Transverse fields

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TE-modes: Once B_z is know, the full field can be found

$$\vec\nabla_\perp\times\vec E_z+ik_z\vec e_z\times\vec E_\perp=i\omega\vec B_\perp$$

$$\vec\nabla_\perp\times\vec B_z+ik_z\vec e_z\times\vec B_\perp=-i\omega\tfrac{1}{c^2}\vec E_\perp$$

$$k_z\vec E_\perp+\omega\vec e_z\times\vec B_\perp=\vec e_z\times(\vec\nabla_\perp\times\vec E_z)=-i\vec\nabla_\perp E_z$$

$$\omega\tfrac{1}{c^2}\vec E_\perp+k_z\vec e_z\times\vec B_\perp=i\vec\nabla_\perp\times\vec B_z$$

$$(\tfrac{\omega^2}{c^2}-k_z^2)\vec E_\perp=i(k_z\vec\nabla_\perp E_z+\omega\vec\nabla_\perp\times\vec B_z)$$

$$(\tfrac{\omega^2}{c^2}-k_z^2)\vec e_z\times\vec B_\perp=-i(\tfrac{\omega}{c^2}\vec\nabla_\perp E_z+k_z\vec\nabla_\perp\times\vec B_z)$$

$$\vec E_\perp=\frac{i}{\frac{\omega^2}{c^2}-k_z^2}(k_z\vec\nabla_\perp E_z+\omega\vec\nabla_\perp\times\vec B_z)$$

$$\vec B_\perp=\frac{i}{\frac{\omega^2}{c^2}-k_z^2}(k_z\vec\nabla_\perp B_z-\tfrac{\omega}{c^2}\vec\nabla_\perp\times E_z)$$

Rectangular Wave Guide

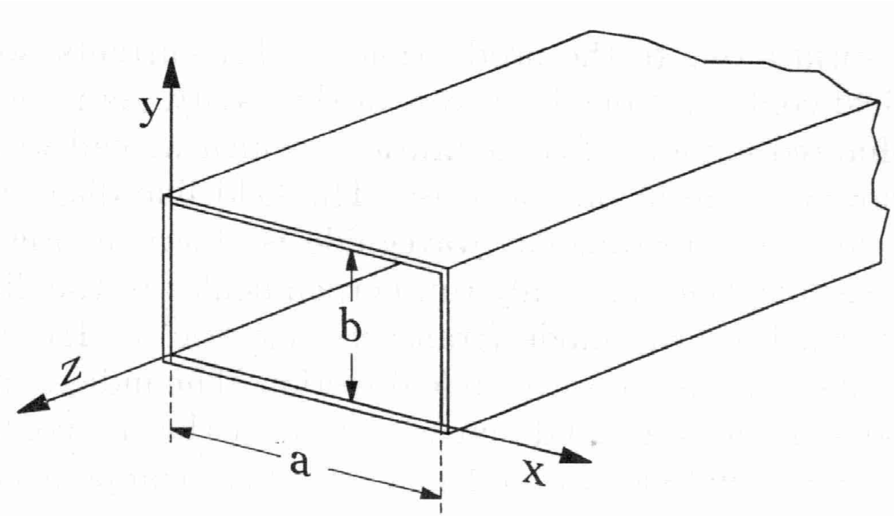
Boundary conditions:

$$E_z(\vec{x}_0) = 0 \quad \vec{\nabla}_\perp^2 E_z = [k_z^2 - (\tfrac{\omega}{c})^2] E_z$$

$$E_z(\vec{x}) = E_0 \sin(\tfrac{n\pi}{a} x) \sin(\tfrac{m\pi}{b} y)$$

$$(\tfrac{\omega}{c})^2 - k_z^2 = k_{nm}^{(B)2} = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2$$

$$\partial_r B_z(\vec{x}_0) = 0 \quad \vec{\nabla}_\perp^2 B_z = [k_z^2 - (\tfrac{\omega}{c})^2] B_z$$



$$B_z(\vec{x}) = B_0 \cos(\tfrac{n\pi}{a} x) \cos(\tfrac{m\pi}{b} y)$$

$$(\tfrac{\omega}{c})^2 - k_z^2 = k_{nm}^{(E)2} = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2$$

TE and TM modes happen to have the same eigenvalues.

