D-branes and gauge theory

Charged particle in $\mathbb{R}^{8,1} \times S^1 \times \mathbb{C} \times \mathbb{C}^*$, radius $R$

$$S = \int d\tau \left( \frac{1}{2} \dot{X}^M \dot{X}_M - \frac{i}{2} m^2 - ig A^M \dot{X}^M \right)$$

In the gauge field by

$$A_{10}(x^M) = - \frac{\Theta}{2\pi R} \quad \Theta = \text{const}$$

Here

$$A_{10} = - i \frac{\partial}{\partial x^{10}} \wedge \lambda = \exp \left( -i \frac{\Theta}{\pi R} x^{10} \right)$$

and

$$F = 0$$

Here,

$$Z = \int dx^{10} A_{10} = - \Theta$$

Nonrelativistic warmup:

$$H = \frac{1}{2m} \left( \frac{\partial}{\partial q} - qA^0 \right)^2$$

for $A_0 = - \frac{\Theta}{2\pi R}$

$$E \psi = \frac{1}{2m} \left( -i \partial_0 + \frac{q}{\Theta} \Theta \right)^2 \psi$$

periodicity in $x^{10}$: $\psi(x^{10}) = \psi(x^{10} + 2\pi R)$

$$\Rightarrow E_n = \frac{1}{2m} \left( \frac{n}{R} + \frac{q}{2\pi R} \right)^2$$

Some works in rel. case, using

$$P^M_{\text{non}} = P^M_{\text{on}} - q A^M$$
Now strings on D-branes have $\mathbb{CP}$ factors

\[ |N_{ij}\rangle \]

with charge $1$ under $U(1)_i$:

\[ U(1)_j \]

\[\partial_j P_{10} = P_{10}^{km} - \frac{\Theta_i + \Theta_j}{\partial \Theta_R \partial \Theta_R} \frac{\partial \Theta_R}{\partial \Theta_R} \]

\[ \theta = \frac{\partial \theta}{\partial \Theta_R} \]

\[ M^2 = \left( \frac{\Theta_j - \Theta_i}{\partial \Theta_R} \right)^2 + \frac{1}{\alpha'} (N-1) \]
Without Wilson lines, T-duality acts on open strings as:

\[ X' = \frac{\pi}{\alpha'} R \]

With Wilson lines, the action is modified by:

\[ X'(\pi) - X'(0) = R'[2\pi n + \Theta_j - \Theta_i] \]
Consider a D1-brane on $S^1$

$$A = - \frac{\theta}{\ell_{\text{pl}} R}$$

For $\theta = \text{const}$, T-dual is $\circ \leftrightarrow \bigcirc$

but for $\theta = \Theta(t)$, T-dual is $\bigcirc$

Now $\partial_t A = F_{010}$ \quad ($F_{MN} = \partial_M A_N - \partial_N A_M$)

so electric flux is T-dual to momentum.
For $N$ coincident D-branes

\[ U(N) \]

Consider a Wilson line $W \in U(N)$

\[ \text{NB since } UU^\dagger = U^\dagger U, \text{ we can write } U = A + iB \]

hence, diagonalizable by unitary transformation.

\[ \text{(i.e. gauge rotation)} \]

\[ \Rightarrow \text{ wlog, } U(N) \to UW^N \]

\[ W = \exp \left[ -i \text{ diag } [\Theta_1, \ldots, \Theta_N] \right] \]

\[ = \text{ diag } [e^{-i\Theta}] \]

T-dualize:
Worldvolume fields on D-branes

1) Isolated Dp-brane has on its WV,

- a Maxwell field in \((p+1)\)d WV,
  \[ A_\mu (\delta^0 \cdots \delta^p) \]
- a massless scalar for each \(\perp\) dir,
  \[ \Phi^\alpha (\delta^0 \cdots \delta^p) \]

So a \(\Phi^\alpha\) excitation is a 'bump' or ripple.

\[ \uparrow^x \quad \rightarrow^x \]

