The energy time uncertainty relation

Wave at fixed time $t$:

Wave traveling passed a fixed position $x$:

\[
\begin{align*}
\Delta t & \approx \frac{\Delta x}{v_{\text{group}}} \\
\Delta \omega & \approx \Delta k \frac{d}{dk} \omega = \Delta k v_{\text{group}}
\end{align*}
\]

\[\Delta t \Delta \omega \approx \Delta x \Delta k \geq 1/2\]

\[\Delta t \Delta E \geq \frac{\hbar}{2}\]

This uncertainty relation is again due to a Fourier transform, here between time and frequency:

\[
\Psi(x, t) = \int_{\text{all } \omega} A_{\omega}(x) \frac{e^{-i\omega t}}{\sqrt{2\pi}} d\omega \quad \Leftrightarrow \quad A_{\omega}(x) = \int_{\text{all } t} \Psi(x, t) \frac{e^{i\omega t}}{\sqrt{2\pi}} dt
\]
The shape and width of energy levels

The time-energy uncertainty relation relates more things than the simple picture suggest:

1) (a) Lifetime of excited states of atoms and nuclei $\tau$ with $N(t) = N_0 e^{-t/\tau}$

(b) Frequency spread $\Delta \omega$ of the emitted radiation or energy spread $\Delta E$ of the emitted particles

2) (a) The probability of the excited particle to have dropped back to its ground state energy, and therefore the probability to measure an emitted photon is $|\psi(t)|^2 \propto e^{-t/\tau}$

(b) The probability to find a photon with frequency $\omega$ is $|A_\omega|^2$ where $A_\omega$ is related to

$$\psi(t) = e^{-\frac{t}{2\tau}} e^{-i\omega_0 t}$$

by a Fourier transform:

$$A_\omega \propto \int_0^\infty e^{-\frac{t}{2\tau} - i\omega_0 t} e^{i\omega t} dt = \left[\frac{1}{i(\omega - \omega_0) - \frac{1}{2\tau}} e^{t\left(i(\omega - \omega_0) - \frac{1}{2\tau}\right)}\right]_0^\infty = \frac{1}{\frac{1}{2\tau} - i(\omega - \omega_0)}$$

This is called a Lorentz curve around $\omega_0$ with full width half max of $\Delta \omega = \frac{1}{2\tau}$

0.4 $\cdot 10^{-23}$ s $\approx \frac{10^{-15} \text{ m}}{c}$

Activity of a radiating sample

Mass of the $\rho$ meson

770MeV

170MeV

770MeV

Georg.Hoffstaetter@Cornell.edu
9) Particle scattering and barrier penetration

Scattering of a particle

The particle is described by a wave packet.

In analogy to water waves: Part of the wave passes the barrier, part is reflected.

A wave packet consists of monoenergetic waves.

Scattering of an infinite mono energetic wave

Rather than to look at time dependent wave packets, one finds the time independent scattering result of a mono energetic wave.