $\sigma_{Brem} = 4$ barn

Nuclear scattering (protons only).

Protons can be lost through nuclear absorption reactions with nuclei of the residual gas. Even most nuclear elastic scattering reactions will cause losses, as the typical elastic scattering angles are larger than the machine angular acceptance. Consequently, we simply take the total proton-nucleus cross section $\sigma_{nuclear}$ as the cross section for proton loss. Curves are given in the "Handbook", p. 216. Typical values of $\sigma_{nuclear}$ for protons on nitrogen are in the range of 0.4 barn, roughly independent of energy from 3 GeV to more than a TeV.

Beam lifetime from residual gas interactions (protons)

For protons, we use $\sigma = \sigma_{Coulomb} + \sigma_{Nuclear}$ in $\frac{1}{\tau} = nc\sigma$. The molecular density of the (ideal) residual gas is given by $n_{mol} = \frac{p}{kT}$, with p=pressure, T=absolute temperature, and k=Boltzmann's constant. Numerically, this is

$$n_{mol}\left[\mathrm{m}^{-3}\right] = 9.66 \times 10^{24} \frac{p[\mathrm{Torr}]}{T\left[{}^{\mathrm{o}}\mathrm{K}\right]}$$

which gives for the lifetime, for a diatomic gas,

$$\tau[\mathrm{hr}] = \frac{0.474T\left[{}^{\mathrm{o}}\mathrm{K}\right]}{p[\mathrm{nTorr}]\sigma[\mathrm{barn}]}$$

For proton storage rings, a typical requirement on the lifetime is τ > 20 hrs. This implies a residual gas pressure

$$p[\mathrm{nTorr}] \le \frac{0.474T\left[{}^{\mathrm{o}}\mathrm{K}\right]}{20\sigma[\mathrm{barn}]}$$

Examples: Fermilab antiproton accumulator p=8 GeV/c. Then $\sigma_{Coulomb} = 0.08$ barn, $\sigma_{Nuclear} = 0.4$ barn, T=293$^{\mathrm{o}}$ K=> p<1.7 x10^{-8} Torr

Fermilab Tevatron p=1000 GeV/c. Then $\sigma_{Coulomb}$ is negligible, $\sigma_{Nuclear} = 0.4$ barn, T=4$^{\mathrm{o}}$ K=> p<2.3x10^{-10} Torr.

Beam lifetime from residual gas interactions (electrons)

For electrons, we use $\sigma = \sigma_{Coulomb} + \sigma_{Brem}$. The equation for the lifetime is more complicated than for protons, because of *photodesorption*. The synchrotron radiation photons produced by the electrons, striking the walls of the vacuum chamber, desorb substantial quantities of gas. This amounts to a contribution to the residual gas density that is proportional to the beam intensity:

$$n = n_0 + GN$$

in which n_0 is the atomic gas density due to beam-unrelated gas sources, such as thermal outgassing, and G is the density produced by photodesorption by one electron. The equation for beam loss then becomes

$$\frac{dN}{dt} = -Nnc\sigma = -Nc\sigma\left(n_0 + GN\right)$$

The lifetime at t=0, $\tau = \frac{1}{n_{eff}c\sigma}$, $n_{eff} = n_0 + GN_0$, depends on the initial beam intensity N_0. For electron storage rings, a typical requirement on the lifetime is τ > 10 hrs. This implies a residual gas pressure

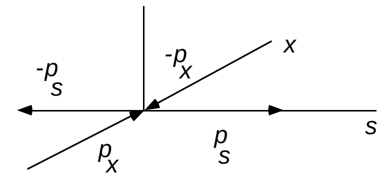
$$p_{eff}[\mathrm{nTorr}] \le \frac{0.474T\left[{}^{\mathrm{o}}\mathrm{K}\right]}{10\sigma[\mathrm{barn}]}$$

in which $p_{eff} = kTn_{eff}/2$ includes the effects of the photodesorbed gas (assumed to be diatomic).

Example: CESR p=5 GeV/c. Then $\sigma_{Coulomb} = 0.13$ barn, $\sigma_{Brem} = 4$ barn, T=293° K=> p_{eff}<3.4 x10^{-9} Torr

2. Loss due to scattering of one particle by another particle in the bunch (Touschek effect):

In an elastic Coulomb scattering event between two particles in the same bunch, there may be an exchange of transverse momenta for longitudinal momenta.



The above figure is in the rest frame of the bunch. In the laboratory frame, the momenta in the s direction get Lorentz boosted to $\gamma p_s \approx \gamma p_x \approx \gamma x' p$.

