Signatures of Axion Monodromy Inflation

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Outline

1. Motivations
2. Linear Potential from String Theory
3. Modulations and Phenomenology
4. Microscopic Constraints
Inflation

- Afterglow Light Pattern 400,000 yrs.
- Dark Ages
- Development of Galaxies, Planets, etc.
- Quantum Fluctuations
- 1st Stars about 400 million yrs.
- Big Bang Expansion 13.7 billion years

Motivations
- Linear Potential from String Theory
- Modulations and Phenomenology
- Microscopic Constraints

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Motivations
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Brief Introduction to Inflation

A period of accelerated expansion

\[ ds^2 = -dt^2 + e^{2Ht} \, d\vec{x}^2 \quad H \approx \text{const} \]

Generic Predictions:
- Scalar perturbations are approximately scale-invariant: \( n_s \approx 1 \)
- Tensor to scalar ratio \( r = 16\epsilon \), where \( \epsilon \equiv -\frac{\dot{H}}{H^2} \).
- Scalar perturbations are approximately Gaussian.
Cosmological Data

- Planck was launched on May 14th 2009 and started collecting data since August
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Planck is able to detect $r > 0.05$ and constrain $f_{NL}^{loc} < 6$. 
r and the Field Excursion

\[ \mathcal{L} = \frac{1}{2}(\partial \phi)^2 - V(\phi) \]
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r and the Field Excursion

\[ \mathcal{L} = \frac{1}{2}(\partial \phi)^2 - V(\phi) \]

\[ r = 16\epsilon = 8 \left( \frac{\dot{\phi}}{H} \right)^2 \text{ with } dN = Hdt \]

\[ \frac{\Delta \phi}{M_p} = \int_{N_{CMB}}^{N_{end}} dN \sqrt{\frac{r(N)}{8}} \]

\(^1\)Lyth, 1996
r and the Field Excursion

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- $r = 16\epsilon = 8 \left( \frac{\dot{\phi}}{H} \right)^2$ with $dN = Hdt$

$$\frac{\Delta \phi}{M_p} = \int_{N_{CMB}}^{N_{end}} dN \sqrt{\frac{r(N)}{8}}$$

$$\frac{\Delta \phi}{M_p} \geq O(1) \left( \frac{r}{0.01} \right)^{1/2}$$

\footnote{Lyth, 1996}
r and the Field Excursion

- \[ \mathcal{L} = \frac{1}{2}(\partial \phi)^2 - V(\phi) \]
- \[ r = 16 \epsilon = 8 \left( \frac{\dot{\phi}}{H} \right)^2 \] with \( dN = Hdt \)
- \[ \frac{\Delta \phi}{M_p} = \int_{N_{CMB}}^{N_{end}} dN \sqrt{\frac{r(N)}{8}} \]
- \[ \frac{\Delta \phi}{M_p} \geq O(1) \left( \frac{r}{0.01} \right)^{1/2} \]
- Detectable gravitational waves suggests superplanckian field variation: \( \Delta \phi > M_p \)

\(^1\text{Lyth, 1996} \)
An inflaton potential in EFT:

\[ V = \frac{1}{2} m^2 \phi^2 + \frac{1}{4\lambda} \phi^4 + \phi^4 \sum_{p=1}^{\infty} \lambda_p \left( \frac{\phi}{\Lambda} \right)^{2p} \]
UV Sensitivity

- An inflaton potential in EFT:
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Still not satisfactory b/c EFT is ignorant about Planck scale.

We need a candidate UV completed theory, string theory.
Axions are pseudoscalar fields with only derivative couplings. Continuous shift symmetry: $S(a) = S(a + \text{const})$. 

\[ V(\phi) = \Lambda^4 \left( 1 - \cos \left( \frac{\phi}{f_0} \right) \right)^2 \] 

\[ \text{Freese, Frieman and Olinto, 1990} \]
Axions are pseudoscalar fields with only derivative couplings. Continuous shift symmetry: $S(a) = S(a + \text{const})$. Non-perturbative effect breaks it to a discrete shift symmetry, $a \rightarrow a + 2\pi$.

