# Orthogonality for different Energies



It is a feature of the wave functions of any potential **V(x)** that the stationary wave functions are orthogonal to each other. $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Phi_E + V(x)\Phi_E = E\Phi_E$

$$\int_{-\infty}^{\infty} E_2\Phi^*_{E_1}\Phi_{E_2}dx = \int_{-\infty}^{\infty}\Phi^*_{E_1}[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Phi_{E_2} + V(x)\Phi_{E_2}]dx$$

$$\int_{-\infty}^{\infty}\Phi^*_{E_1}\frac{\partial^2}{\partial x^2}\Phi_{E_2}dx = \underbrace{\left[\Phi^*_{E_1}\frac{\partial}{\partial x}\Phi_{E_2}\right]_{-\infty}^{\infty}}_{0} - \int_{-\infty}^{\infty}\frac{\partial}{\partial x}\Phi^*_{E_1}\frac{\partial}{\partial x}\Phi_{E_2}dx$$

$$\int_{-\infty}^{\infty} E_2\Phi^*_{E_1}\Phi_{E_2}dx = \int_{-\infty}^{\infty}[\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\Phi^*_{E_1}\frac{\partial}{\partial x}\Phi_{E_2} + \Phi^*_{E_1}V(x)\Phi_{E_2}]dx$$

$$\int_{-\infty}^{\infty} E_1\Phi^*_{E_1}\Phi_{E_2}dx = \int_{-\infty}^{\infty}[\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\Phi^*_{E_1}\frac{\partial}{\partial x}\Phi_{E_2} + \Phi^*_{E_1}V(x)\Phi_{E_2}]dx$$

$$(E_1 - E_2)\int_{-\infty}^{\infty}\Phi^*_{E_1}\Phi_{E_2}dx = 0 \quad \rightarrow \quad \boxed{\int_{-\infty}^{\infty}\Phi^*_{E_n}\Phi_{E_m}dx = \delta_{nm}} = \begin{cases} 0 & \text{if } E_n \neq E_m \\ 1 & \text{if } E_n = E_m \end{cases}$$



Georg.Hoffstaetter@Cornell.edu

# 6 Photons and quantum states



A electromagnetic plain wave can be uniquely defined by specifying four things:

1. Frequency
2. Direction of propagation
3. Polarization
4. Amplitude of the electric field (and thereby the energy in the field)

Similarly the state of a photon is uniquely defined by specifying three things:

1. Frequency (and therefore the energy of the photon **h**$\nu$)
2. Direction of motion
3. Polarization state

The energy in the wave is then determined by the number of photons.

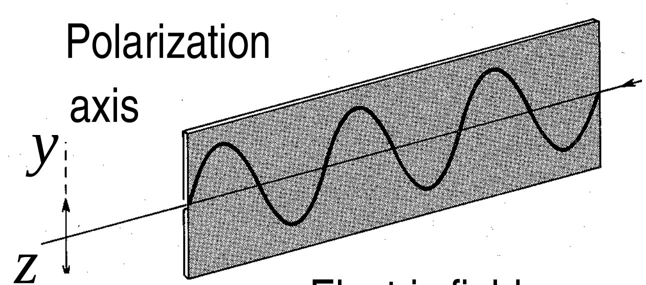
The **state of a photon** completely determines a photon in the following sense:
Everything that can be known about a photon is specified.

Cornell

Georg.Hoffstaetter@Cornell.edu

# Linearly polarized waves

A beam of light is linearly polarized if its electric Vector lies in a single plane that includes the beam (the plane of polarization). The polarization axis is parallel to direction of the electric field.




$$\vec{E} = \vec{E}_{pol}\cos(kz - \omega t + \varphi_0) = \mathrm{Re}[\vec{E}_{pol}e^{i\varphi_0}e^{i(kz-\omega t)}]$$

Any plane wave with wave number $k = \frac{2\pi}{\lambda}$ and angular frequency $\omega = 2\pi\nu = ck$ can be described by a **superposition** of two orthogonally polarized waves:

