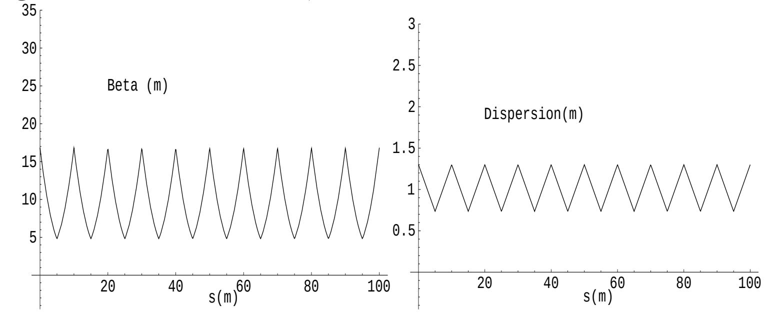
LECTURE 7

Lattice design: insertions and matching
Linear deviations from an ideal lattice:
Dipole errors and closed orbit deformations

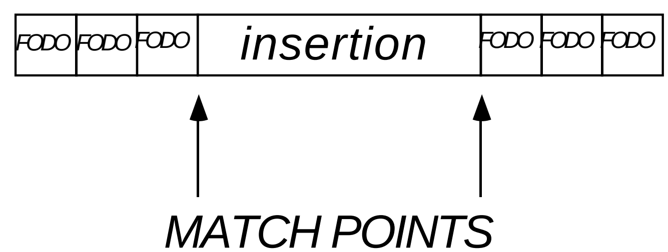
Lattice design: insertions and matching

The “backbone” of an accelerator lattice is the FODO cell. A machine composed entirely of identical FODO cells has very regular lattice functions: (this is 100 m of our 500 m accelerator)



However, such a machine is lacking long straight sections for injection and extraction. The dispersion is non-zero everywhere, which is unfavorable for the location of RF cavities. There is no low-β for colliding beam luminosity enhancement. There is no room for wigglers or undulators, or for beam collimation systems.

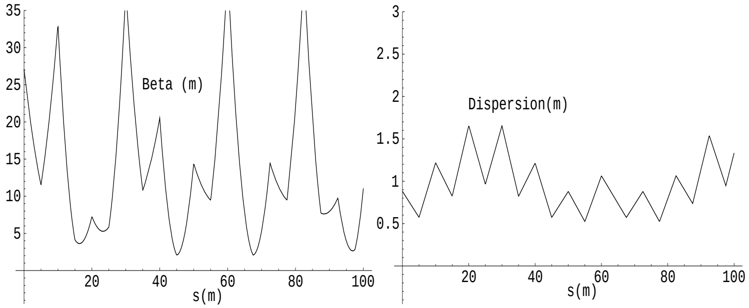
To allow for such devices, we create *insertions* in the otherwise regular FODO lattice. An insertion is a break in the FODO lattice into which a different configuration of magnets is placed, to allow for some of the functions mentioned above.



Ideally, we would like to leave *unchanged* the lattice functions in the part of the machine outside the insertion. In order to do this, the optics of the insertion must be designed such that the one turn matrix of the machine, with the insertion included, gives the same lattice functions at the match points as the original unperturbed lattice.

Failure to do this is termed a *"mismatch"* and the resulting perturbations to the lattice functions are sometimes called "beta beats".

Example: increase the drift space in the FODO cell at s=60 m from 5 m to 7.5 m:

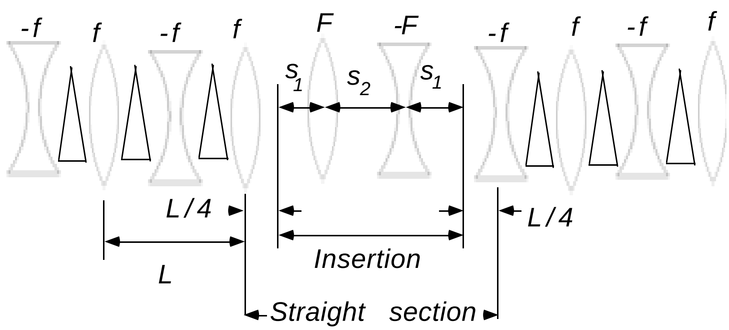


This produces major perturbations to the lattice functions (beta more than doubles) and is highly undesirable. The insertion needs to be *matched*. We'll discuss several simple types of matched insertions.