2) \(N\) coincident Dp-branes have

- a YM field \(A_\mu \lambda_{ij}\)
- \(U(N)\) gauge group

- transverse scalars in adjoint of \(U(N)\)

\[ \Phi^\alpha \lambda_{ij} \]
In flat $\mathbb{R}^{9,1}$ with $B=0$:

**D9-branes**

**D=10 SYM**

\[
S = \int d^{10}x \text{ Tr} \left( -\frac{1}{4g_{\text{YM}}^2} F_{MN} F^{MN} \right)
\]

dim. red on one direction (or on $p$ directions)

\[
A_M \rightarrow A_\mu, \quad \Phi^\alpha
\]

\[
S = \frac{1}{4g_{\text{YM}}^2} \int d^{p+1}x \text{ Tr} \left( -F_{\mu\nu} F^{\mu\nu} - 2 D_\mu X^\alpha D^\mu X_\alpha + [X_\alpha, X_\beta] [X^\alpha, X^\beta] \right)
\]

\[
D_\mu X^\alpha = \partial_\mu X^\alpha - i[A_\mu, X^\alpha]
\]
The $L^2$ is a potential.

When $[\lambda^a, \lambda^b] = 0$, we may diagonalize

$$X^a \rightarrow \text{diag}(X_1^a, \ldots, X_N^a)$$

here

$$N = \# \text{D}-\text{branes}$$

The eigenvalue $X_i^a$ governs the location of the $i$th brane in $X^a$ direction.

But if $[\lambda^a, \lambda^b] \neq 0$, we cannot simultaneously specify all the D-brane positions!
How large is a D-brane?

'only one scale, \( \alpha' \)': incorrect.

\[ T_{NS5} = \frac{\alpha'}{g_s} \cdot \frac{1}{\text{string units}} \]

\[ G_\mu = 8\pi G_N T_{NS} \approx g_s^2 T_{NS5} \sim O(1) \text{ in } \alpha'/\text{units} \]

But D-brane are lighter (for \( g_s < 1 \)),

\[ T_{D5} = g_s T_{NS5} \]

\( \Rightarrow \) BR occurs at smaller distances?

Better idea: if all strings have size \( \sim l_s = \sqrt{\alpha'/2g_s} \), then use one D-brane to probe another!
Problem: string hole.

Recall, a highly boosted string has many virtual d.o.f. visible.

\[ \leadsto \]

So hard to scatter is nearly no good.

Need to slip inside the hole.

So take V fixed, \( M_{\text{bo}} = \frac{M_s}{g_s} \to \infty \) by \( g_s \to 0 \).

Strings less important.
So let's compute

\[ A = \int_0^\infty \frac{dt}{at} \sum_{i,k} \exp(-2i\pi\alpha' + (k^2 + m_i^2)) \]

\[ m_i^2 = \frac{b}{(\alpha')^2} + \frac{K^2}{\alpha'} \quad \text{(for } v=0) \]

Summing up expando & small v, we get

\[ A = \int_0^\infty \frac{dt}{1+8e^{-\frac{b^2}{2\alpha'}}} \sqrt{f(t,v)} \]

Can expand this

\[ \delta = \frac{1}{\alpha}(\# \text{NN denotes}) \]

*** At small b, integrand is \( f(v) \) at \( t \to 0 \)

(i.e. no \( \frac{1}{b^2} \) terms).

Any singularity comes from \( t \to 0 \) regime.

\( \Rightarrow \) lightest open strings become massless
For $r \gg l_s$,

dominant effect is exchange of massless closed strings (i.e., supergravity).

For $r < l_s$,

lightest states have

$m_W = M_s\left(\frac{r}{l_s}\right) < M_s$

so $WW\ QFT$ contains (almost) light fields governed by dynamics.

(excited states $\Rightarrow > 2$ perturbations in QFT)

So as $r \to 0$,

governed by IR behavior of WW QFT!

(To compare)

1. DBI probe action

in SUGRA 

sourced by 6D brane

2. LEFA for

open string gauge

(ep. metric on $M$)
This (it's) [studies] D0-brane dynamics, with $r \ll l_s$, using only the lightest states between the branes.