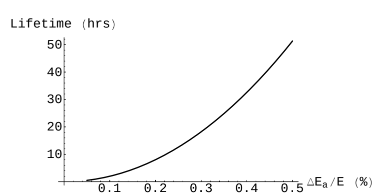
If $\gamma x' > \frac{\Delta E_a}{E}$, the relative energy acceptance of the machine, the particles will be lost.

The expression for the Touschek lifetime is quite complex. It depends on the details of the lattice, the machine energy acceptance, the particle energy, and the emittance of the beam. The

formula for the lifetime is given in "Handbook", p. 125-126. The basic structure is

$$\frac{1}{\tau_{Touschek}} \propto \frac{r_0^2 c N_b}{\gamma^4 \varepsilon_x \varepsilon_y \varepsilon_L} \left\langle f\left(\beta_x, \beta_y, \varepsilon_x, \varepsilon_y, \delta, \eta_x, \Delta E_a / E\right)\right\rangle$$

The following is a plot of the Touschek lifetime for the CESR operating parameters, as a function of the relative energy acceptance $\Delta E_a/E$:



(Electron machines only): quantum lifetime

The quantum fluctuations due to photon radiation may cause a particle to exceed the energy aperture or physical aperture of the machine. The resulting lifetime is called the "quantum lifetime".

Let's look at the loss process in the horizontal transverse plane first. When we discussed synchrotron radiation damping, we saw that the balance between damping and quantum excitation (due to photon emission in dispersive regions) led to an equilibrium horizontal emittance. The process which leads to this equilibrium involves the random emission of large numbers of photons; such a random process lead to a Gaussian distribution, in both x and x'.

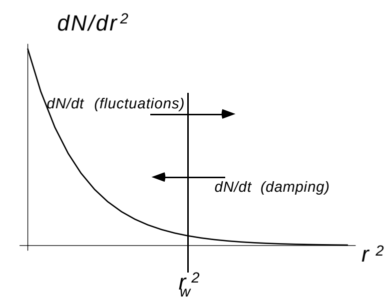
In phase-amplitude variables (r,ϕ), the Gaussian phase space distribution can be written as

$$\frac{dN}{rdrd\phi} = \frac{N}{2\pi\varepsilon}\exp\left[-\frac{r^2}{2\varepsilon}\right],$$

in which ε is the rms equilibrium horizontal emittance.

Integrating over ϕ, the number of electrons in an interval of amplitude squared dr^2 is $\frac{dN}{dr^2}dr^2$, where $\frac{dN}{dr^2} = \frac{N}{2\varepsilon}\exp\left[-\frac{r^2}{2\varepsilon}\right]$

Consider the following plot of $\frac{dN}{dr^2}$:



The number of particles per unit time (the particle *flux*) crossing a line at $r^2 = r_w^2$ due to damping is given by

$$\left.\frac{dN}{dt}\right|_{damping} = \left.\frac{dN}{dr^2}\right|_{r_w^2}\left.\frac{dr^2}{dt}\right|_{damping}$$

in which $\left.\frac{dr^2}{dt}\right|_{damping} = -\left.\frac{r^2}{\tau_{r^2}}\right|_{r_w^2} = -\frac{2r_w^2}{\tau_x}$. So

$$\left.\frac{dN}{dt}\right|_{damping} = -\frac{Nr_w^2}{\varepsilon\tau_x}\exp\left[-\frac{r_w^2}{2\varepsilon}\right]$$

But since the distribution is in equilibrium, the total

$\frac{dN}{dt} = \left.\frac{dN}{dt}\right|_{fluctuations} + \left.\frac{dN}{dt}\right|_{damping} = 0$. Thus, we know that

$$\left.\frac{dN}{dt}\right|_{fluctuations} = \frac{Nr_w^2}{\varepsilon\tau_x}\exp\left[-\frac{r_w^2}{2\varepsilon}\right].$$

Now imagine that we have an aperture limit at r_w. Since there are no particles at $r>r_w$, there will no longer be an inward damping flux, only the outward flux due to fluctuations, which constitutes beam loss. The distribution function must go to zero at this point, so it will not longer be Gaussian. However, if $r_w^2 >> \varepsilon$, the distribution will not change very much, and we can use the above expression to estimate the rate of beam loss:

$$\frac{dN}{dt} \approx -\frac{Nr_w^2}{\varepsilon\tau_x}\exp\left[-\frac{r_w^2}{2\varepsilon}\right] = -\frac{N}{\tau_q}$$

which gives for the quantum lifetime τ_q

$$\tau_q = \tau_x\frac{\varepsilon}{r_w^2}\exp\left[\frac{r_w^2}{2\varepsilon}\right]$$