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Non-perturbative effect breaks it to a discrete shift symmetry, $a \rightarrow a + 2\pi$.

Natural Inflation\(^2\) with potential $V(\phi) = \Lambda^4 \left(1 - \cos\left(\frac{\phi}{f}\right)\right)$

\(^2\text{Freese,Frieman and Olinto, 1990}\)
Problem with Natural Inflation: Axions with super-Planckian decay constant$^3$ are elusive in controllable string theory setup.

$^3$Banks, Dine, Fox and Gorbatov, 2003; Svrcek and Witten, 2006
$^4$Dimopoulos, Kachru, McGreevy and Wacker, 2005
Problem with Natural Inflation: Axions with super-Planckian decay constant\textsuperscript{3} are elusive in controllable string theory setup.

A good idea: N-flation \textsuperscript{4} use $N \sim 10^3$ axions collectively to inflate.

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Our proposal: recycle an axion \(N\) times via monodromy

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Real-life Monodromy: Waterride

A waterride I have taken when I was young
Real-life Monodromy: Spiral Stairs

or the spiral stairs once walked on
Summary of Motivations

- Large field inflation is both interesting for observations and theoretical concerns.
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- **Large** field inflation is both interesting for observations and theoretical concerns.
- Our model = An **axion** from **string theory** with **shift symmetry**
  + Its non-periodic potential from **monodromy**
  + **Modulations** computed with string theory setup
Axions in String Theory

- String theory provides many axions:

\[ b(x) = \int_{\Sigma_2} B_2, \quad c(x) = \int_{\Sigma_p} C_p \]

\(^5\text{Wen and Witten, 1986}\)
Axions in String Theory

- String theory provides many axions:

\[ b(x) = \int_{\Sigma_2} B_2, \quad c(x) = \int_{\Sigma_p} C_p \]

- Worldsheet vertex operator\(^5\):

\[ V(k) = \int_{\text{WS}} d^2\xi \exp(ik \cdot X(\xi)) \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}(X) \]

\[ V(k = 0) = \int_{\Sigma_2} B_2 = 0 \text{ indicates axion } b \text{ can only have derivative couplings.} \]

\(^5\)Wen and Witten, 1986
We need to break shift symmetry to obtain an inflaton potential.
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**From D-branes**

The presence of worldsheet boundaries such as D-branes. We use wrapped branes to create a monodromy.

**From instantons**

Worldsheet instantons, or D-brane instantons introduce sinusoidal modulations.
Consider a D5-brane wrapping on a two cycle

\[
S_{DBI} = - T_5 \int d^4 x \sqrt{g_4} \int d^2 \xi \sqrt{\text{det}(G^{ind} + B^{ind})}
\]

\[
V(b) = T_5 \sqrt{l^4 + b^2}
\]

Take large b limit: \(V(\phi) = \mu^3 \phi\)
Consider a D5-brane wrapping on a two cycle $6$,

$$S_{DBI} = -T_5 \int d^4x \sqrt{g_4} \int d^2 \xi \sqrt{\text{det}(G^{\text{ind}} + B^{\text{ind}})}$$

$$V(b) = T_5 \sqrt{l^4 + b^2}$$

Take large $b$ limit: $V(\phi) = \mu^3 \phi$
Fixing Parameters

A linear potential

\[ V = \mu^3 \phi \text{ with } \epsilon = \frac{1}{2\phi^2}, \eta = 0 \]

- \( N(\phi) = \int_{\phi_{\text{end}}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon}} \) 60 efolds indicates \( \phi_* = 11M_p \)
- COBE normalization: \( P_s = \frac{1}{24\pi^2} \frac{V}{\epsilon} = (5.4 \times 10^{-5})^2 \) fixes \( \mu = 6 \times 10^{-4} M_p \)
Predictions of the Linear Potential

\[ V = \mu^3 \phi \]

