Homework for Physics 456/656

Introduction to Accelerator Physics and Technology (Hoffstaetter)

Due Date: Thursday, 10/02/03 - 11:40 in 110 Rockefeller Hall

Exercise 1:

(a) The field in a bending magnet has usually two symmetries: Midplane symmetry since the upper and lower part of the magnet are built identically, and a mirror symmetry with respect to the vertical plane, since each pole is built with right/left symmetry when viewed along the beam pipe. Which multipoles, in addition to the main dipole component, satisfy this symmetry and can therefore be associated with such a bending magnet.

(b) Similarly, a focusing magnet has C_2 and midplane symmetry. Which multipoles, in addition to the main quadrupole term, satisfy this symmetry and can therefore appear when such a magnet is built.

(c) Generalize your observation to a magnet which is built with exact C_n symmetry and midplane symmetry. Which multipole terms can the field have?

Exercise 2:

(a) Suppose an air-coil magnet has four wires parallel to a beam pipe with the (x, y) coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$, and $(0, -a)$. The fist and the third wire have the current I, the second and fourth have $-I$. What multipole components Ψ_ν will be created in the center of the beam pipe?

(b) Given an electron beam with $2GeV$ energy and $a = 5$cm. How much current would one need to create a quadrupole component of $k_1 = 0.01m^{-2}$?

Exercise 3:

(a) A matrix $\underline{M}$ is symplectic when it satisfies $\underline{MJM}^T = \underline{J}$. Using $\underline{J}^{-1} = -\underline{J}$ and $\underline{J}^T = -\underline{J}$, show that the following properties are also satisfied:

$$\underline{M}^{-1} = -\underline{J}\,\underline{M}^T\underline{J}\ , \quad \underline{M}^T\underline{J}\,\underline{M} = \underline{J}\ . \tag{1}$$

(b) The linear transport map of a quadrupole is given by

$$\begin{pmatrix} x \\ p_x \end{pmatrix} = \begin{pmatrix} \cos(\sqrt{k}s) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}s) \\ -\sqrt{k}\sin(\sqrt{k}s) & \cos(\sqrt{k}s) \end{pmatrix} \begin{pmatrix} x_0 \\ p_{x0} \end{pmatrix} \tag{2}$$

when k is the strength of the quadrupole field. Derive the generating function $F_1(x, x_0, s)$ that represents this map.