(Pretty amazing!)

\[ S = \int dt \frac{1}{2g_s} \text{Tr} F^2 - i \text{Tr} \overline{\psi} D^\mu D_\mu \psi \]

\[ F_{0i} = 2 \phi_i + [A_0, \phi_i] \]
\[ F_{ij} = [\phi_i, \phi_j] \]
\[ D_0 \psi = \partial_0 \psi + [A_0, \psi] \]
\[ D_i \psi = [\phi_i, \psi] \]

$\psi \phi$ for $i + 0$ only $(\text{FT on } \text{D0})$

To absorb $g_s$, rescale:

\[ t = g_s^{-\frac{1}{3}} t_{(11)} \]
\[ A_0 = g_s^{-\frac{1}{3}} A_{(11)} \Rightarrow S = \int dt \, (F^{(11)})^2 \]
\[ \phi_i = g_s^{-\frac{1}{3}} \phi_i^{(11)} \]
So characteristic time is \( t_{\text{ch}} = \frac{-\frac{1}{3} + 5}{g_5^{1/3} M_5} \)

Mass is \( \frac{g_5^{1/3} M_5}{g_5^{1/3}} \)

Now slow D0-branes (\( v \ll 1 \)) have \( M_{\text{D0}} v^2 \sim g_5^{1/3} M_5 \)

\( \frac{v^2}{g_5} \sim g_5^{1/3} \)

\( v \sim g_5^{2/3} \ll 1 \)

(Quick way: \( \int dt \left( \frac{F}{g_5} \right)^2 : [g_5] = M_5^3 \)

so \( g_5^{1/3} M_5 \) is natural scale.)

All this is valid when \( v a \ll 1 \).
A single D0 string is not allowed, for then the endpoint charges are **uncancelled** (Gauss's law).
But also the mass of the 1-2 string depends on the separation $|x_0 - x_1| = r$

$$m_{1a}^2 = \left(\frac{1}{x_{1a}}\right)^2$$

If I began with

$$\#1, \#2$$

and adiabatically moved $\#2$,

then nothing exciting occurs.

But if $\#2$ moves rapidly/suddenly

$$m_{1a}(t) = \frac{1}{\text{rate}} \cdot r(t) \; \text{does} \; \text{change abruptly.}$$
Recall from QM that if we begin with a square well ground state and expand the well suddenly, the wavefunction \( \Psi(x) \) is momentarily unchanged, meaning it is some nontrivial combination of the eigenstates of the new Hamiltonian. Something occurs in QFT with a changing background, e.g. a rapidly expanding universe. Here, the state of no. 12 strings, separation to

gets transported into some state other than

\[ a \quad \quad b \quad \quad \text{separation new} \]

i.e. strings are present!
So \[ \rightarrow \] (eq:

(one can compute occupation number)

In realizing the sudden change of separation via a near miss scattering event,

\[ \rightarrow \]

\[ \leftarrow \]

\[ \rightarrow \]

\[ \leftarrow \]

\[ \leftarrow \]

meaning that D5-branes interact to produce stretched strips that gradually decay.
We've reduced D0-brane scattering to QM. Simplify further to a (fairly good) toy model we consider

\[ H = -\frac{1}{a} \phi^s (\nabla^2 + \nabla^2) + \frac{1}{2 \sigma} x^2 \sigma + \begin{pmatrix} 0 & x_1 - i x_2 \\ x_1 + i x_2 & 0 \end{pmatrix} \]

for \( \Phi(x, x, w, w) \) a 2-component wavefunction

Here \( x_1, x_2 \) \( \approx \) positions

\[ \langle x_2(p) \rangle = b \]
\[ \langle x_1(p) \rangle = v^+ \]

while \( W_1, W_2 \) are massive (stretch) strings.

