

Quantum amplitudes for circular polarization

The field of any plane light wave with frequency ν can be written as

$$\vec{E} = \text{Re}\left[\left(\frac{\vec{e}_x - i\vec{e}_y}{\sqrt{2}} B_R + \frac{\vec{e}_x + i\vec{e}_y}{\sqrt{2}} B_L\right) e^{i(kz - \omega t)}\right]$$

The Intensity in the **R** channel after an **R/L** analyzer:

$$\begin{aligned} I_R &\propto 2 \left\langle \text{Re}\left[\frac{\vec{e}_x - i\vec{e}_y}{\sqrt{2}} B_R e^{i(kz - \omega t)}\right]^2 \right\rangle_t \\ &= 2 \left\langle \frac{1}{4} [2B_R B_R^* \frac{\vec{e}_x - i\vec{e}_y}{\sqrt{2}} \frac{\vec{e}_x + i\vec{e}_y}{\sqrt{2}}] \right\rangle_t = B_R B_R^* \end{aligned}$$

Intensity in the **L** channel: $I_L \propto B_L B_L^*$

The corresponding state vector describes a photon which is in the state $|R\rangle$ with probability $|B_R|^2$ and in the state $|L\rangle$ with probability $|B_L|^2$.

The interferences of the photons correspond to the interferences of the field when the phases and the probability amplitudes are chosen according to the amplitudes and phases of the field components by using complex quantum amplitudes

$$|\Psi\rangle = |R\rangle B_R + |L\rangle B_L$$

Georg.Hoffstaetter@Cornell.edu



Formal correspondence to polarization states

Polarization states

Orthogonality

$$\langle x | y \rangle = 0, \quad \langle R | L \rangle = 0$$

Stationary states of the wave function

Orthogonality

$$\int_{-\infty}^{\infty} \Phi_n^*(x) \Phi_m(x) dx = \delta_{nm}$$

Completeness: Every states can be described in terms of the basis states.

$$|\Psi\rangle = |x\rangle A_x + |y\rangle A_y$$

$$f(x) = \sum_{n=0}^N \Phi_n(x) A_n$$

The expansion amplitudes are found by projections; they are projection amplitudes.

$$A_x = \langle x | \Psi \rangle$$

$$A_n = \int_{-\infty}^{\infty} \Phi_n^*(x) f(x) dx$$

Due to this correspondence, also the stationary states of the wave function are written as **ket vector**:

$$\Phi_n(x) \rightarrow |n\rangle \text{ with } \langle n | m \rangle = 0$$

$$|f\rangle = \sum_{n=0}^N |n\rangle A_n \text{ with } A_n = \langle n | f \rangle$$



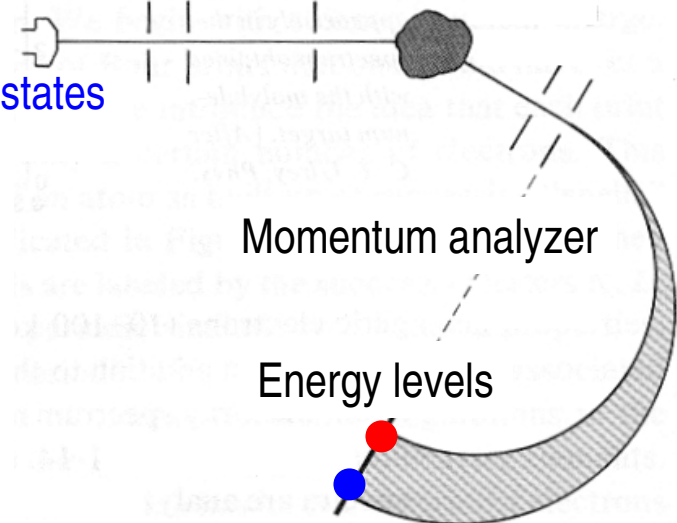
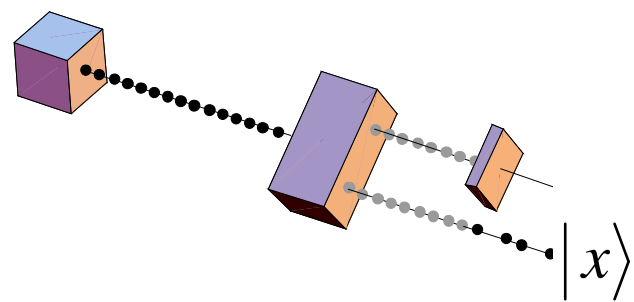
Physical correspondence to polarization states

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Polarization states

States of the wave function

Analyzers: can separate the components of the different basis states



Measurement: changes the state of the quantum particle.

$$|\Psi\rangle = |x\rangle A_x + |y\rangle A_y$$

$$|x\rangle$$

$$f(x) = \sum_{n=0}^N \Phi_n(x) A_n$$

$$\Phi_n(x)$$

With probability

$$P_x = A_x A_x^* = |\langle x | \Psi \rangle|^2$$

With probability

$$P_n = A_n A_n^*$$

$$\sum_{n=0}^N |A_n|^2 = 1$$

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