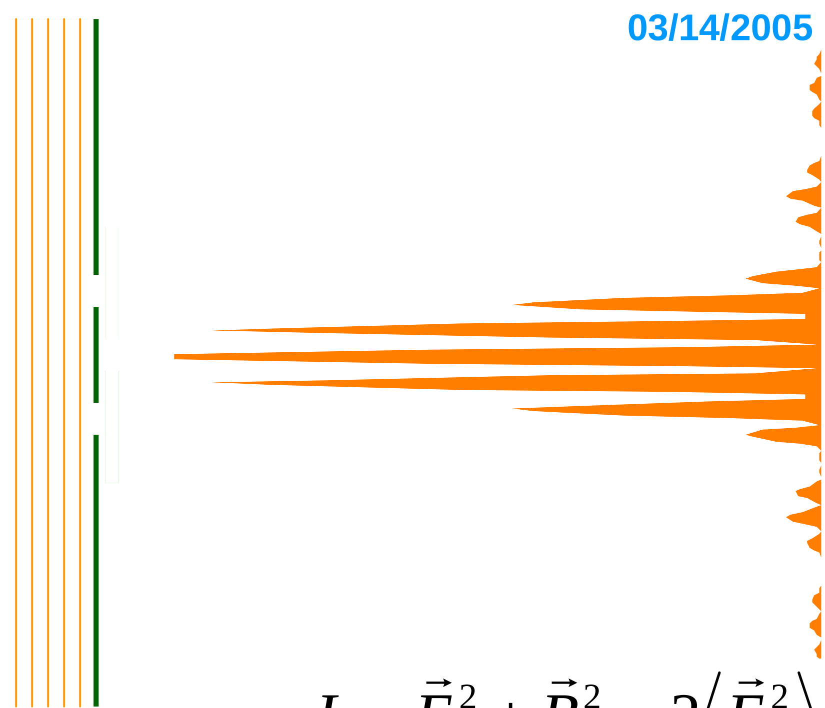
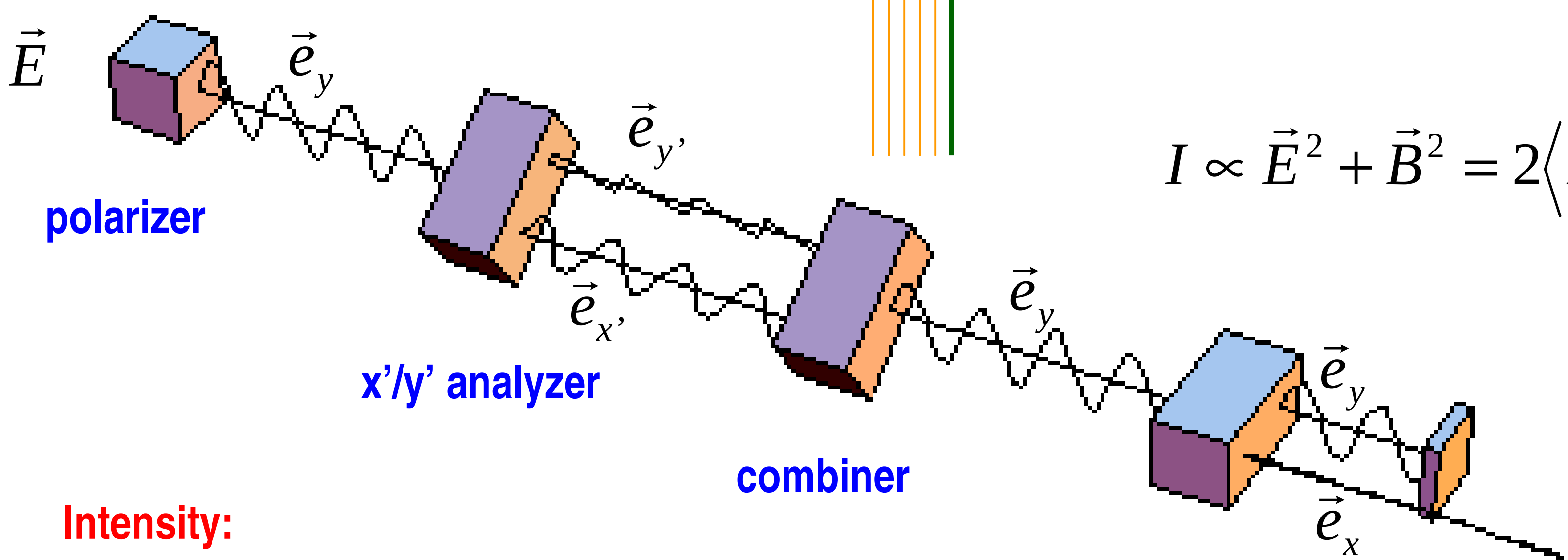
7) Quantum amplitudes and state vectors