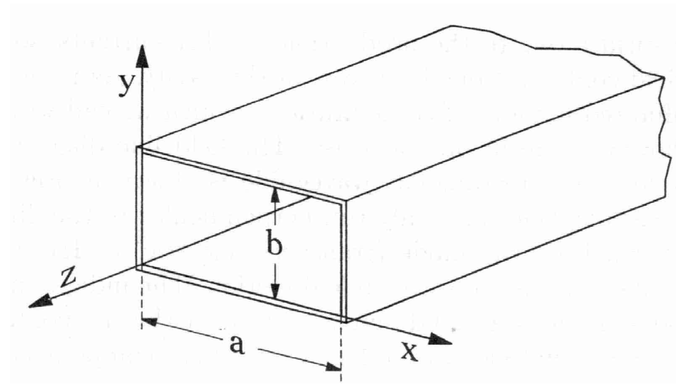
For simplicity one still looks at TE and TM modes separately.

Rectangular TE Modes

$$E_z(\vec{x}) = 0\,, \quad B_z(\vec{x}) = B_0 \cos(\tfrac{n\pi}{a}x)\cos(\tfrac{m\pi}{b}y)$$

$$\vec{E}_\perp = \frac{i}{\frac{\omega^2}{c^2}-k_z^2}(k_z\vec{\nabla}_\perp E_z + \omega\vec{\nabla}_\perp \times \vec{B}_z)$$

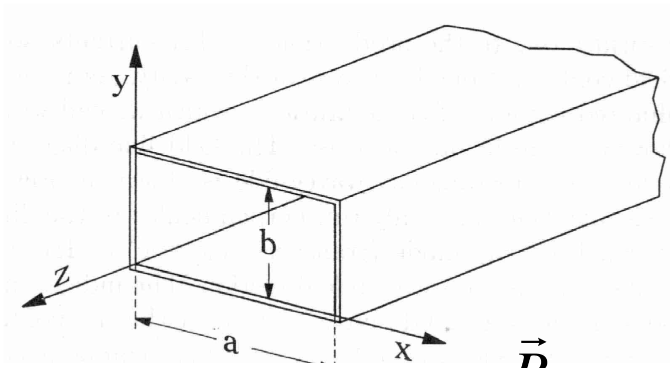
$$\vec{B}_\perp = \frac{i}{\frac{\omega^2}{c^2}-k_z^2}(k_z\vec{\nabla}_\perp B_z - \tfrac{\omega}{c^2}\vec{\nabla}_\perp \times \vec{E}_z)$$

