LECTURE 9

Single Particle Acceleration:
Standing wave structures
Travelling wave structures

In our discussion of structures used to provide electric fields for the acceleration of particles, we'll focus on RF fields: AC electric fields in the frequency range of 10 MHz to 30 GHz, with accelerating gradients from 1 MV/m to 100 MV/m.

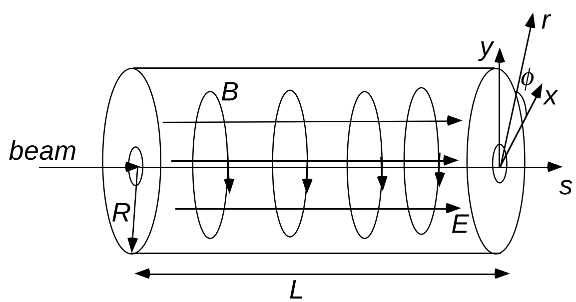
There are two principal types of RF accelerating structures in use in accelerators:

1. Standing wave structures: resonant cavities
Used in both linacs and synchrotrons

2. Travelling wave structures: waveguides
Used primarily in linacs

RF Cavities

The prototypical example is the "pillbox cavity"



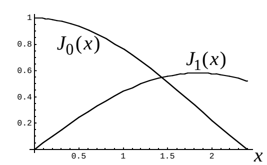
The cavity is operated in the TM_{010} mode: longitudinal E field, transverse B field.

The solutions to Maxwell's equations in the cavity, with the boundary conditions $E_{\|} = B_{\perp} = 0$ at the cavity walls, are

$$E_s(r,t) = E_0 J_0(kr)\cos\omega t;\ \sigma = \varepsilon_0 E_{\perp}$$

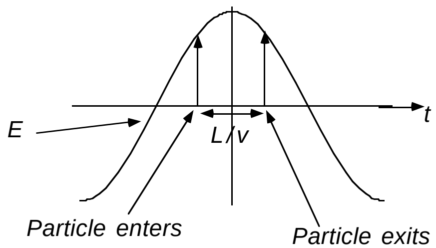
$$B_{\phi}(r,t) = \frac{E_0}{c} J_1(kr)\sin\omega t;\ K = \frac{B_{\|}}{\mu_0}$$

with $k = \frac{2\pi}{\lambda}$ and $\omega = ck$. $J_0(x)$ and $J_1(x)$ are Bessel functions, with $J_0(2.405) = 0$. σ and K are surface charge and current.



The boundary condition requires $kR = 2.405$, so the cavity radius determines the wavelength.

Example: $R = 10$ cm $\Rightarrow \lambda = 26$ cm => $f = 1.15$ GHz



Energy gain for a particle of velocity v:

$$eV_{acc} = e\int_{-L/2}^{L/2} ds E_0 \cos\omega t \Rightarrow$$

$$V_{acc} = \int_{-L/2}^{L/2} ds E_0 \cos\frac{\omega s}{v} = E_0 \frac{2v}{\omega}\sin\frac{\omega L}{2v}$$

“Transit time factor”:

$$T = \frac{V_{acc}}{E_0 L} = \frac{\sin u}{u}, \quad u = \frac{\omega L}{2v}$$

This limits the length of the cavity: e.g., for T=0.9 and v~c, L/R~2/3=>L=6.7 cm

Figures of merit for an RF cavity

1. Quality factor: a measure of how lossy the cavity is.

$$Q = 2\pi\frac{stored\ \ energy}{energy\ \ loss\ \ in\ \ 1\ \ cycle} = \frac{\omega W_s}{P_l}$$

Stored energy: $W_s = \dfrac{1}{2\mu_0}\displaystyle\int_{\substack{cavity\\volume}} B^2 dV$

Power loss: $P_l = \dfrac{1}{2}\displaystyle\int K^2 R_w dS = \dfrac{R_w}{2\mu_0^2}\int_{\substack{cavity\\surface}} B^2 dS$

in which $R_w = \dfrac{\rho_c}{\delta}$ is the surface resistivity, ρ_c is the volume resistivity, and $\delta = \sqrt{\dfrac{2\rho_c}{\mu_0\omega}}$ is the skin depth.

