Signatures of Modulations in Axion Monodromy Inflation

Gang Xu, Cornell University

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Outline

1 Motivation
   - Inflation
   - Generating non-Gaussianity

2 Our Model
   - More Motivation
   - The Potential

3 Analysis
   - Calculation Steps
   - Constraints
   - Results

4 Conclusion
Inflation

Generating non-Gaussianity

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Signatures of Axion Monodromy Inflation
A brief introduction to Inflation

A period of accelerated expansion

\[ ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \]

\[ H \approx \text{const} \]

Explains why universe is so large, flat and empty (Guth, 1981)

Predictions:
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- Tensor fluctuations \( r = 16\epsilon \), where \( \epsilon \equiv -\frac{\dot{H}}{H^2} \).
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**Predictions:**

- Amplitude $P_s$ of primordial scalar fluctuations.
- Scalar fluctuations are approximately scale-invariant: $n_s \approx 1$
- Tensor fluctuations $r = 16\epsilon$, where $\epsilon \equiv -\frac{\dot{H}}{H^2}$.
- Scalar fluctuations are approximately Gaussian.
Why non-Gaussianity?

\[ \phi = \phi_G + f_{NL} \phi_G^2 \]

\(^1\) WMAP5 Komatsu et.al arXiv: 0803.0547[astro-ph]
Why non-Gaussianity?

- $\phi = \phi_G + f_{NL} \phi_G^2$ \(^1\)
- $P_s$, $n_s$, $r$ are numbers; $f_{NL}$ is usually a function of the scales

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\[ \begin{align*}
    k_1 & \quad k_2 \\
    k_3 & \quad \text{Equilateral}
\end{align*} \quad \begin{align*}
    k_1 & \quad k_2 \\
    k_3 & \quad \text{Local}
\end{align*} \]

- WMAP5 data gives: 
  \[ -9 < f_{NL}^{\text{local}} < 111 \]
  \[ -151 < f_{NL}^{\text{equil}} < 253 \]

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\end{array} & \quad \begin{array}{c}
\text{Local} \\
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- WMAP5 data gives: \(-9 < f_{local}^{NL} < 111 \)  
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- Would be a truly remarkable discovery
- In five years, Planck \( f_{loc}^{NL} < 6 \)
- Let’s hope we are lucky!

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Current Situation

- Generic single field slow roll: \( f_{NL} \sim O(\epsilon) \), \( \epsilon \sim 10^{-2} \)
- Exceptions: non-inflation models, DBI inflation or

\[
\begin{align*}
\text{bump} & \\
\text{sharp feature}
\end{align*}
\]

- For a potential with oscillatory modulations, when \( H < \omega < M_p \) the non-Gaussianity will be dominated by \(^2\)

\[
f_{\text{res}} \sim \epsilon \dot{\eta} \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H}
\]

Motivation Summary

- **Inflation** can deliver a flat, homogeneous universe.
- **Non-Gaussianity** can tell us a lot about this inflationary era.
- A resonant production mechanism can help us to achieve an observable non-Gaussianity.
Why string theory?

- Promising candidate for UV completion
- Easy to find a scalar to act as the inflaton
**Why axions?**

**Axion**: a pseudo-Goldstone boson with a shift symmetry,

\[ S(a) = S(a + f), \]

\(^3\)McAllister, Silverstein and Westphal arXiv:0808.0706[hep-th]
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What are the axions \(^3\) in our model?

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Why monodromy?

Monodromy:

\[
\text{DBI} = -\int dp^+ \xi (2\pi)^p \alpha' - \left( p + \frac{1}{2} \right) e^{-\Phi} \sqrt{\det (G_{MN} + B_{MN})} \partial \alpha X^M \partial \beta X^N \]

\[
V(b) = \epsilon g_s (2\pi)^{5/2} \alpha' \sqrt{l_4^4 + b^2}
\]

Large field inflation:
\[
V(\phi) \approx \mu^3 \phi
\]

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Why monodromy?

Monodromy:

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V(b) = \frac{\epsilon}{g_s (2\pi)^5 \alpha'^2} \sqrt{l^4 + b^2}
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Large field inflation: \( V(\phi) \approx \mu^3 \phi \)
Parameter Fixing

$$V = \mu^3 \phi + \Lambda^4 \cos \left( \frac{\phi}{f} \right)$$

- 60 e-folds: $\phi_0 \approx 11 M_{pl}$
- COBE: $\mu = 6 \times 10^{-4} M_{pl}$
- CMB: $\frac{\Lambda^4}{\mu^3} < 3.3 \times 10^{-4} M_{pl}$
Calculation steps and Predictions

1. **Background evolution:** solve $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

2. **Calculate spectrum:** solve Mukhanov-Sasaki equation

   $$u''_k + \left(k^2 - \frac{z''}{z}\right) = 0, \quad z \equiv \frac{a\dot{\phi}}{H}$$

3. **Calculate bispectrum:** following Chen et al.

**Predictions**

$$n_s \approx 0.975 \text{ and } r \approx 0.07$$
f the decay constant

\[ f_{\text{res}} \simeq \frac{9\Lambda^4}{4\mu^3 \phi_0^{3/2} f^{5/2}}, \]

Where \( f = \frac{c_0}{2\pi} \sqrt{\frac{g_s}{T_L}} \quad 4 \), and \( f_{\text{res}} \simeq 2 \times 10^{-3} \left( \frac{T_L}{c_0^2 g_s} \right)^{5/4} \)

\(^4\)Svrcek and Witten[arXiv:hep-th/0605206]
The decay constant $f$ can be calculated as

$$f_{\text{res}} \simeq \frac{9 \Lambda^4}{4 \mu^3 \phi_0^{3/2} f^{5/2}}.$$

Where $f = \frac{c_0}{2\pi} \sqrt{\frac{g_s}{T_L}}^4$, and $f_{\text{res}} \simeq 2 \times 10^{-3} \left(\frac{T_L}{c_0^2 g_s}\right)^{5/4}$

$g_s \ll 1$

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1. \( g_s \ll 1 \)
2. \( S_{WS} > 2 \) and \( S_{ED3} > 2 \)

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3. \( V < U_{\text{mod}} \simeq \frac{1}{T_L^3} e^{-4\pi T_L / N_L} \)

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3. \( V < U_{\text{mod}} \simeq \frac{1}{T_L^3} e^{-4\pi T_L/N_L} \)
4. Taste bound: \( N_L < 50 \)

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### Table of Results

<table>
<thead>
<tr>
<th>$T_L$</th>
<th>$N_L$</th>
<th>$g_s$</th>
<th>$f_{res}$</th>
</tr>
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<tr>
<td>30</td>
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<td>0.03</td>
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<tr>
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</table>
We have studied the observational signatures of an axion monodromy inflation model. This model predicts observable non-Gaussianity.