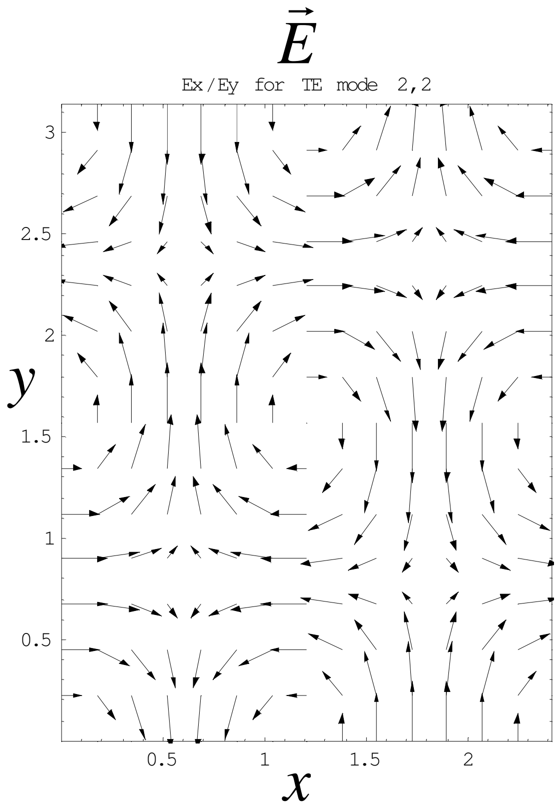


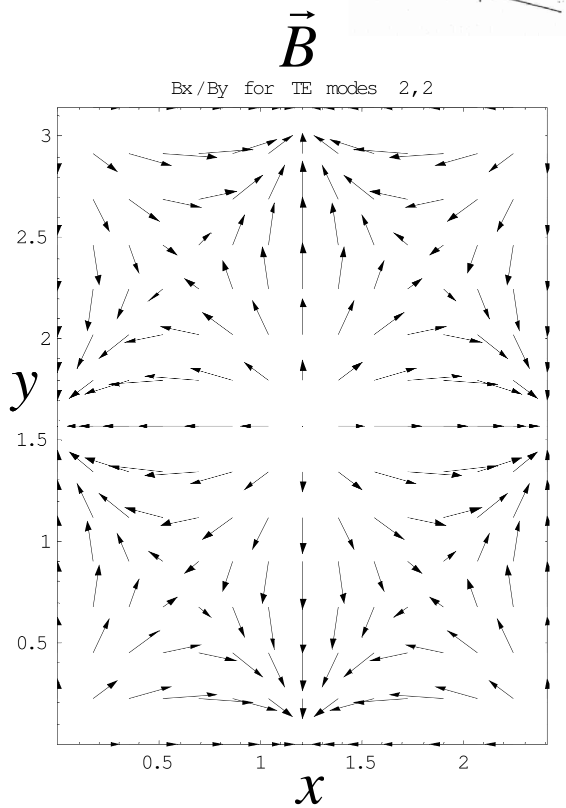
$$\vec{E}(\vec{x}) = \frac{\omega}{k_{nm}^{(E)2}} B_0 \begin{pmatrix} \frac{m\pi}{b}\cos(\frac{n\pi}{a}x)\sin(\frac{m\pi}{b}y)\sin(k_z z-\omega t) \\ -\frac{n\pi}{a}\sin(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)\sin(k_z z-\omega t) \\ 0 \end{pmatrix}$$

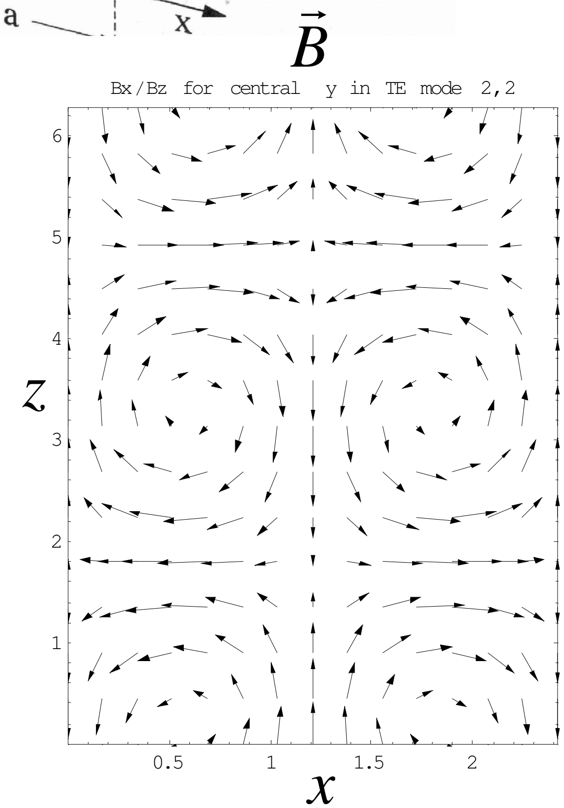
$$\vec{B}(\vec{x}) = \frac{k_z}{k_{nm}^{(E)2}} B_0 \begin{pmatrix} \frac{n\pi}{a}\sin(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)\sin(k_z z-\omega t) \\ \frac{m\pi}{b}\cos(\frac{n\pi}{a}x)\sin(\frac{m\pi}{b}y)\sin(k_z z-\omega t) \\ \frac{k_{nm}^{(E)2}}{k_z}\cos(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)\cos(k_z z-\omega t) \end{pmatrix}$$