The 2 components correspond to \( W_1, W_2 \), Fermi superpartners of \( W_1, W_2 \).
The massless fields are $W_i$ mass $\sqrt{b^2 + A^2}$ and $\chi_i$ mass $\pm \sqrt{b^2 + v^2}$ (plus excited states after this).

When $\frac{\omega}{\omega^2} \ll 1$ can use adiabatic approximation (Born-Oppenheimer).

$$\frac{\omega}{\omega^2} = \frac{1}{\omega} \cdot \omega^2 = \frac{v^2 x^2}{x^2} = \frac{v^2}{x^2}$$

so for $|x| > v_2$, $\sqrt{}$.

$\rightarrow$

\[ \begin{array}{c}
\left( \begin{array}{c}
\sqrt{v}^rac{1}{2} \\
\sqrt{v}^rac{1}{2}
\end{array} \right)
\end{array} \]

One can check that Bose d.o.f. $W_i$ are unchanged even for smaller $x$ (since $H_{\text{bose}}(t) = H_{\text{bose}}(-t)$).
But the fermions $\phi$: 

$$E$$

Result: straight fermionic strings nearly always created in near-miss scattering.

Max separation: $4 \frac{m_0 v^2}{\kappa} \approx \frac{L}{\kappa}$

$$L \approx \frac{b v^2}{g_s}$$

Lifetime of the resonance: $\frac{\nu b}{g_s}$

Radiation of RR photons is small, $P \propto a^2$ and $a \approx g_s$
Can work out resonance energies by WKB:

\[ p = \sqrt{2m\left(E-ax\right)} \]
\[ \int p\,dx = n - \frac{a}{2} \propto n \]
\[ \sqrt[\lambda/3]{(E-ax)^{3/2}} \propto n \]

\[ E_n \sim n^{\frac{a}{3}} m^{1/3} \lambda \]
\[ \lambda = \frac{M_s^2}{m} \left(\frac{a}{\omega_0^2}\right) \]
\[ E_n \sim g_s^{1/3} M_s n^{a/3} \sim g_s^{1/3} M_s \]

\[ \Rightarrow M_s v^2 \sim g_s^{1/3} M_s \]
\[ v^2 \sim g_s^{4/3} \quad v \sim g_s^{2/3} \]

So size is \( L = \frac{h v}{g_s} \sim \sqrt{\frac{1}{g_s^{1/3}}} \).

Note: \( h^{[10D]} \) given by

\[ g_s \alpha = \left(\frac{h}{m}\right)^3 \]
\[ \frac{h^{[10D]}}{s} = \frac{h^{[14D]}}{g_s s} \]
\[ h^{[14D]} \sim g_s s \]

So resonance \( \ll s \)

but resonance \( \gg h^{[10D]} > h^{[14D]} \).
M. Preary
Toroidal Compactification

\[ D = d+1 \quad x^d \text{ periodic, } x^d = x^d + 2\pi R \]

Metric

\[ ds^2 = G^{MN} dx^M dx^N \]

\[ = G_{\mu \nu} dx^\mu dx^\nu + G_{dd} (dx^d + A_d dx^d)^2 \]

- invariant under \( x^d \) translations \( (G_{dd} = G_{dd}(x^d)) \)
- \( d \)-dim reparam.
- \( x^d = x^d + \lambda(x^M) \)

... provided \( A_\mu = A_\mu - \partial_\mu \lambda \)

so \( dx^d + A_d dx^d \rightarrow dx^d - 2\pi dx^d \)

\[ + A_\mu dx^\mu + 2\pi A_d dx^d \checkmark \]

Simple case, \( G_{dd} = 1 \).

