Homework for Physics 456/656

Introduction to Accelerator Physics and Technology (Hoffstaetter)

Due Date: Thursday, 10/23/03 - 11:40 in 110 Rockefeller Hall

Exercise 1:
Consider that the leading order aberrations of a solenoid, which for example have the dependence $w_0^2\bar{w}_0$, to show that a solenoid that is shifted by a complex displacement $\Delta e^{i\theta}$ has additional aberrations with dipole symmetry (C_1) in first order of Δ and with quadrupole symmetry (C_2) in second order of Δ.

Exercise 2:
(a) Given the Twiss parameters α, β, γ: specify the transformation from the amplitude and phase variables J and ϕ to the Cartesian phase space variables x and x'.
(b) Specify the inverse transformation.
(c) Given the Gaussian beam distribution in amplitude and phase variables, $\rho(J,\phi) = \frac{1}{2\pi\epsilon}e^{-\frac{J}{\epsilon}}$. What is the projection $\rho(x)$ of this distribution on the x axis. Check that the rms width of this distribution leads to $\sqrt{< x^2 >} = \sqrt{\beta\epsilon}$.

Exercise 3:
Find the general form of the beta function in a drift:
(a) by solving the differential equation for $\beta(s)$ with the initial conditions $\beta(0) = \beta_0$ and $\alpha(0) = \alpha_0$.
(b) by propagating the matrix of initial Twiss parameters by the transport matrix of a drift according to

$$\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \underline{M} \begin{pmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{pmatrix} \underline{M}^T \ . \tag{1}$$

(c) Find the general form of a beta function in a quadrupole of focusing strength k. Do not use the thin lens approximation.