Rectangular TE_{22} Mode









Rectangular TM Modes

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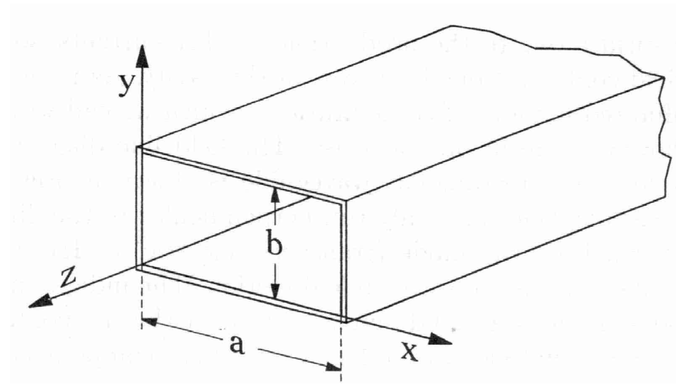
$$E_z(\vec{x}) = E_0 \sin(\tfrac{n\pi}{a} x)\sin(\tfrac{m\pi}{b} y)\,, \quad B_z = 0$$

$$\vec{E}_\perp = \frac{i}{\frac{\omega^2}{c^2} - k_z^2}(k_z \vec{\nabla}_\perp E_z + \omega \vec{\nabla}_\perp \times \vec{B}_z)$$

$$\vec{B}_\perp = \frac{i}{\frac{\omega^2}{c^2} - k_z^2}(k_z \vec{\nabla}_\perp B_z - \tfrac{\omega}{c^2} \vec{\nabla}_\perp \times E_z)$$

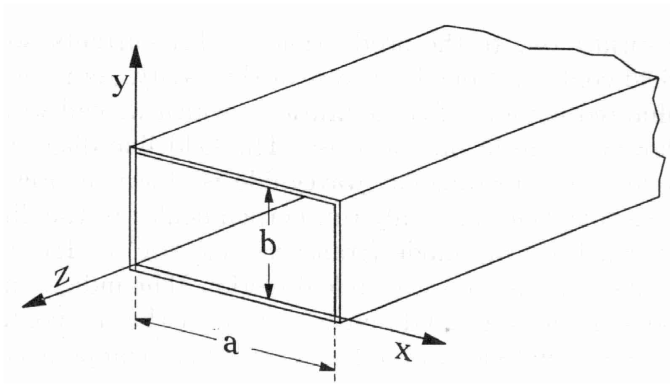


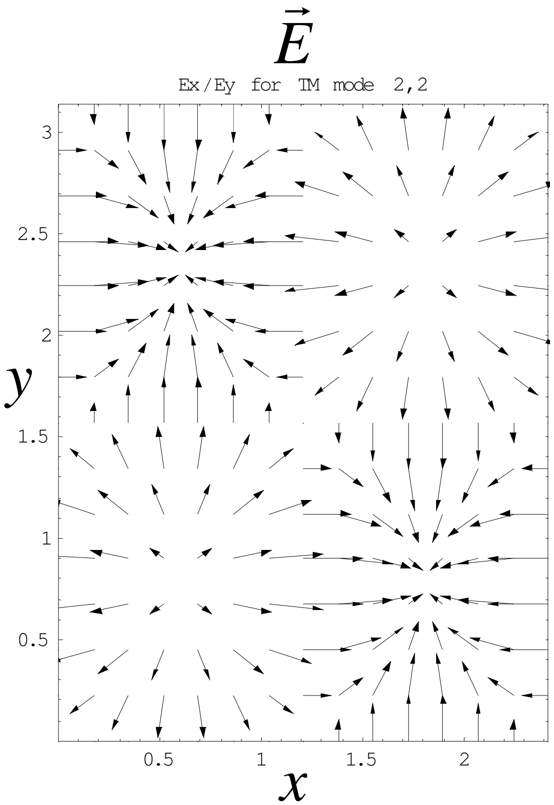
$$\vec{E}(\vec{x}) = \frac{k_z}{k_{nm}^{(E)2}} E_0 \begin{pmatrix} -\frac{n\pi}{a}\cos(\frac{n\pi}{a}x)\sin(\frac{m\pi}{b}y)\sin(k_z z - \omega t) \\ -\frac{m\pi}{b}\sin(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)\sin(k_z z - \omega t) \\ \frac{k_{nm}^{(E)2}}{k_z}\sin(\frac{n\pi}{a}x)\sin(\frac{m\pi}{b}y)\cos(k_z z - \omega t) \end{pmatrix}$$