(massless) Scalar \( \phi(x^M) = \sum_{n=-\infty}^{\infty} \phi_n(x^d) \exp(inx^d/R) \)

then \( \partial^M A_M \phi = 0 \)

\[ \equiv \partial^M A_M \phi_n(x^M) e^{inx^d/R} \quad + \left( \frac{i\alpha}{R} \right) \phi_n(x^d) e^{-inx^d/R} \]

or \( \partial^M A_M \phi_n(x^M) = \left( \frac{i\alpha}{R} \right) \phi_n(x^M) \)

\[ m^2 = \left( \frac{n\pi}{R} \right)^2 \]
Also we'll need:

\[ R^{(d)} = R^{(d)} - 2e^{-\sigma} \nabla e^\sigma - \frac{1}{4} e^{2\sigma} F_{\mu \nu} F^{\mu \nu} \]

(\( F_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \))
Bosonic action of IIA:

$$S_{10} = \frac{1}{2k_{10}^2} \int d^{10} \sqrt{G} \left\{ e^{-2\Phi} \left( R + 4 \Phi \Box \phi \phi^{*} - \frac{1}{2} |H_{3}|^2 \right) \right\}$$

$$- \frac{1}{4k_{10}^2} \int d^{10} \sqrt{G} \left( |F_{2}|^2 + |F_{4}|^2 \right)$$

$$- \frac{1}{4k_{10}^2} \int B_{2} \wedge F_{4} \wedge F_{4}$$

where

\[
\begin{align*}
H_{3} &= dB_{0} \\
F_{2} &= dA_{1} \\
F_{4} &= dA_{3} - A_{1} \wedge F_{3} \\
F_{7} &= dA_{3}
\end{align*}
\]

Bosonic action of Cremmer-Julia 11D SUGRA:

$$S_{11} = \frac{1}{2k^{2}} \int d^{11} \sqrt{-G} \left( R^{(11)} - \frac{1}{2} |F_{4}|^{2} \right) - \frac{1}{12k^{2}} \int A_{3} \wedge F_{4} \wedge F_{4}$$
Now take
\[ ds^2 \equiv G^{(\mu \nu)} \, dx^\mu \, dx^\nu = G^{(\mu \nu)} \, dx^\mu \, dx^\nu + \frac{\kappa}{2} (x^\mu) [\, dx^{10} + A(x^\mu) \, dx^\nu \,]^{2(\mu \nu)} \]

Then \[ R^{(\mu \nu)} = R^{(\mu \nu)} - 2 \varepsilon^{\mu \nu \sigma \rho} \varepsilon^{\rho \sigma} \frac{1}{4} e^{(\sigma \rho)} F_{\mu \nu} F^{\mu \nu} \]

Joe's convention: \[ F^p_q = \frac{1}{p!} F_{\mu_1 \cdots \mu_p} \, F^{\mu_1 \cdots \mu_p} \]

define \[ k_{10} = \frac{k^2}{2m - R_{10}} \]

\[ S_{11} = \frac{k_{10}}{2m - R_{10}} \int d^{10} x \, \sqrt{-G^{(10)}} \, e^{(\frac{1}{2})} \left[ \frac{1}{16} \, F_{\mu \nu \rho} \, F^{\mu \nu \rho} + \frac{1}{4 \pi} \, F_{\mu \nu} \, F^{\mu \nu} \right] \]

\[ \left[ \text{now} \quad F^{10}_{\mu \nu} = G^{10 \mu} F_{10 \nu} + G^{10 \nu} F_{10 \mu} \right. \]
\[ = e^{(\rho \sigma)} F_{\mu \nu}^{\rho \sigma} + \]
\[ F_{\mu \nu}^{\rho \sigma} = G^{10 \rho \sigma} F_{\mu \nu}^{\rho \sigma} + G^{10 \mu} F_{10 \nu}^{\rho \sigma} \]
Remember \( G^{(10)}_{\mu} \neq G_{\mu} \).

\[
F_{\mu \nu} F^{\mu \nu} = G^{(10)}_{\mu} F^{\mu}_{\alpha \beta \gamma} F^{\alpha \beta \gamma}_{\nu} + G^{(10)}_{\mu 0} F^{\mu}_{\alpha \beta \gamma} F^{\alpha \beta \gamma}_{0} .4
\]

\[
+ 2 G^{(10)}_{\mu \nu} F^{\mu}_{\alpha \beta \gamma} F^{\alpha \beta \gamma}_{\nu} .4
\]