Interference of light



Interference of waves:

There are places which have intensity when either one of the slits is open, but nevertheless have no intensity when both slits are open.



$$I \propto \vec{E}^2 + \vec{B}^2 = 2\left\langle \vec{E}^2 \right\rangle_t$$

x/y analyzer

$$I \propto 2\left\langle \vec{E}^2 \right\rangle_t$$

Intensity:

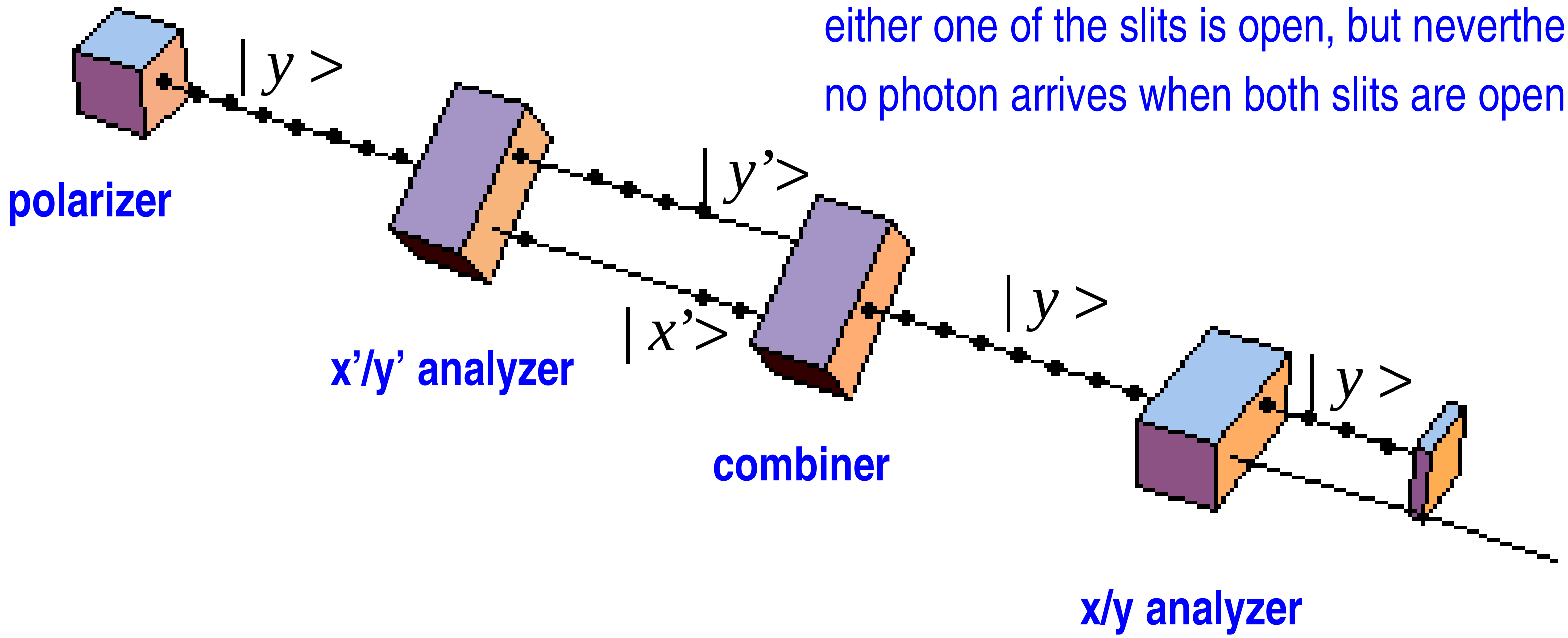
To obtain the intensity one has to square the filed vector. The field is obtained by the vector sum of the fields for the different path that components of the wave can take.