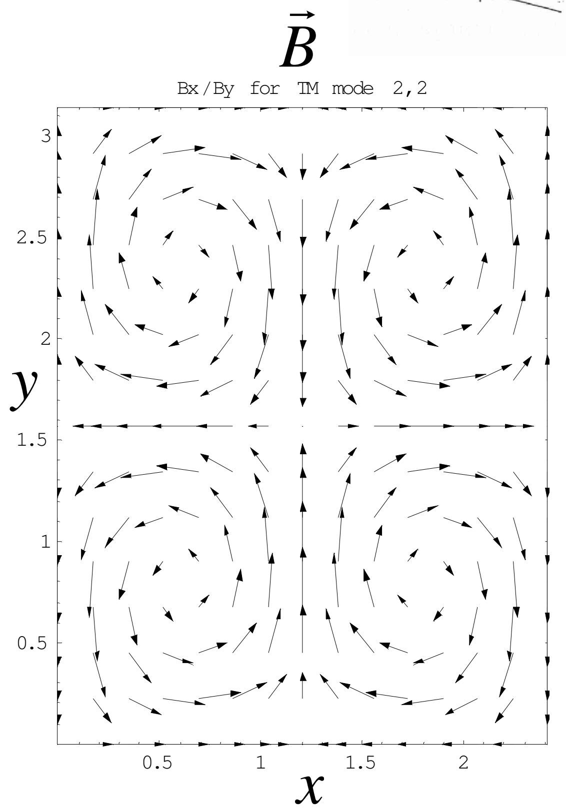
$$\vec{B}(\vec{x}) = \frac{\omega}{c^2 k_{nm}^{(E)2}} E_0 \begin{pmatrix} \frac{m\pi}{b}\sin(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)\ \sin(k_z z - \omega t) \\ -\frac{n\pi}{a}\cos(\frac{n\pi}{a}x)\ \sin(\frac{m\pi}{b}y)\ \sin(k_z z - \omega t) \\ 0 \end{pmatrix}$$