Joe's Convention:

\[
|p^2| = p^0 p^1 p^2 p^3
\]

\[
\text{but } \left\{ \begin{array}{l}
G^{(10)}_{\mu} = G^{(10)}_{\mu} + e^2 A_{\mu} \\
G^{(10)}_{\mu 0} = e^2 A_{\mu} \\
G^{(10)}_{\mu \nu} = e^2 A_{\mu} A_{\nu}
\end{array} \right.
\]

we get

\[
\frac{1}{2} R_{10} \int \frac{1}{x^2} \left[ \frac{1}{4!} G^{(10)}_{\mu} F^{\mu}_{\alpha \beta \gamma} F^{\alpha \beta \gamma}_{\nu} + \frac{1}{4!} e^2 A_{\mu} A_{\nu} F^{\mu}_{\alpha \beta \gamma} F^{\alpha \beta \gamma}_{\nu} + \frac{1}{4!} 2 e^2 A_{\mu} F^{\mu}_{\alpha \beta \gamma} F^{\alpha \beta \gamma}_{0} .4 \right.
\]

\[
\left. + \frac{1}{4!} e^2 F^{10}_{\alpha \beta \gamma} F^{0 \alpha \beta \gamma} .4 \right] \}
\]

\[
= - \frac{1}{4 \times 10^8} \int d^4 x \sqrt{-g^{(10)}} \left\{ e^5 |F_{\mu} F^\mu| + \ldots \right\}
\]
When one is careful about raising + lowering indices, this assembles into

\[
S_{10}^{\text{from } l_{13}^2} = -\frac{1}{4\kappa_8^2} \int d^4x \sqrt{g} \left( V_{(10)} + V_{(12)} \right)
\]

where:

\[
A_{\mu\nu}^{(10)} = A_{\mu\nu}^{(11)}
\]

\[
B_{\mu\nu}^{(10)} = A_{10,\mu\nu}^{(11)}
\]

\[
F_{\mu\nu}^{(10)} = F_{\mu\nu}^{(11)}
\]

\[
F_{3,\mu\nu}^{(10)} = F_{4,\mu\nu}^{(11)}
\]

\[
V_{(10)} = \frac{\alpha_s}{\kappa_8} V_{(12)}
\]

\[
V_{(12)} = d A_{8}^{(10)}
\]

\[
V_{(12)} = d B_{5}^{(10)}
\]
and, \( F_4 = dA_3 - A_1 \wedge F_3 \)

\( A_1 : A_{\mu
\nu} \) from \( G_{10} \).

Finally,

\[
S_{10}^{\text{from AFF}} = - \frac{1}{12k_1^2} \int A_3 \wedge F_4 \wedge F_4
\]

if \( A_3 \ll d\chi^{10} \),

\[
A_\mu^{10} \wedge A_\nu^{10} \Longrightarrow B_{\mu} \wedge B_{\nu} \wedge F_4^{10} \wedge F_4^{10}
\]

if \( F_4 \ll d\chi^{10} \),

\[
F_{\mu\nu}^{(11)} \wedge A_3 \wedge F_3 \wedge F_4
\]

related by parts \( \int F_3 \wedge dA_3 = \int A_3 \wedge dB_2 \equiv \int b_3 \wedge F_3 \).

Three terms to choose, \( \& \) get \( x^3 \)

\[
S_{10}^{\text{from AFF}} = - \frac{1}{4k_1^2} \int B_2 \wedge F_4 \wedge F_4
\]

Can rescale fields to make the \( K_{10} \) as

other terms are quadratic in \( (dA_3) \).
So from $GMN \rightarrow \begin{cases} \text{Metric} & G_{\mu \nu} \\ \text{1-form} & A_{\mu} \\ \text{Scalar} & \phi \end{cases}$

$AMNP \rightarrow \begin{cases} \text{3-form} & A_{\mu \nu \rho} \text{ with } F_{\mu \nu} \text{ field strength} \\ \text{1-form} & B_{\mu} \end{cases}$

After appropriate rescalings (i.e. redefinitions) we arrive at $S_{IIA}$!