1. Collins insertion

A simple scheme involving two quadrupoles can be used in a straight section to provide beta function matching. The Collins insertion does not provide dispersion matching, but the combination of a dispersion suppressor and a Collins insertion results in a zero-dispersion straight section in which all the lattice functions are matched.

Collins insertion:



The transfer matrix of the insertion is

$$\mathbf{M}_I(s_2,s_1,F)=\begin{pmatrix}1 & s_1\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & s_2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -\frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & s_1\\ 0 & 1\end{pmatrix}$$

In order to match the lattice functions, we require that

$$\mathbf{M}_I(s_2,s_1,F)=\begin{pmatrix}\cos\mu_I+\alpha\sin\mu_I & \beta\sin\mu_I\\ -\gamma\sin\mu_I & \cos\mu_I-\alpha\sin\mu_I\end{pmatrix}$$

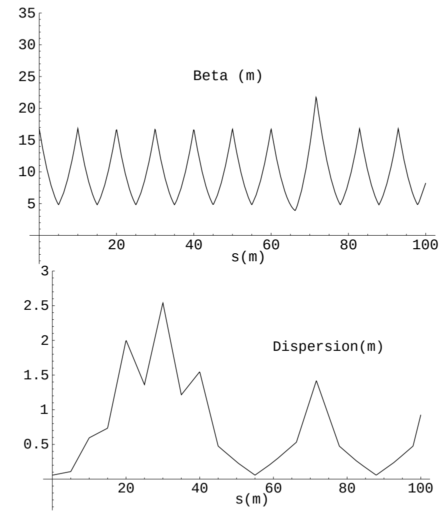
in which α, β, and γ are the regular FODO cell lattice functions at the match point, and μ_I is the phase advance across the insertion (a free parameter)

The result is

$$s_1 = \frac{\tan\frac{\mu_I}{2}}{\gamma} \quad s_2 = \frac{\alpha^2 \sin\mu_I}{\gamma} \quad F = -\frac{\alpha}{\gamma}$$

Typically, to maximize the length of the straight section s_2, we choose $\mu_I = \pi/2$, so that $s_1 = \frac{1}{\gamma}, \quad s_2 = \frac{\alpha^2}{\gamma}, \quad s_2 + s_1 = \beta$. For this insertion to match in both x and y, we need to have $\alpha_x = -\alpha_y$, which will be the case for a thin-lens FODO cell.

The insertion raises the tune of the machine by 1/4.
This insertion matches the beta function but does nothing for the dispersion mismatch:
17.8 m long Collins insertion, starting at s=60

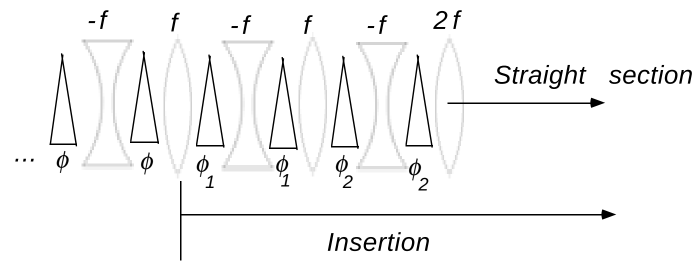


To fix the dispersion mismatch, we need another type of insertion:

2. Dispersion suppressor

This insertion is used to create a zero-dispersion straight section. There are many possible variants: we will only discuss the simple "missing-magnet" scheme.

"Missing magnet" dispersion suppressor



The insertion starts with two regular-strength FODO cells, in which the dipoles are operated at different bend angles, ϕ_1 and ϕ_2, than the dipoles in the rest of the lattice (which have a bend angle ϕ). These cells are followed by a straight section, in which the dispersion will be zero. The insertion is symmetric about the center of the straight section.