- Tensor mode: \( r = 0.07 \). We will soon know!
- Tilt: \( n_s = 0.975 \)
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Modulations

- $V(\phi) = \mu^3 \phi + \mu^3 b f \cos\left(\frac{\phi}{f}\right)$
- Monotonicity: $b < 1$
- Number of oscillations in CMB: $\frac{1}{10f}$
Background Evolution

Solve $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ perturbatively in $b$

$\phi = \phi_0 + \phi_1$

Zeroth order solution: $\phi_0(t) = (\phi_*^{3/2} - \frac{\sqrt{3}}{2} \mu^{3/2} t)^{2/3}$

First order solution: $\phi_1 \approx -3bf^2\phi_0 \sin \left( \frac{\phi_0}{f} \right)$
Slow roll parameters

\[ \epsilon = \epsilon_0 + \epsilon_{osci} \approx \frac{1}{2\phi_*^2} - \frac{3bf}{\phi_0 f} \cos \left( \frac{\phi_0}{f} \right) \]

\[ \eta = \eta_0 + \eta_{osci} \approx 0 - 6b \sin \left( \frac{\phi_0}{f} \right) \]
Resonant Mechanism

- Background oscillates at frequency $\omega = \frac{\dot{\phi}}{f}$
- Perturbation mode of comoving momentum $k$ oscillates at frequency $\frac{k}{a}$ until freezes at $k = aH$
Resonant Mechanism

- Background oscillates at frequency $\omega = \frac{\dot{\phi}}{f}$
- Perturbation mode of comoving momentum $k$ oscillates at frequency $\frac{k}{a}$ until freezes at $k = aH$
- So when $H < \omega < M_p$, every mode will resonate with the background at some time.
Oscillations in the power spectrum

\[ P_s = A_s \left( \frac{k}{k_*} \right)^{n_s-1} \left( 1 + \delta n_s \cos \left( \frac{\phi_k}{f} \right) \right) \text{ where } \phi_k \simeq \phi_* - \frac{\ln k/k_*}{\phi_*} \]
Solution: \( \delta n_s = \frac{12b}{\sqrt{1 + (3f \phi_*)^2}} \sqrt{\frac{\pi}{8} \coth \left( \frac{\pi}{2f \phi_*} \right)} f \phi_* \sim 24b \sqrt{f} \)
Predicted Angular Power Spectrum
Compare with Unbinned WMAP
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Improvement from Planck

Planck Collaboration, 2006
Figure of merit:

\[ bf < 10^{-4} \text{ for } f < 0.01 \]
Lightning review of non-Gaussianity

- Simple case: \( \phi(x) = \phi_G(x) + f_{NL}^{local} \phi_G^2(x) \)
- \( f_{NL} \), amplitude of bispectrum, is usually a function of three momenta

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8Komatsu and Spergel, 2001
9Komatsu et. al. WMAP5 2008
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![Diagram](https://example.com/scheme.png)

Equilateral

Local

- WMAP5 data gives: \(-9 < f_{NL}^{local} < 111\), \(-151 < f_{NL}^{equil} < 253\)
- Planck will constraint \( f_{NL}^{local} < 6 \)

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\( f_{NL} \) from Inflation Models

- Canonical single-field slow-roll\(^{10}\): \( f_{NL} \sim O(\epsilon), \quad \epsilon \sim 10^{-2} \)
- Curvaton, non-canonical kinetic terms (k-inflation, DBI) or features in the potential:

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\(^{10}\) Maldacena, 2003
Resonant non-Gaussianity condition\textsuperscript{11}: \( H < \omega < M_p \)
for linear potential: \( 2.4 \times 10^{-6} < f < 0.09 \).

