Homework for Physics 456/656

Introduction to Accelerator Physics and Technology (Hoffstaetter)

Due Date: Thursday, 10/30/03 - 11:40 in 110 Rockefeller Hall

Exercise 1:

(a) Find the Twiss parameters of the phase space ellipse that is periodic for a FODO Cell in thin lens approximation, i.e. all particles that enter a FODO cell on this phase space ellipse exit the cell on the same ellipse. Let the focusing and defocussing quadrupole have the strength k and $-k$. Furthermore, let the cell start with half a focusing quadrupole, and let the distance between quadrupoles be $L/2$ so that the transport matrix of the cell is given by

$$\underline{M} = \underline{Q}(\frac{k}{2})\underline{D}(\frac{L}{2})\underline{Q}(-k)\underline{D}(\frac{L}{2})\underline{Q}(\frac{k}{2}) \ . \tag{1}$$

(b) Characterize how this phase space ellipse changes along the FODO cell by drawing ellipses in phase space at various points along the cell. Do this for the horizontal and the vertical plane separately.

(c) Compute the periodic dispersion (η, η') for this FODO cell, assuming that there is a thin lens dipole with bending angle ϕ in the center between both quadrupoles.

(d) For what betatron phase advance (in degree) along the FODO is the maximum beta function in the FODO the smalest?

(e) Let the FODO have quadrupoles of 1m length, 5cm bore radius, and a pole tip field of 1T for protons of 40GeV. How long does the FODO have to be to obtain 60° phase advance?

Exercise 2:

Characterize Twiss parameters by $\{\beta(s), \alpha(s), \psi(s)\}$. Imagine two sections of a beam line where the first section transports Twiss parameters $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_1, \alpha_1, \psi_1\}$ and the second transports $\{\beta_1, \alpha_1, 0\}$ to $\{\beta_2, \alpha_2, \psi_2\}$. Show that the total beam-line transports $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_2, \alpha_2, \psi_1 + \psi_2\}$.