The bend angles ϕ_1 and ϕ_2 are chosen to make the dispersion function and its slope zero at the beginning of the straight section.

Let η_c and η_c' be the values of the dispersion and its slope at the beginning of the insertion. These are just the regular FODO cell values.
The dispersion propagates through the two FODO cells according to

$$\begin{pmatrix} \eta \\ \eta' \\ 1 \end{pmatrix} = \mathbf{M}_c(\phi_2,\mu)\mathbf{M}_c(\phi_1,\mu)\begin{pmatrix} \eta_c \\ \eta_c' \\ 1 \end{pmatrix}$$

where the FODO cell matrices depend on the cell phase advance μ and on the dipole bend angles ϕ_1 and ϕ_2

To get zero dispersion in the straight section, we solve the equation

$$\begin{pmatrix} \eta \\ \eta' \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \mathbf{M}_c(\phi_2,\mu)\mathbf{M}_c(\phi_1,\mu)\begin{pmatrix} \eta_c \\ \eta_c' \\ 1 \end{pmatrix}$$

for the dipole bend angles and find the simple results

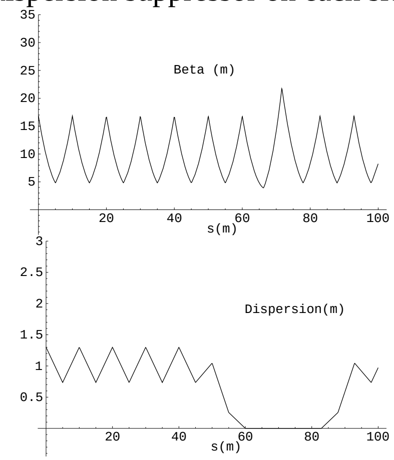
$$\phi_1 = \phi\left(1 - \frac{1}{4\sin^2\left(\frac{\mu}{2}\right)}\right); \quad \phi_2 = \frac{\phi}{4\sin^2\left(\frac{\mu}{2}\right)}$$

For $\mu \geq 60^o$, the bends need to have reduced strength relative to the normal FODO cells; the strength depends on the cell phase advance.

For $\mu = 60^o$, $\phi_1 = 0$ and $\phi_2 = \phi$. In this case, we just leave out two magnets in the first FODO cell of the insertion, and run the next cell as normal. This is the origin of the term "missing magnet". Even for general phase advance, this scheme is easy to implement and widely used.

This guarantees that the dispersion is both zero in the straight section, and unperturbed in the rest of the lattice. In combination with the Collins insertion, we get a long straight section into which we can put devices to perform some of the utility functions mentioned above.

17.8 m long Collins insertion, starting at s=60 m, with two-cell dispersion suppressor on each side

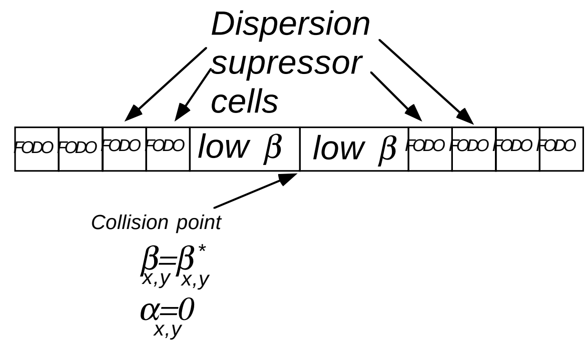


Many other, more complex types of insertions are possible. Among these are the so-called π and 2π insertions.

The π insertion has a transfer matrix equal to the negative of the unit matrix, and hence automatically provides lattice function matching. The insertion phase advance is π. Such an insertion does not match the dispersion.

A 2π insertion has a 3x3 unit matrix and matches all the lattice functions.

A particularly important type of insertion for colliding beam machines is the *low-β insertion*



Two symmetric insertions are used, to match from the FODO lattice to the collision point. Dispersion suppression is also required.