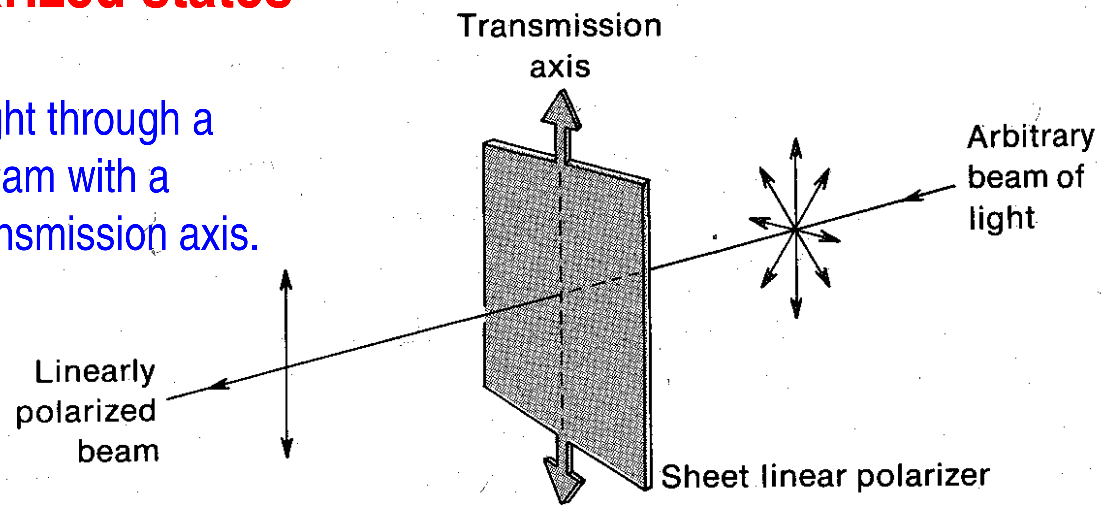
$$\begin{aligned}\vec{E} &= \mathrm{Re}[\vec{a}e^{i(kz-\omega t+\alpha)} + \vec{b}e^{i(kz-\omega t+\beta)} + \vec{c}e^{i(kz-\omega t+\chi)} + \ldots] \\ &= \vec{e}_x\,\mathrm{Re}[(a_1e^{i\alpha} + b_1e^{i\beta} + c_1e^{i\chi} + \ldots)e^{i(kz-\omega t)}] \\ &+ \vec{e}_y\,\mathrm{Re}[(a_2e^{i\alpha} + b_2e^{i\beta} + c_2e^{i\chi} + \ldots)e^{i(kz-\omega t)}] \\ &= \mathrm{Re}[\vec{e}_x a_x e^{i\varphi_x}e^{i(kz-\omega t)} + \vec{e}_y a_y e^{i\varphi_y}e^{i(kz-\omega t)}]\end{aligned}$$

CORNELL

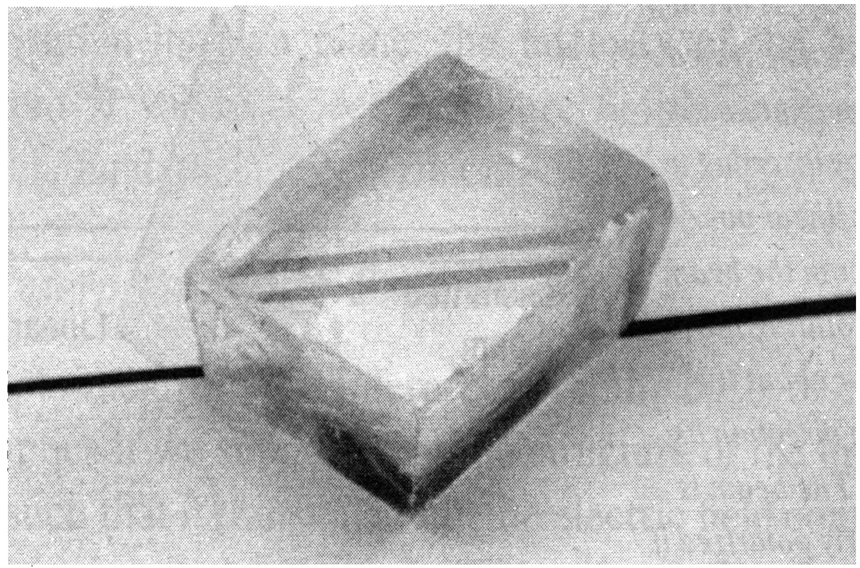
Georg.Hoffstaetter@Cornell.edu

# Production of linearly polarized states

1) Passing of an arbitrary beam of light through a **polarizer** produces a polarized beam with a polarization axis parallel to the transmission axis.




2) A **calcite crystal** has a different refractive index for different polarization directions and therefore splits an arbitrary beam into two beams with two perpendicular polarization directions.



These polarizers can also be used to determine whether a beam is in a linearly polarized state and the polarization direction can be found.



Georg.Hoffstaetter@Cornell.edu