This can be written in terms of the limiting value of the aperture, $x_a^2 = \beta r_w^2$, and the mean square horizontal beam size $\sigma^2 = \beta\varepsilon$, as

$$\tau_q = \tau_x \frac{\sigma^2}{x_a^2} \exp\left[\frac{x_a^2}{2\sigma^2}\right]$$

Note the extremely rapid dependence of the lifetime on the ratio $\frac{x_a^2}{\sigma^2}$. To obtain quantum lifetimes of 10 hours, for typical damping times of order 10 ms, we need to have

$$\frac{\tau_q}{\tau_x} = \frac{\sigma^2}{x_a^2} \exp\left[\frac{x_a^2}{2\sigma^2}\right] \geq 4\times 10^6 \Rightarrow \frac{x_a^2}{\sigma^2} \approx 24 \Rightarrow x_a \approx 5\sigma.$$

Going below this limit reduces the lifetime extremely rapidly. In this regime, the lifetime will be very sensitive to the aperture.

Usually one designs for $x_a \geq 10\sigma$ to provide adequate safety margin.

The equivalent set of considerations with regard to energy fluctuations leads to the following result for the quantum lifetime due to energy fluctuations:

$$\tau_q = \tau_\varepsilon \frac{\sigma_E^2}{\Delta E_a^2} \exp\left[\frac{\Delta E_a^2}{2\sigma_E^2}\right]$$

in which τ_ε is the energy damping time, σ_E is the rms energy spread, and ΔE_a is the energy aperture (due either to the bucket height, or to aperture limits at a dispersive point). Again, one typically designs for $\Delta E_a \geq 10\sigma_E$.

4. Beam loss at the interaction point in colliders

Electron-positron colliders: beam loss occurs through radiative Bhabha scattering ($e^+ + e^- \to e^+ + e^- + \gamma$), in which the final energy of one of the electrons falls outside the energy aperture.

The differential cross section for radiative Bhabha scattering is given in “Handbook”, p. 220. When integrated to give the cross section corresponding to an energy loss sufficient to leave the machine, the result is a slowly varying function of the relative energy aperture and the beam energy. For a wide range of energies and apertures, the cross section is in the range of $\sigma_{RBS} \approx 2-3\times 10^{-25}$ cm^2.

The lifetime for beam loss from this process depends on the luminosity, since the loss occurs due to beam-beam collisions.

From the definition of the luminosity, the loss rate for one species of particle will be

$$\frac{dN}{dt} = \mathcal{L}\sigma_{RBS}$$

Since $\mathcal{L} = kN^2$, where k is a constant if the cross sectional areas of the beams don’t change, we have

$$\frac{d\mathcal{L}}{dt} = 2kN\frac{dN}{dt} = 2kN\mathcal{L}\sigma_{RBS} = -\frac{\mathcal{L}}{\tau_{\mathcal{L}}}$$

which gives for the initial luminosity lifetime $\tau_{\mathcal{L}}$,

$$\tau_{\mathcal{L}} = \frac{1}{2kN_0\sigma_{RBS}} = \frac{N_0}{2\mathcal{L}_o\sigma_{RBS}}$$

in which L_0 is the initial luminosity, and N_0 is the initial number of particles.

Example: Consider a high luminosity electron-positron collider, for which L_0=3x10^{34} cm^{-2} s^{-1}, and for which N_0 = 5x10^{13}. The luminosity lifetime due to radiative Bhabha scattering will be about an hour.

Hadron colliders: beam loss occurs through inelastic reactions

$$p + p(\bar{p}) \ -> \ hadrons$$

At very high energies, this cross section is slowly varying with energy, and is about $\sigma_{pp} \approx 100$ mbarn = 10^{-25} cm^{2}. The luminosity lifetime will be

$$\tau_L = \frac{N_0}{2 L_o \sigma_{pp}}$$

Example: The LHC, for which L_0=10^{34} cm^{-2} s^{-1}, and for which N_0 = 3.6x10^{14}. The luminosity lifetime due to pp collisions will be about 50 hours.

Mechanisms for emittance growth

1. Scattering from atoms of the residual gas in the beam vacuum chamber

Elastic Coulomb scattering: Random, small angle scattering (multiple Coulomb scattering) causes transverse emittance growth

To discuss this, we will start by deriving a general relation for amplitude growth from any source of random angular kicks delivered to the beam.

Let θ_n be an angular kick delivered to a particle at a particular location in the ring, on the *n*th turn. Let's look at the change in the phase-amplitude variables at that point in the ring.