The box labeled "low-β" will contain at least a quadrupole doublet, together with a straight section of length L_0 on each side of the collision point, to provide space for experiments. In this drift space the beta function varies like

$$\beta(s)=\beta^*\left(1+\left(\frac{s}{\beta^*}\right)^2\right)$$

The phase advance across the straight section is

$$\Delta\Phi=\frac{1}{\beta^*}\int_{-L_0}^{L_0}\frac{ds}{\left(1+\left(\frac{s}{\beta^*}\right)^2\right)}=2\tan^{-1}\left[\frac{L_0}{\beta^*}\right]$$

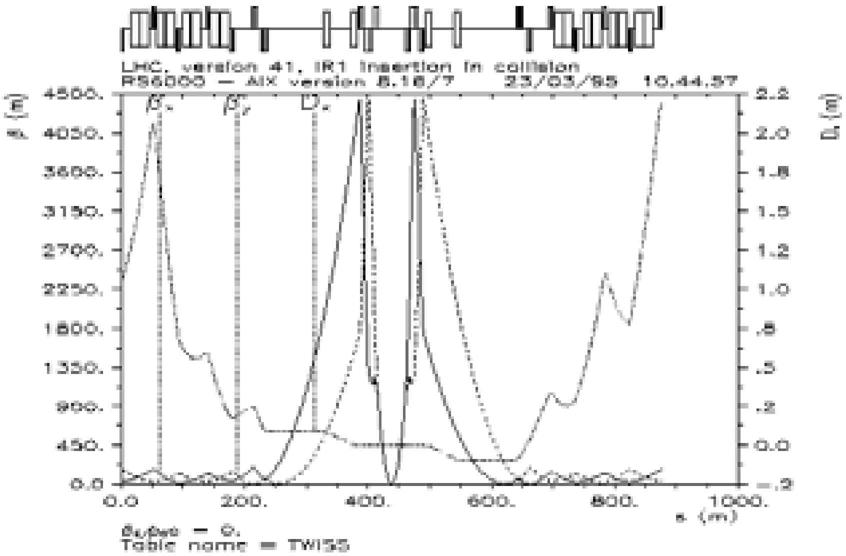
which is close to π, for $L_0 >> \beta^*$.

The phase advance across the straight section dominates that of the insertion. Thus, the machine tune increases by about 0.5 when a low-β insertion is added.

The rapid increase of the β-function in the straight section leads inevitably to a large value, β_{max}, of the β function somewhere in the insertion, before the lattice function can be matched to the FODO lattice. Typically, β_{max} in the low-β insertion is the maximum value of β in the machine. Since, as we'll see, errors tend to have effects proportional to $\sqrt{\beta}$ or β at their location, the low-β insertion is usually the most sensitive region of the machine.

A rough "rule of thumb": $\beta^*\beta_{max} \propto L_0^2$

Example: LHC low-β insertion, β^* =0.5 m



Linear deviations from an ideal lattice: Dipole errors and closed orbit deformations

We now begin to examine the results of field errors: the differences between the real fields in a machine, and the idealized fields on which the lattice design is based.

We'll start with the simplest kind of field errors: those due to dipole fields.

Dipole field errors can come from a variety of sources. Some of them are:

- Dipole fields due to quadrupoles not being aligned on the reference orbit (this is usually the biggest source of error)
- Differences between the idealized dipole field and the true dipole field, due to fabrication errors in the magnets, and/or due to remnant field effects
- Horizontal dipole fields (causing vertical orbit errors) due to rotated dipole magnets
- Dipole field errors due to misalignments of combined function magnets
- Stray fields on the reference orbit from other accelerator components

From Lecture 3, p 7: The trajectory equations, to lowest order in dipole field errors, are

$$x'' + x\left(k + \frac{1}{\rho^2}\right) = -\frac{\Delta B_y(s)}{B_0\rho}; \quad y'' - yk = \frac{\Delta B_x(s)}{B_0\rho}$$

