Stationary states in the square well

\[ \alpha = \sqrt{\frac{2m}{\hbar^2}} (V_0 - E) \]

\[ k = \sqrt{\frac{2m}{\hbar^2}} E, \quad k_v = \sqrt{\frac{2m}{\hbar^2}} V_0 \]

\[ z = \frac{L}{2} : \quad kA e^{-\alpha \frac{k}{2}} = kB \sin(k \frac{L}{2} + \varphi) \]

\[ -\alpha A e^{-\alpha \frac{k}{2}} = kB \cos(k \frac{L}{2} + \varphi) \]

\[ \pm \frac{k}{k_v} = \sin(k \frac{L}{2} + \varphi) \]

\[ \mp \alpha \frac{k}{k_v} = k \cos(k \frac{L}{2} + \varphi) \]

\[ \frac{k}{k_v} = -\text{sign}[\cos(k \frac{k_v L}{2} + \varphi)] \sin(k \frac{k_v L}{2} + \varphi) \]
Graphical determination of possible wave numbers

\[ k = \frac{2}{L} \left( \frac{kL}{2} \right) \]

\[ \frac{k}{k_v} = \left| \sin\left(\frac{kL}{2} + \varphi\right) \right| \]

and

\[ 0 > \tan\left(\frac{kL}{2} + \varphi\right) \]

with

\[ \varphi = 0 \quad \text{or} \quad \varphi = \frac{\pi}{2} \]
Square well versus Infinite well

Stationary states for the finite well:

\[ \frac{k}{k_V} = \cos\left(\text{mod}\left[\frac{kL}{2}, \frac{\pi}{2}\right]\right) \]

Stationary states for the infinite well:

\[ k = n \frac{\pi}{L}, \quad \text{mod}\left[\frac{kL}{2}, \frac{\pi}{2}\right] = 0 \]

- The finite potential well with height \( V_0 \) has one stationary state more than the infinite potential well has states with energies below \( V_0 \).

- Number of solutions = \( \text{int}\left[\frac{k_V L}{\pi}\right] + 1 = \text{int}\left[\frac{L}{\pi}\sqrt{\frac{2m}{\hbar^2} V_0}\right] + 1 \)

- There is at least one bound state, and it is symmetric.
Graphic determination of states for the square well

Finite well:

\[ k = k_v \cos(\text{mod}[\frac{kL}{2}, \frac{\pi}{2}]) \]

Infinite well:

\[ k = n \frac{\pi}{L} \]

\[ E = \frac{\hbar^2}{2m} k^2 \]