\textsuperscript{11}Chen, Easther, and Lim, 2008
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Resonant non-Gaussianity condition\textsuperscript{11}: $H < \omega < M_p$ for linear potential: $2.4 \times 10^{-6} < f < 0.09$. 

$$\langle R(\tau, k_1)R(\tau, k_2)R(\tau, k_3) \rangle = \frac{p_s^2}{(k_1k_2k_3)^2} f_{\text{res}} \sin \left( \frac{1}{\phi f} \ln K + \text{phase} \right) \delta^3(K) (2\pi)^7, \text{ } \text{\textsuperscript{12}}$$

$$f_{\text{res}} \approx \frac{3 \dot{\eta}_{\text{osci}}}{8H \sqrt{\phi f}},$$

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Non-Gaussianity: Oscillatory

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  for linear potential: \( 2.4 \times 10^{-6} < f < 0.09 \).

- \( \langle R(\tau, k_1)R(\tau, k_2)R(\tau, k_3)\rangle = \frac{p^2_s}{(k_1k_2k_3)^2} f_{\text{res}} \sin\left( \frac{1}{\phi f} \ln K + \text{phase} \right) \delta^3(K)(2\pi)^7 \), \(^{12}\)

- \( f_{\text{res}} \approx \frac{3 \dot{\eta}_{\text{osci}}}{8H\sqrt{\phi f}} \),

- Our model:

\[
 f_{\text{res}} \approx \frac{9b}{4\phi_0^{3/2}f^{3/2}} = \frac{9}{4} b \left( \frac{\omega}{H} \right)^{3/2}
\]

\(^{11}\) Chen, Easther, and Lim, 2008

\(^{12}\) Flauger and Pajer, to appear
Summary of the Phenomenology

- $r=0.07$: Gravitational waves detectable by Planck.
- Potentially detectable oscillations in scalar power spectrum.
- Potentially detectable resonantly enhanced non-Gaussianity.
Parameter Space

\[ \log_{10} b \]

\[ \log_{10} f \]

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Signatures of Axion Monodromy Inflation
Cartoon of Embedding
The Plan

Calculate the decay constant $f$ and amplitude of modulation $bf$ and constrain them.
The Plan

Calculate the decay constant $f$ and amplitude of modulation $bf$ and constrain them

**Decay constant $f$**

Calculate the decay constant from the kinetic term: $-\frac{1}{2}f^2(\partial c)^2$

Recall in SUSY we have $\mathcal{L} = K_a\bar{a} \partial \Phi^a \partial \bar{\Phi}^{\bar{a}}$ for superfields
The Plan

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Size of the modulation

Stabilize the moduli following KKLT

Scalar potential: $V = e^K(|DW|^2 - 3|W|^2)$

Need to know $D=4 \mathcal{N} = 1$ data.
Under orientifold action, cohomology classes split, $H_{r,s} = H_{r,s}^+ + H_{r,s}^-$.

Corresponding two cycle bases $\omega_A$ also split into $\omega_a$, $a = 1, \cdots, h_{1,1}^+$, and $\omega_\alpha$, $\alpha = 1, \cdots, h_{1,1}^+$.

\[^{13}\text{Grimm and Louis, 2004}\]
**Motivations**

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**$D = 4, \mathcal{N} = 1$ data from Type IIB O3/O7 orientifolds**

Under orientifold action, cohomology classes split, $H^{r,s} = H^{r,s}_+ + H^{r,s}_-$

Corresponding two cycle bases $\omega_A$ also split into $\omega_a$, $a = 1, \cdots, h^{1,1}_-$, and $\omega_\alpha$, $\alpha = 1, \cdots, h^{1,1}_+$

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**Chiral multiplet**

$$G^a \equiv \frac{1}{2\pi} (c^a - i \frac{b^a}{g_s})$$

$$T_\alpha \equiv i \rho_\alpha + \frac{1}{2} c_{\alpha\beta\gamma} v^\beta v^\gamma + \frac{g_s}{4} c_{\alpha bc} G^b (G - \bar{G})^c$$

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Chiral multiplet

\[
G^a \equiv \frac{1}{2\pi} (c^a - i \frac{b^a}{g_s}) \\
T_\alpha \equiv i \rho_\alpha + \frac{1}{2} c_{\alpha\beta\gamma} \nu^\beta \nu^\gamma + \frac{g_s}{4} c_{\alpha bc} G^b (G - \bar{G})^c
\]