How could we guess this from $III$ in $10D$?

Consider a $D0$-brane.

\[ \text{Mass } M = \frac{1}{g_{S} \sqrt{\alpha'}} \]

$N$ $D0$-branes mutually BPS, so

\[ M = \frac{N}{g_{S} \sqrt{\alpha'}}. \]

As $g_{S} \rightarrow 0$, spacing in spectrum gets small,

cf. \( M_{KK} = (\frac{n}{R_{10}})^{2} \)

For \( \hat{R}_{10} = g_{S} \sqrt{\alpha'} \), the spectra match.
\[ g_s = \frac{R_{10}}{\sqrt{\alpha_1}} \]

and \[ K_{10} = \frac{1}{\pi} (2\pi)^7 g_s^2 \alpha_1^{14} \]

so \[ K_{11} = 2\pi R_{10} K_{10} \]

\[ = \frac{1}{2} (2\pi)^8 g_s^3 \alpha_1^{19/6} \]

define \[ M_{11} = \frac{1}{\sqrt{\alpha_1}} g_s^3 \]

so \[ 2K_{11} = (2\pi)^8 (M_{11})^{-1} \]

while correspondingly \[ l_{11} = \frac{1}{M_{11}} = g_s^{-3/2} \sqrt{\alpha_1} \]

recall also \[ l_4^2 = \frac{1}{K_{10}} \int_{\text{internal}} \frac{d^4x}{(2\pi)^4} = \frac{L^2}{K_{10}} \]

\[ l_4 = \frac{L}{\sqrt{2}} (2\pi)^{-7/2} g_s \alpha_1^{1/12} \]

so \[ l_4 \propto g_s \sqrt{\alpha_1} \]

therefore at small \( g_s \), fixed \( L \), \[ l_4 \ll l_{11} \ll l_s = \sqrt{\alpha_1} \]

\[ g_s^{3/4} \rightarrow g_s \cdot \]
Now let's think about charged objects.

III:  DO

A1

D2

A3

(D5)

D4

A3

D6

(A7)

A1

NB. If we use $*F_{(E)} = F_{(M)}$

then $A_p^{(E)} = A_p^{(M)}$.

DD couples to $A_1$, $A_1 \Leftrightarrow \text{G_2}$ ("KK $\delta$"),

DD $\Rightarrow$ P10

metric couples to momentum.

Also here $E \Leftrightarrow B$

F1 $\Rightarrow B_a$

NS5

B2
Where do these D-branes (and NS-branes) come from in 11D?

Postulate: \( E = B \)

\[ M5 \quad A_3 \]

\[ M5 \quad A_3 \]

\[ \left( \int_{\text{5+ around M5-brane}}^{A_3} dA_j = 1 \right) \]

In general, \( dF_p = (\ast j)_{p+1} \)

In D dim, \( (\ast j)_{p+1} \Rightarrow j \) is a D-\( p \)-form corresponding to a (D-p+l)-brane.

\[ g: \text{DO} \Rightarrow \text{form} \quad p=8 \]

and the magnetic potential is then a p-1 form for a D-(p-1)-brane or k-form for a D-k-brane.

so 10D: k-form \( \sim 7-k \)-brane

11D: k-form \( \sim 8-k \)-brane
(M2)_{\mu:} D2

(M2)_{\mu:} F1

(M5)_{\mu:} NSS

\[ dF = \ast j \mu \]

\[ \text{check: } M2 \leftrightarrow A_3 \]
\[ \downarrow \quad \downarrow \]
\[ F1 \leftrightarrow B_2 \]

\[ \text{check: } M2 \leftrightarrow A_3 \]
\[ \downarrow \quad \downarrow \]
\[ NS \leftrightarrow B_2 \]

\[ \text{check: } M2 \leftrightarrow A_3 \]
\[ \downarrow \quad \downarrow \]

\[ \text{D4} \leftrightarrow A_3 \]

\[ \text{if } A_6 \text{ has only } x^{10} \]