CORNELL

Georg.Hoffstaetter@Cornell.edu

Interference for a photon state

Interference of photons:
Photons arrive at the output when either one of the slits is open, but nevertheless no photon arrives when both slits are open.

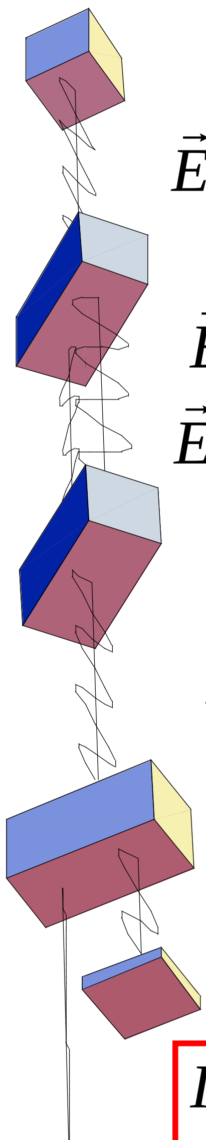


The output **intensity** of a light wave corresponds to the number of photons and therefore to the **probability** that a photon is found in the output channel. To have photon probabilities correspond to wave intensities, one introduces a **state vector** that describes photons and **can interfere** like the field vector of the wave.

Georg.Hoffstaetter@Cornell.edu

Projection amplitudes and probability

03/14/2005



$$\vec{E} \propto \vec{e}_y = \sin\vartheta\,\vec{e}_{x'} + \cos\vartheta\,\vec{e}_{y'}$$

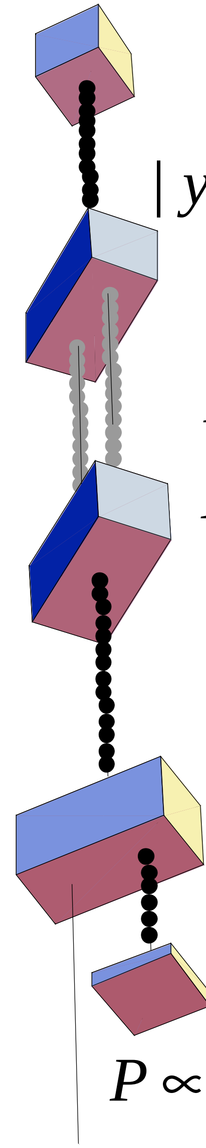
$$\vec{E}_1 \propto \vec{e}_{x'}(\vec{e}_{x'}\cdot\vec{e}_y),\ I_1 \propto |\vec{e}_{x'}\cdot\vec{e}_y|^2$$

$$\vec{E}_2 \propto \vec{e}_{y'}(\vec{e}_{y'}\cdot\vec{e}_y),\ I_2 \propto |\vec{e}_{y'}\cdot\vec{e}_y|^2$$

$$\vec{E}_1 + \vec{E}_2$$

$$\vec{E} \propto \vec{e}_x$$

$$I \propto |(\vec{e}_x\cdot\vec{e}_{x'})(\vec{e}_{x'}\cdot\vec{e}_y) + (\vec{e}_x\cdot\vec{e}_{y'})(\vec{e}_{y'}\cdot\vec{e}_y)|^2$$



$$|y>$$

$$P_1 = |<y'|y>|^2$$

$$P_2 = |<x'|y>|^2$$

$$P \propto |<x|x'><x'|y> + <x|y'><y'|y>|^2$$

Georg.Hoffstaetter@Cornell.edu

To keep in touch:

- Read French&Taylor, An Introduction to Quantum Physics, Section 6 and 7.
  (As mentioned on the Homework handout)

$$z = a + ib \;\rightarrow\; \mathrm{Re}\{z\} = a, \quad \mathrm{Im}\{z\} = b$$

$$\mathrm{Re}\{i\,z\} = \mathrm{Re}\{i(a+ib)\} = -b = -\mathrm{Im}\{z\}$$

$$\mathrm{Im}\{i\,z\} = \mathrm{Im}\{i(a+ib)\} = a = \mathrm{Re}\{z\}$$

$$\boxed{e^{i\varphi} = \cos\varphi + i\sin\varphi} \;\rightarrow\; \cos\varphi = \mathrm{Re}\{e^{i\varphi}\}, \quad \sin\varphi = \mathrm{Im}\{e^{i\varphi}\}$$

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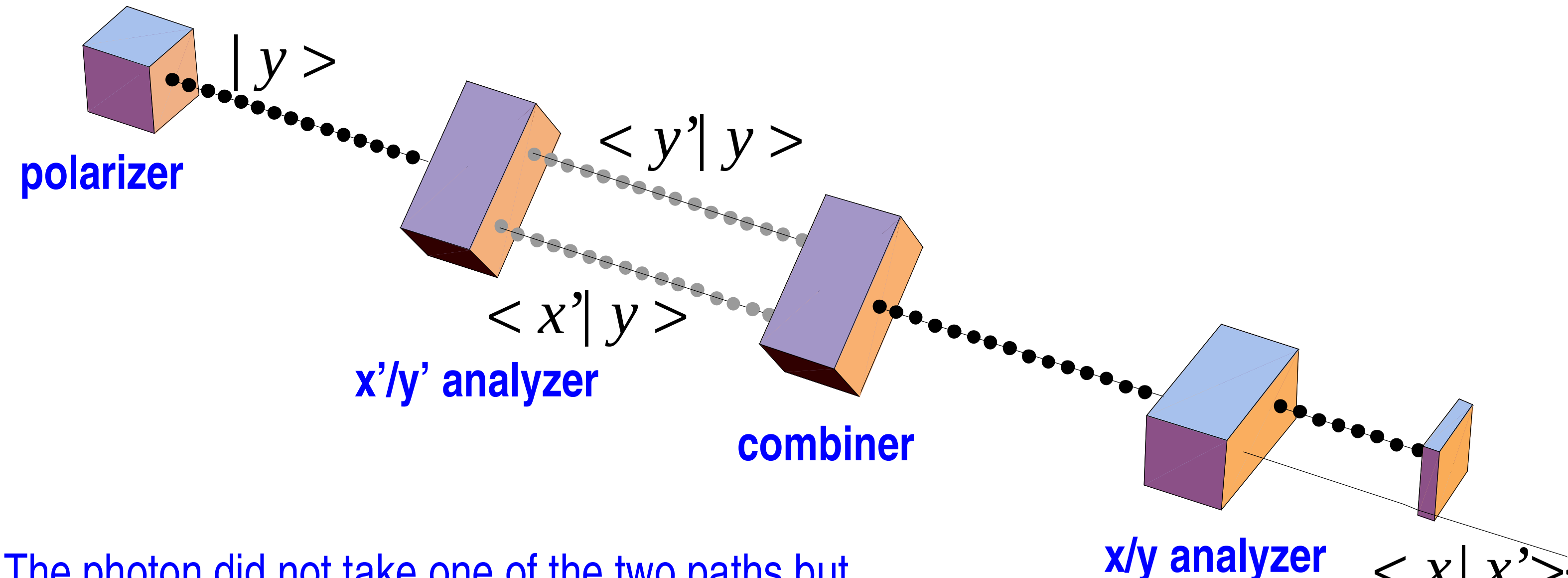
Georg.Hoffstaetter@Cornell.edu

