Homework for Physics 456/656

Introduction to Accelerator Physics and Technology (Hoffstaetter)

Due Date: Thursday, 10/09/03 - 11:40 in 110 Rockefeller Hall

Exercise 1:
Given a reference trajectory that is a helix around the z-axis with

$$\vec{R}(z) = r\cos(kz)\vec{e}_X + r\sin(kz)\vec{e}_Y + z\vec{e}_Z, \tag{1}$$

with the Cartesian coordinate vectors $\vec{e}_X$, $\vec{e}_Y$ and $\vec{e}_Z$.
(a) Show that z is not the pathlength s with which the reference trajector is parametrized. Then compute the path length $s(z)$ and specify $\vec{R}(s)$ so that $|d\vec{R}| = ds$ and compute $\vec{e}_s$, $\vec{e}_\kappa$, and $\vec{e}_b$.
(b) Compute $\vec{e}_x$ and $\vec{e}_y$ of the curvilinear system and check that $\frac{d}{ds}\vec{e}_x$ and $\frac{d}{ds}\vec{e}_y$ are what they are specified to be in the handouts.

Exercise 2:
What symmetry property does the Hamilton function $H(x, a, y, b, \tau, \delta, s)$ have when the motion is mid-plane symmetric?

Exercise 3:
Let the linearized particle transport from initial phase space coordinates $\vec{z}_i$ to final phase space coordinates $\vec{z}_f$ be:

$$\begin{pmatrix} x_f \\ a_f \\ \tau_f \\ \delta_f \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & 0 & D_x \\ M_{21} & M_{12} & 0 & D_a \\ T_x & T_a & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ a_i \\ \tau_i \\ \delta_i \end{pmatrix} . \tag{2}$$

The zeros in the matrix show that the particle motion is independent of the starting time and that the energy is independent of the starting conditions.
(a) Describe the meaning of the coefficients D_x, D_a, T_x, and T_a.
(b) Use the fact that this 4×4 matrix is symmplectic to show that the top left-hand 2×2 sub-matrix is symplectic.
(c) Show how T_x and T_a can be computed when this top left-hand sub-matrix and the dispersion D_x and its slope D_a are known.