\[ \text{A}_3 \text{ does not, } \]

\[ \partial A_3 = \ast dA_6 \]

\[ \text{where is 9D D0-brane?} \]

\[ \text{well, D6 } B A_1 \text{ is dual to D0-brane} \]

\[ \text{D0 carries KK E charge } \int A_{KK} dx \]

\[ \text{so D0 must carry KK B charge, i.e., be} \]

\[ \text{a KK monopole!} \]

\[ \text{NB monopole for 1-form potential is} \]
\[ \text{< point in 4D, or a 6-monopole in 10D.} \]
Tensions:

\[ T_{\text{Fl}} = \frac{1}{2\pi \alpha'} \quad \text{with} \quad \alpha' = \frac{r_0}{1} \]

\[ T_{\text{M2}} = \frac{2\pi}{(2\pi l_\text{Pl})^3} \quad l_\text{Pl} = g_5^2 l_6 \]

Then \( 2\pi T_{\text{M2}} \cdot R_n = \frac{r_0}{2\pi} \cdot g_5^2 \cdot l_6 \cdot \alpha' \]

\[ = \frac{2\pi}{(2\pi l_\text{Pl})^3} = T_{\text{Fl}} \]


What is the KK monopole?

\[ ds_{10}^2 = ds_{10\text{RS}}^2 - dt^2 + ds_{10\text{TN}}^2 \]

\[ ds_{10\text{TN}}^2 = (1 + \frac{R}{r}) (dr^2 + r^2 d\Omega_3^2) + \]

\[ \frac{1}{(1 + \frac{B}{r})} (dy + R \sin^2(\theta) d\phi)^2 \]

with \( B = \nabla \times A_\phi \quad A_\phi = R \sin^2(\theta) B \)

for \( R = g_6 l_6 \quad (R = R_\text{Pl}) \)

we can compute \( \int T_{10} dS = T_{10\text{Fl}} \)
Matrix Theory

We've seen that low-\(v\) collisions of D0-branes know about 11D. Can we go further?

Consider a state in IIA with

\[ P_{10}, q, \text{ mass } m, \text{ momentum in } R^9 \]

Then, \( E = \left( P_{10} + q^2 + m^2 \right)^{1/2} \).

When \( P_{10} \gg q, m \) we have

\[ E \approx P_{10} + \frac{q^2 + m^2}{2P_{10}} \]

\[ = \frac{N}{R_{10}} + \frac{P_{10}}{2N} \left(q^2 + m^2\right) \quad n = \# \text{D0-branes} \]

ie D0 charge

\( \leftrightarrow \) momentum in 10th direction

So if we start with a reasonable state eg some D0's moving around, some excited strings --

\[ \text{etc.} \]

\[ \text{etc.} \]
...and boost in $R_{10}$ direction,

Thus in the new frame, if we constructed similarly as before, we can only use $D0$-branes (with $q \ll P_{10}$)
i.e. nonrelativistic $D0$-branes)

and massless strings on/between them.
(massive strings would give $E \propto 0(1)$.)

Another way to understand this:

If we boost in one direction $(x_0)$

then the dynamics is $\text{Galilean} (NR)$ rather than Lorentzian

\[ E = \frac{P_{10}}{P_{10}} + \frac{q^2}{2P_{10}} + \frac{m^2}{P_{10}} \]

$P_{10}$ serves as a mass for otherwise massless particles.

Boost choice breaks Lorentz invariance, since we're selecting an increase of momentum in $x_0$ direction.
Residual inv: Galilean.
The boost renders the frequencies of excitations of the D0-branes very large (since time is dilated).
So I get three other dust knobs leaving only D0-branes and straight strings.

Upshot: IIA with huge boost in $x_0$ 
$\Leftrightarrow$ system of D0-branes and straight strings
with relativistic dispersion in 10D.

D0-brane QM.

Finally: 1D Lorentz invariance $\Rightarrow$ can describe any M-theory configuration in this way!

QM of D0-branes $\Leftrightarrow$ M-theory?