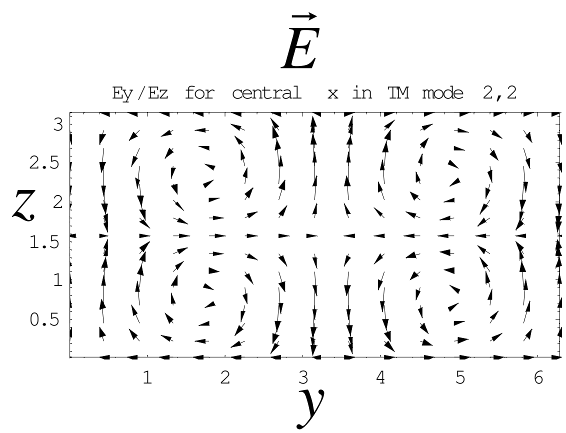
Rectangular TM_{22} Mode

CHESS & LEPP

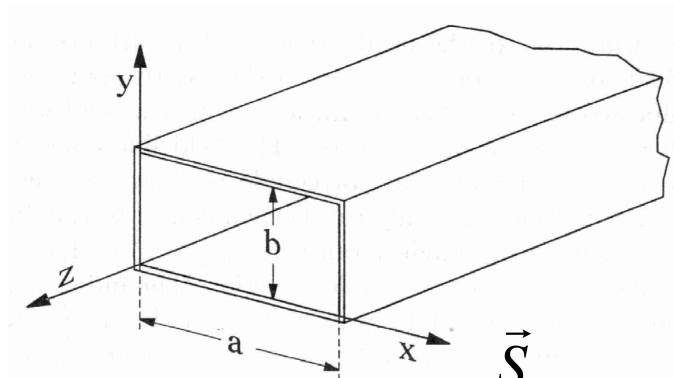


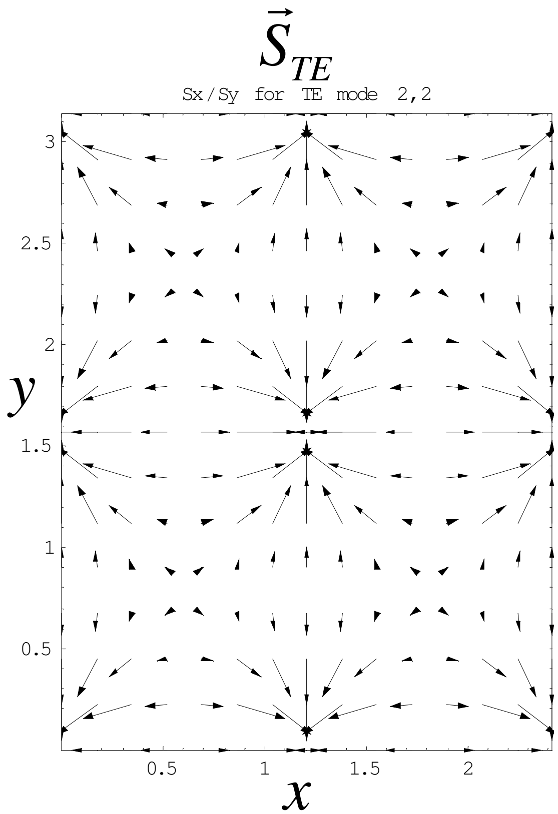


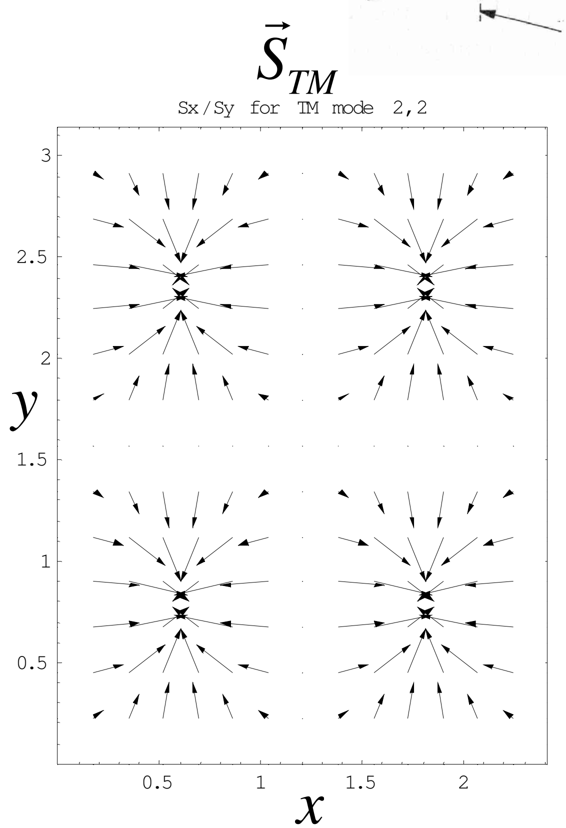