- $K = \log \left( \frac{g_s}{2} \right) - 2 \log \mathcal{V}_E$
- $\mathcal{V}_E = \frac{1}{6} c_{\alpha\beta\gamma} \nu^\alpha(T, G) \nu^\beta(T, G) \nu^\gamma(T, G)$, note specifically $\nu = \nu(\text{Re } T, \text{Im } G)$

\[\text{Grimm and Louis, 2004}\]
Calculation of Axion Decay Constant

$$-\frac{1}{2}f^2(\partial c)^2 \subset K_G \bar{G} \partial G \partial \bar{G}$$

$$\frac{f^2}{M_p^2} = \frac{g_s}{8\pi^2} \frac{c_\alpha - v^\alpha}{V_E}$$
Calculation of Axion Decay Constant

- \(-\frac{1}{2} f^2 (\partial c)^2 \subset K_G \bar{G} \partial G \partial \bar{G}\)

\[
\frac{f^2}{M_p^2} = \frac{g_s}{8\pi^2} \frac{c_\alpha - \nu^\alpha}{\mathcal{V}_E}
\]

Validity of \(\alpha'\) expansion: use worldsheet instanton to estimate: \(e^{-S_{WS}} < e^{-2}\)

\[
2 < \frac{1}{2\pi\alpha'} \int \sqrt{g_{\text{string}}} = \sqrt{g_s} \nu^\alpha 2\pi
\]
Calculation of Axion Decay Constant

\[-\frac{1}{2} f^2 (\partial c)^2 \subset K_G \bar{G} \partial G \partial \bar{G}\]

\[
\frac{f^2}{M_p^2} = \frac{g_s}{8\pi^2} \frac{c_\alpha - \nu^\alpha}{\mathcal{V}_E}
\]

Validity of $\alpha'$ expansion: use worldsheet instanton to estimate: $e^{-S_{WS}} < e^{-2}$

\[
2 < \frac{1}{2\pi \alpha'} \int \sqrt{g_{\text{string}}} = \sqrt{g_s} \nu^\alpha 2\pi
\]

Lower bound on $f$:

\[
\frac{f^2}{M_p^2} > \frac{\sqrt{g_s}}{(2\pi)^3 \mathcal{V}_E}
\]
Warmup: Review of KKLT

Non-perturbative superpotential:

\[ W = W_0 + \sum_{\alpha=1}^{h_+^{1,1}} A_\alpha e^{-a_\alpha T_\alpha}, \quad a_\alpha = \frac{2\pi}{N_\alpha} \]
Warmup: Review of KKLT

- Non-perturbative superpotential:

\[
W = W_0 + \sum_{\alpha=1}^{h^{1,1}} A_\alpha e^{-a_\alpha T_\alpha}, \quad a_\alpha = \frac{2\pi}{N_\alpha}
\]

- \( D_\alpha W \equiv \partial_{T_\alpha} W + W \partial_{T_\alpha} K = -A_\alpha a_\alpha e^{-a_\alpha T_\alpha} - W \frac{v_\alpha}{2V_E} = 0 \)
stabilizes the volume of all 4-cycles
Non-perturbative superpotential:

\[ W = W_0 + \sum_{\alpha=1}^{h_{1,1}} A_\alpha e^{-a_\alpha T_\alpha}, \quad a_\alpha = \frac{2\pi}{N_\alpha} \]

- \( D_\alpha W \equiv \partial_{T_\alpha} W + W \partial_{T_\alpha} K = -A_\alpha a_\alpha e^{-a_\alpha T_\alpha} - W \frac{v_\alpha}{2V_E} = 0 \)
  stabilizes the volume of all 4-cycles
- \( D_a W = -iW \frac{c_{a\alpha c} v_\alpha b^c}{4\pi V_E} = 0 \) stabilizes \( b^a = 0 \)
Find $bf$: instanton correction