Both of the form

$$z'' + Kz = \frac{\Delta B(s)}{B_0\rho}$$

We will treat dipole errors in the "kick approximation": Write the above equation as

$$\Delta z' + Kz\Delta s = \frac{\Delta B(s)\Delta s}{B_0\rho} = \frac{\Delta(BL)}{B_0\rho}$$

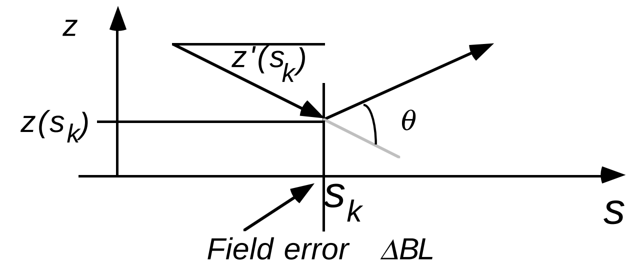
in which the field error is taken to be highly localized over a length *L*. Then, as *Δs*->0 with *Δ*(*BL*) finite, we have

$$\Delta z' = \theta = \frac{\Delta(BL)}{B_0 \rho}$$

The field error, in this approximation, just causes a change in the slope of the trajectory, by the angle θ, at the location of the error.

The trajectory of a particle, which would otherwise be on the reference orbit but for the field error, must be a closed curve, just like the reference orbit, since the kick is periodic with period C. How do we find the equation of this curve, relative to the reference orbit?

Let the field error be located at $s=s_k$



The one-turn transfer matrix $\mathbf{M}(C+s_k, s_k)$ takes us from s_k where the kick θ occurs, around one circumference. Hence we have

$$\mathbf{M}(C+s_k, s_k)\begin{pmatrix} z(s_k) \\ z'(s_k)+\theta \end{pmatrix} = \begin{pmatrix} z(C+s_k) \\ z'(C+s_k) \end{pmatrix} = \begin{pmatrix} z(s_k) \\ z'(s_k) \end{pmatrix}$$

where the last follows from the fact that the trajectory is closed.

In terms of the lattice functions, we have

$$\begin{pmatrix} z(s_k) \\ z'(s_k) \end{pmatrix} =$$

$$\begin{pmatrix} \cos 2\pi Q + \alpha(s_k)\sin 2\pi Q & \beta(s_k)\sin 2\pi Q \\ -\gamma(s_k)\sin 2\pi Q & \cos 2\pi Q - \alpha(s_k)\sin 2\pi Q \end{pmatrix}\begin{pmatrix} z(s_k) \\ z'(s_k)+\theta \end{pmatrix}$$

Solving this gives

$$z(s_k) = \frac{\beta(s_k)}{2}\theta \cot \pi Q; \quad z'(s_k) = -\frac{\theta}{2}\left(1+\alpha(s_k)\cot \pi Q\right)$$

$$z'(s_k)+\theta = \frac{\theta}{2}\left(1-\alpha(s_k)\cot \pi Q\right)$$

Comparing with the trajectory equations in the form

$$z(s_k) = a\sqrt{\beta(s_k)}\cot\delta \sin\delta$$

$$z'(s_k) = -\frac{a\sin\delta}{\sqrt{\beta(s_k)}}\left(1+\alpha(s_k)\cot\delta\right)$$

we can identify

$$a = -\frac{\theta}{2}\frac{\sqrt{\beta(s_k)}}{\sin\delta}; \quad \delta = -\pi Q$$

so that the closed trajectory as a function of *s* has the form

$$z(s) = a\sqrt{\beta(s)}\cos\left(\Phi(s)-\Phi(s_k)+\delta\right) =$$

$$= \frac{\theta}{2}\frac{\sqrt{\beta(s)}\sqrt{\beta(s_k)}}{\sin \pi Q}\cos\left(\Phi(s)-\Phi(s_k)-\pi Q\right)$$

The above form applies when $s>s_k$. A form valid for all s is

$$z(s) = \frac{\theta}{2}\frac{\sqrt{\beta(s)}\sqrt{\beta(s_k)}}{\sin \pi Q}\cos\left(\left|\Phi(s) - \Phi(s_k)\right| - \pi Q\right)$$