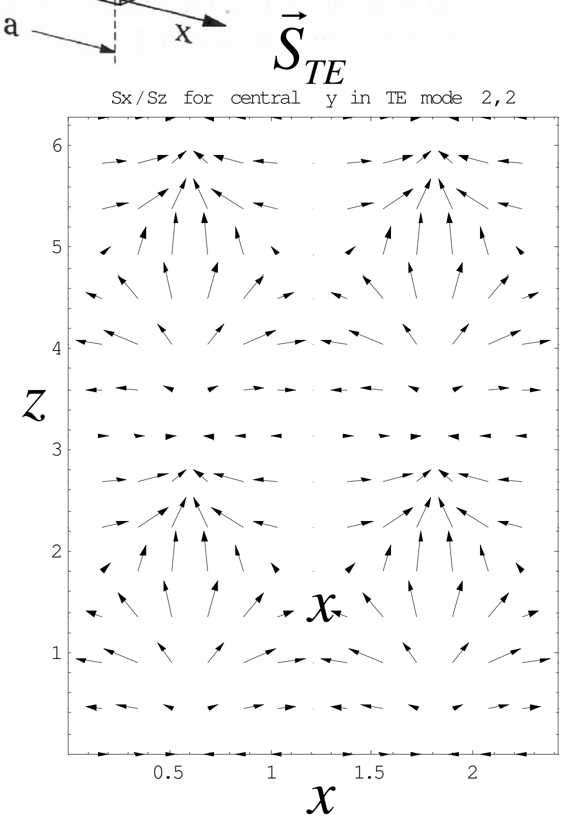


Pointing Vector

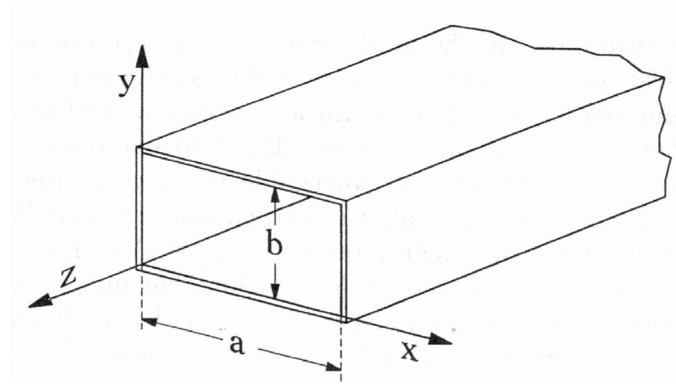




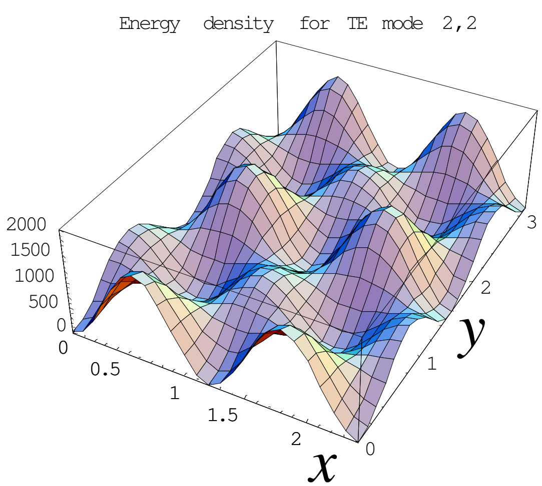




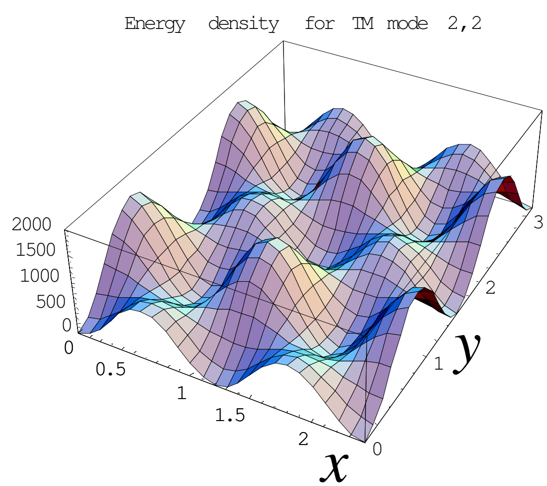
Energy Density



U_{TE}



U_{TM}



U_{TM}