- Educated guess: $K = -2\log(\mathcal{V}_E + e^{-S_{ED1}} \cos(c))$
Find $bf$: instanton correction

- Educated guess: $K = -2 \log(\mathcal{V}_E + e^{-S_{ED1}} \cos(c))$
- Perturbative moduli stabilization like $\tau = \tau_0 + e^{-S} \tau_1 \cdots$
Find $bf$: instanton correction

- Educated guess: $K = -2 \log(\mathcal{V}_E + e^{-S_{ED1}} \cos(c))$
- Perturbative moduli stabilization like $\tau = \tau_0 + e^{-S_1} \tau_1 \cdots$
- $bf = \frac{U_{\text{mod} \phi}}{\mu^3 \phi} e^{-S_{ED1}} \left( K_{(1)} + 2 \Re \frac{W_{(1)}}{W_{(0)}} \right)$
Find $bf$: instanton correction

- Educated guess: $K = -2\log(\mathcal{V}_E + e^{-S_{ED1}}\cos(c))$
- Perturbative moduli stabilization like $\mathcal{T} = \tau_0 + e^{-S}\tau_1 \cdots$
- $bf = \frac{U_{mod} \phi}{\mu^3 \phi} e^{-S_{ED1}} \left( K_{(1)} + 2\text{Re} \frac{W_{(1)}}{W_{(0)}} \right)$
- $U_{mod} = \frac{g_s}{2} \left( \frac{3|W|^2}{\mathcal{V}_E^2} \right)_{(0)}$ from AdS minimum + uplifting
- $bf < 2c_0 \cdot 10^7 \frac{g_s}{\mathcal{V}_E^2} e^{-2/g_s} \left( \frac{W}{0.1} \right)^2$
Consistency Conditions

- $g_s \ll 1$
- Euclidean instanton: $e^{-a_{\alpha} T_{\alpha}} < e^{-2} \rightarrow \tau_{\alpha} > \frac{N_{\alpha}}{\pi}$
- Back reaction on the throat
  \[ N_w = \frac{\phi}{2\pi f} \ll \frac{R_{\text{perp}}^4 X}{4\pi g_s} \rightarrow \frac{f}{M_p} > \frac{0.09}{\chi^{1/3} V^{2/3}_E} \]
- $V_{\text{inf}} \ll U_{\text{mod}} \rightarrow \tau_{\alpha} \ll 73 - 8 \log \left( \frac{\nu^{\alpha} \pi \sqrt{g_s}}{2g_s} \right)$ where we used $N_{\alpha} < 50$
- Back reaction of the 4-cycle volume.
- Higher derivative terms
Backreaction of 4-cycle volume

Effect of $N_w$ D3 branes\textsuperscript{14}

\textsuperscript{14}Alternative: Berg, Pajer and Sjörs, to appear
Higher derivative terms

- $S_{10D,E} \supset \alpha'^3 \bar{R}^4$
- $\bar{R}_{MN}^{PQ} \supset \nabla_{[M} H_{N]}^{PQ}$ and $H_{[M}^{C[P} H_{N]}^{Q]} + \ldots$, \(^{15}\) where $H = dB$
- $S_{4D} = \int d^4x \left[ -\frac{1}{2} \dot{\phi}^2 + \frac{\dot{\phi}^8}{M_i^{12}} + \frac{\ddot{\phi}^4}{M_{ll}^{8}} \right]$
- $\frac{\omega}{M_i}, \frac{\omega}{M_{ll}}$ at most a few percent.

\(^{15}\) Gross and Sloan, 1987
A Toy Example

\[ V = 1.13e \]
\[ g_s = 0.134 \]
Axion monodromy inflation is falsifiable by its prediction of $r = 0.07$.

In addition: possible detectable oscillations in the power spectrum and resonantly enhanced non-Gaussianity.

We study the microscopic constraints on specific class of models in string theory.
Future directions

- Study of oscillatory type non-Gaussianity with data.
- Study of reheating.
- Realization of chain inflation.
- Other compactification models.