The paths of a photon



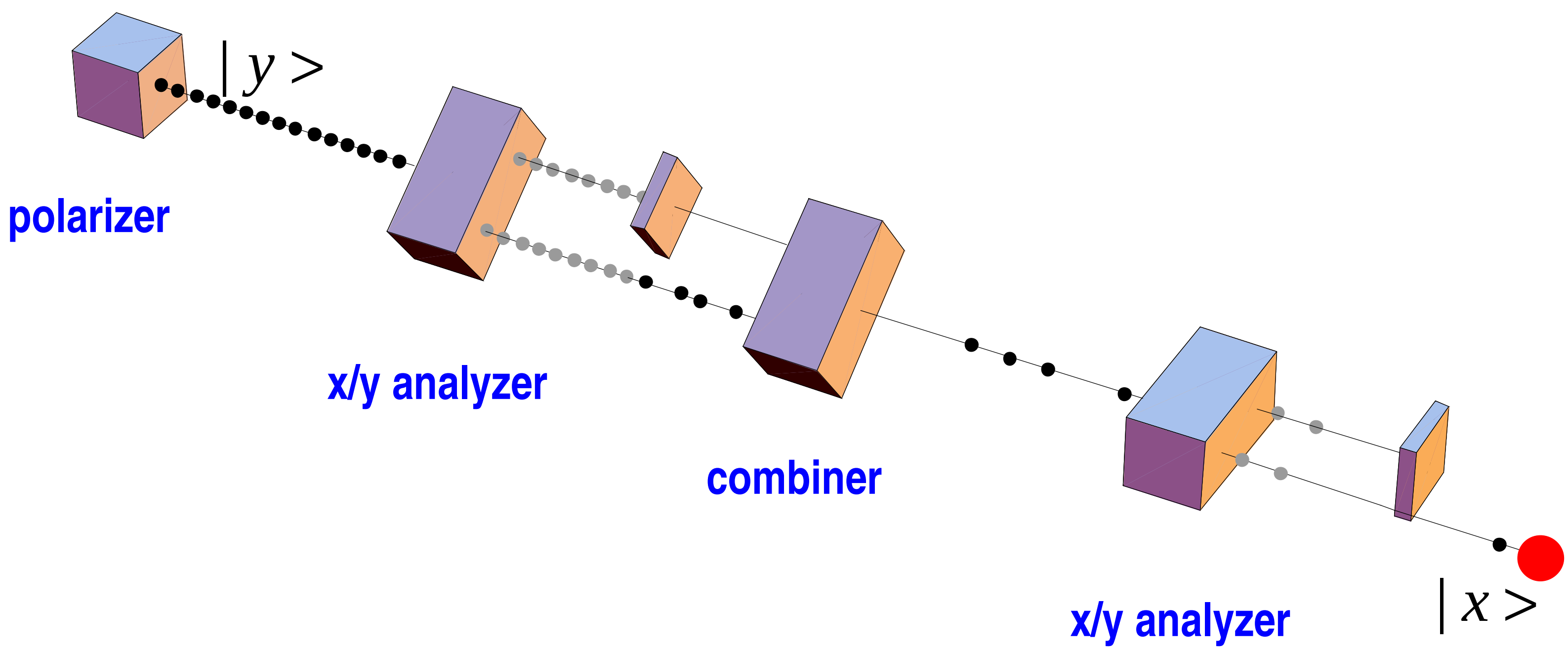
$$< x | x' >< x' | y > + < x | y' >< y' | y >$$

The photon did not take one of the two paths but has a projection amplitude for **both paths**.

Probability and projection amplitude:

1) To obtain the probability that a photon is found in the output state one has to square the projection amplitude for a transition from the input to the output state.
2) This projection amplitude is found by summing the projection amplitudes for the different paths that the photon can take to the output state.
3) The projection amplitude for one path is found by multiplying the projection amplitudes for each step along the path.

Blocking the path of a photon



Blocking one path changes the state of every photon,
since **no photons avoids the blocked path** completely.

Georg.Hoffstaetter@Cornell.edu