Eliminate ρ_c in favor of δ to get $P_l = \dfrac{\delta\omega}{4\mu_0}\displaystyle\int B^2 dS$

then we get

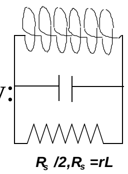
$$Q = \frac{\omega W_s}{P_l} = \frac{2}{\delta}\frac{\int B^2 dV}{\int B^2 dS}$$

Q depends on the geometry of the cavity and on the surface resistance

We’d like Q to be high, to minimize losses

2. Shunt impedance per unit length

$$r = \frac{E_0^2 L}{P_l}\left[T^2\right] = \frac{V_{acc}^2}{P_l L}$$ Cavity:



We'd like r to be high, to get high accelerating voltages with low losses.

3. The ratio r/Q:

$$\frac{r}{Q} = \frac{V_{acc}^2}{\omega W_s L}$$

This ratio depends only on the cavity geometry.

4. Filling time:

Power in the cavity decays according to

$$P_l = -\frac{dW_s}{dt} = \frac{\omega W_s}{Q} \Rightarrow W_s(t) = W_{s0} \exp\left(-\frac{\omega t}{Q}\right)$$

Field decay time

$$t_f = \frac{2Q}{\omega}$$

Example: for a "pillbox" cavity

$$Q = \frac{LR}{\delta(L+R)} \propto \frac{1}{\sqrt{f}}$$

$$\frac{r}{Q} = 2.6\mu_0 f \propto f; \quad r \propto \sqrt{f}$$

For pillbox parameters above

$$Q = 20620, \; r = 77 \; \text{M}\Omega/\text{m}, \; t_f = 5.7 \; \mu\text{sec}.$$

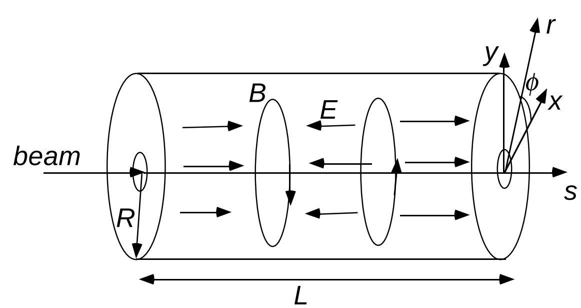
The total shunt impedance of the cavity is about

$R_s = rL$= 77x0.067=5 MΩ,

so that an RF drive power of 500 kW will produce an accelerating voltage of $V_{acc} = T\sqrt{P_l r L}$=1.4 MV.

Travelling wave structures

These structures are essentially cylindrical waveguides :



The waveguide is operated in the TM_{01} mode.

The fields are travelling waves:

$$E_s(r,s,t) = E_0 J_0(k_c r)\cos[ks - \omega t]$$

$$E_r(r,s,t) = E_0 \frac{\sqrt{\omega^2 - \omega_c^2}}{\omega_c} J_1(k_c r)\sin[ks - \omega t]$$

$$B_\phi(r,s,t) = \frac{E_0}{c}\frac{\omega}{\omega_c} J_1(k_c r)\sin[ks - \omega t]$$

where

$$k = \frac{\sqrt{\omega^2 - \omega_c^2}}{c};\ k_c = \frac{\omega_c}{c} = \frac{2.405}{R}$$

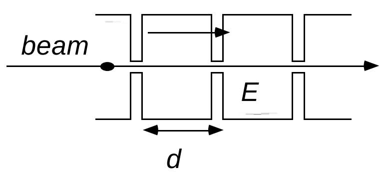
ω_c is called the "cutoff frequency".

The wave velocity is $v_p = \frac{\omega}{k} = \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} > c$

The group velocity (the velocity with which the energy travels) is

$$v_g = \frac{d\omega}{dk} = c\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} < c$$

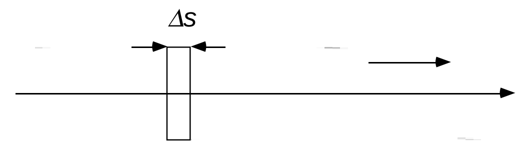
In order to be able to accelerate charged particles over any reasonable distance, the wave and the particle must have the same velocity. The waveguide is "loaded" with periodic structures (called "disks") to make this happen.