Let the initial values of the phase-amplitude variables on turn *n* be r_n and ϕ_n, with corresponding Floquet variables ξ_n and $\dot{\xi}_n$. When x' changes by θ_n, the change in Floquet coordinate is $\Delta\dot{\xi} = Q\sqrt{\beta}\theta_n$. We add this to the original Floquet coordinate and propagate this around the ring once, to find

$$\begin{pmatrix} \xi \\ \dot{\xi} \end{pmatrix}\Bigg|_{n+1} = \begin{pmatrix} \cos 2\pi Q & \dfrac{\sin 2\pi Q}{Q} \\ -Q \sin 2\pi Q & \cos 2\pi Q \end{pmatrix} \begin{pmatrix} \xi_n \\ \dot{\xi}_n + \Delta\dot{\xi} \end{pmatrix}$$

Then we look at the change in the amplitude

$$\frac{dr^2}{dn} = \Delta r^2 = r_{n+1}^2 - r_n^2 = -2\sqrt{\beta} r_n \theta_n \sin\phi_n + \beta\theta_n^2$$

In general, this will depend on the value of θ_n.. But if we average over many turns, and the kicks θ_n are truly random from turn to turn, then the piece linear in θ_n averages to zero, and we will have

$$\frac{d\langle r^2 \rangle}{dn} = \beta \langle \theta^2 \rangle$$

that is, the amplitude (and emittance) will grow proportional to the average of the square of the kick angle. Let us now apply this to the case of multiple Coulomb scattering.

The average scattering angle squared for Coulomb scattering, due to passage through a distance *s* in a medium, is given in the "Handbook", p 213, as

$$\left\langle \theta^2 \right\rangle \approx \left(\frac{13.6 \ \text{MeV}}{\beta pc} \right)^2 \frac{s}{X_0}$$

The quantity X_0 is called the radiation length; a formula is also given on the same page in the "Handbook"

Using these expressions in the formula for the amplitude growth above due to Coulomb scattering in the residual gas, and integrating around the ring, gives

$$\frac{d\left\langle r^2 \right\rangle}{dn} = \langle \beta(s) \rangle \left(\frac{13.6 \ \text{MeV}}{\beta pc} \right)^2 \frac{C}{X_0}$$

in which C= machine circumference.

This emittance growth is generally consequential only in proton machines, for which there is no natural damping mechanism.

2. Coulomb scattering of one particle by other particles in the bunch:

This is called *intrabeam scattering.*

Small angle Coulomb scattering equalizes the beam temperature in all dimensions: it causes a transfer of emittance from one dimension to another. Above transition, the emittance in all three degrees of freedom can grow. The relations for the growth time are quite complex: they are given in p 125-127 of the "Handbook". The growth times have a form very similar to that of the Touschek lifetime:

$$\frac{1}{T_{x,y,\Delta p}} \propto \frac{r_0^2 c N_b}{\gamma^4 \varepsilon_x \varepsilon_y \varepsilon_L} \left\langle f_{x,y,\Delta p}\left(\beta_x, \beta_y, \varepsilon_x, \varepsilon_y, \delta, \eta_x\right) \right\rangle$$

3. Emittance growth from random noise sources:

Random power supply noise, and ground motion, can cause transverse emittance growth

The general relation

$$\frac{d\left\langle r^2 \right\rangle}{dn} = \beta \left\langle \theta^2 \right\rangle$$

is applicable. The source of the random kicks could be dipole power supply noise, producing random field noise with a mean square value $\left\langle (\Delta B)^2 \right\rangle$:

$$\left\langle \theta^2 \right\rangle = \frac{\left\langle (\Delta B)^2 L^2 \right\rangle}{(B_0 \rho)^2}$$

Random ground motion, with a mean square amplitude $\left\langle (\Delta x)^2 \right\rangle$, will produce angular kicks (by misaligning quadrupoles) given by

$$\left\langle \theta^2 \right\rangle = \frac{\left\langle (\Delta x)^2 \right\rangle}{f^2}$$

in which f is the focal length of the quadrupole in question.

Example: If a machine has a revolution period T, then the rms emittance growth rate per unit time due to random quadrupole motion will be

$$\frac{d\varepsilon}{dt} = \frac{1}{2}\frac{\beta}{T}\frac{\left\langle (\Delta x)^2 \right\rangle}{f^2}$$

Consider the Tevatron collider, in which the rms emittance is approximately $2\text{x}10^{-9}$ m-rad, and for which T=21 μs. Let a quadrupole with a focal length of 5 m, at a β of 50 m, suffer random noise vibrations. Let the rms amplitude be 1 nm, and assume that all this vibration is directly reflected in the field. How long will it take for the emittance to double?

Plugging in the numbers, such a vibration would produce an emittance growth time of about $5\text{x}10^{-14}$ m/s. Thus, the emittance would increase by $2\text{x}10^{-9}$ in about 11 hours. Since this is a typical store length, this would be a serious problem.