Typically, for electron linacs, $kd = \frac{\pi}{2}$ or $\frac{2\pi}{3}$; then $v_p \approx c$. The beam particles at $v=v_p$ will ride the travelling wave down the loaded waveguide, accelerating as they go. For protons, where the particle velocity changes as the energy grows, the disk spacing must be varied along the length to adjust the phase velocity to the particle velocity.

Energy is transported down the waveguide in the travelling electromagnetic wave; the accompanying wall currents dissipate energy, so that there is a loss of power as the wave travels.

Let us assume that we have a waveguide in which the cells are all identical. Then the group velocity is constant along the structure. Consider a slab of space, of length Δs:



Conservation of energy requires that

$$P(s+\Delta s) = P(s) - \Delta P_l$$

The Q of that section of the waveguide is

$$Q = \frac{\omega \Delta W_s}{\Delta P_l} \Rightarrow \Delta P_l = \frac{\omega \Delta W_s}{Q}$$

The power (energy transport per unit time) is

$$P(s) = \frac{dW_s}{dt} = \frac{dW_s}{ds}\frac{ds}{dt} = \frac{dW_s}{ds} v_g \Rightarrow$$

$$\Delta W_s = \frac{P(s)}{v_g} \Delta s$$

in which v_g is the group velocity (velocity of energy transport).

So $\Delta P_l = P(s) \frac{\omega}{v_g Q} \Delta s$ and we have

$$P(s + \Delta s) - P(s) = -P(s) \frac{\omega}{v_g Q} \Delta s$$

$$\frac{dP}{ds} = -\frac{\omega}{v_g Q} P$$

This equation describes the attenuation of the power along the waveguide. The solution is

$$P(s) = P_0 \exp[-2\alpha s], \quad \alpha = \frac{\omega}{2Q v_g}$$

The electric field amplitude varies as the square root of the power, so

$$E(s) = E_0 \exp[-\alpha s]$$

The effective accelerating voltage seen by a charged particle traveling on the crest of the wave for a distance L is

$$V_{acc} = \int_0^L E(s) ds = \frac{E_0}{\alpha}(1 - \exp[-\alpha L]) = \frac{E_0 L}{\tau}(1 - \exp[-\tau])$$

in which $\tau = \alpha L$ is the *attenuation factor*. The shunt impedance per unit length

$$r = \frac{E^2 L}{P_l} = -\frac{E^2}{\frac{dP}{ds}} = \frac{E_0^2 L}{2\tau P_0}$$

is constant along the structure, and in terms of it we can write

$$V_{acc} = \sqrt{P_0 r L}\frac{\sqrt{2\tau}}{\tau}(1 - \exp[-\tau])$$

The filling time of the waveguide is $t_f = \frac{L}{v_g} = \frac{\tau}{\alpha v_g} = \frac{2Q\tau}{\omega}$

Example: CESR linac, section 1:

L=3.05 m, r=60 MΩ/m, Q=18400, f=2896 MHz, v_g=0.0088c.

Then we find τ=0.57, t_f=1.15 μsec, $V_{acc} \approx 11\sqrt{P_0}$ kV.

For a peak power input to the section of P_0 = 10 MW,

we get V_{acc} = 34 MV.

Typically such high peak power is only available in pulses of a few μsec in length.

Constant impedance. vs. constant gradient

In the case just discussed, the group velocity was constant along the structure, leading to a constant impedance, and a decreasing electric field. It is also possible to lower the group velocity from cell to cell, which can give a constant electric field along the structure. This generally makes better use of the available power, as higher average fields can be reached.

In this case the accelerating voltage is given by

$$V_{acc} = \sqrt{P_0 r L (1 - \exp[-2\tau])}$$

with $P(L) = P_0 \exp[-2\tau]$