Twiss parameters in accelerating cavities

\[ \alpha = -\frac{1}{2} \beta', \quad \gamma = \frac{1 + \alpha^2}{\beta} \]

\[ a = r' = \sqrt{2J \frac{mc}{p}} \left[ -\frac{2\alpha + \beta \psi'}{2\sqrt{\beta}} \sin(\psi + \phi_0) + \frac{\beta \psi'}{\sqrt{\beta}} \cos(\psi + \phi_0) \right] \]

\[ a' \approx -\frac{1}{p} \left[ r(pK + \frac{1}{2} p'') + ap' \right] \]

\[ a' = -\sqrt{2J \frac{mc}{\beta p}} \left( \frac{(\beta \psi')^2 + \alpha^2}{\beta} + \alpha' - \alpha \frac{p'}{p} + \beta \frac{p''}{2p} - \beta \frac{3p'^2}{4p^2} \right) \begin{pmatrix} \sin(\psi + \phi_0) \\ \cos(\psi + \phi_0) \end{pmatrix} \]

\[ = -\sqrt{2J \frac{mc}{\beta p}} \begin{pmatrix} \beta(K + \frac{1}{2} \frac{p''}{p}) - (\alpha + \beta \frac{p'}{2p}) \frac{p'}{p} \\ \beta \frac{p'}{p} \psi' \end{pmatrix} \begin{pmatrix} \sin(\psi + \phi_0) \\ \cos(\psi + \phi_0) \end{pmatrix} \]

\[ \Rightarrow \quad \psi' = \frac{A}{\beta}, \text{ choice: } A = 1 \]

\[ \alpha' + \gamma = \beta \left[ K + \left( \frac{p'}{2p} \right)^2 \right] \]

\[ \begin{pmatrix} r \\ a \end{pmatrix} = \sqrt{2J \frac{mc}{p}} \begin{pmatrix} \frac{\sqrt{\beta}}{\alpha + \beta \frac{p'}{2p}} & 0 \\ -\frac{1}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \sin(\psi + \phi_0) \\ \cos(\psi + \phi_0) \end{pmatrix} \]
Beta functions in accelerating cavities

\[
\begin{pmatrix}
  r \\
  a
\end{pmatrix}
= \sqrt{2J_n} \frac{mc}{p} \begin{pmatrix}
  \sqrt{\beta} & 0 \\
  -\frac{\bar{\alpha}}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}}
\end{pmatrix} \begin{pmatrix}
  \cos(\psi + \phi_0) \\
  \sin(\psi + \phi_0)
\end{pmatrix}, \quad \bar{\alpha} = \alpha + \beta \frac{p'}{2p}
\]

For systems with changing energy one uses the normalized Courant-Snyder invariant \( J_n = J \beta_r \gamma_r \)

\[
\begin{pmatrix}
  \frac{1}{\sqrt{\beta}} & \bar{\alpha} \\
  0 & \sqrt{\beta}
\end{pmatrix} \begin{pmatrix}
  \frac{1}{\sqrt{\beta}} & 0 \\
  \bar{\alpha} & \sqrt{\beta}
\end{pmatrix} \begin{pmatrix}
  r \\
  a
\end{pmatrix} \frac{p}{2mc} = \begin{pmatrix}
  \frac{1+\bar{\alpha}^2}{\beta} & \bar{\alpha} \\
  \bar{\alpha} & \beta
\end{pmatrix} \begin{pmatrix}
  r \\
  a
\end{pmatrix} \frac{p}{2mc} = J_n
\]

Reasons:

1. \( J \) is the phase space amplitude of a particle in \((x, a)\) phase space, which is the area in phase space (over 2p) that its coordinate would circumscribe during many turns in a ring. However, \( a = p_x/p_0 \) is not conserved when \( p_0 \) changes in a cavity. Therefore \( J \) is not conserved.

2. \( J_n = J p_0/mc \) is therefore proportional to the corresponding area in \((x, p_x)\) phase space, and is thus conserved.
The normalized emittance

Remarks:

1. The phase space area that a beam fills in \((x, a)\) phase space shrinks during acceleration by the factor \(p_f/p\). This area is the emittance \(\varepsilon\).

2. The phase space area that a beam fills in \((x, p_x)\) phase space is conserved. This area (divided by \(mc\)) is the normalized emittance \(\varepsilon_n\).

\[
\varepsilon = \frac{\varepsilon_n}{\beta_r \gamma_r}
\]
The Klystron as Power Source

- I up to > 10A

- Power < 1.5MW

- Power < 40MW pulsed

\[ P = \eta U_0 I_{\text{beam}}, \quad \eta \leq 65\% \]

- DC acceleration to several 10kV, 100kV pulsed

- Energy modulation with a cavity

- Time of flight density modulation

- Excitation of a cavity with output coupler

Time of flight bunching

Only works for non-relativistic electrons

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Penning Principle
(of the Philips Ion Gage)
- Magnetic field of about 0.01T.
- Pressurized gas is inserted at <100Pa (10-3Atm)
- Gas is ionized and remains ionized since electrons are accelerated in the E and circle in the B-field.
- Positive ions are accelerated through a hole in the cathode to several 100V.
A thermionic cathode produces free electrons.

An earthed anode accelerates them through an aperture into a linac.

The cathode is not flat but curved (Pierce Cathode) to produce a force that counters Coulomb expulsion (the Space Charge Force).

Typical voltages are 100-150kV, typical peak currents are a few Ampere.

Due to power limits, only short pulses can be produced (> a few μs long).

A thyratron is used as fast high-current switch and capacitors provide the short pulse.

The pulse from the capacitors is magnified (by about 10) in a transformer to reach the 100-150kV.
\[ \begin{align*}
\partial_z^2 \Phi &= -\frac{1}{\varepsilon_0} \rho \\
I &= \rho \dot{z} \\
\partial_z I &= \partial_t \rho = 0 \\
m\gamma c^2 + q\Phi &= mc^2 \\
\Phi(0) &= 0, \quad \Phi(d) = V
\end{align*} \]
Other Electron and Positron Sources

Photo-Cathode Sources

- A laser shines on a high voltage cathode, which emits photo electrons.
- These are accelerated either through an aperture in an anode (DC source), or in an RF field (RF photo-cathode source).
- With GaAs as cathode and with a polarized laser, polarized electrons are produced.
- Bunches can be as short as a few ps.
- Peak currents of a few 100A can be achieved.

Positron Source

- Electrons are accelerated to about 200MeV in a linac and hit a tungsten target.
- Pair production leads to e+/e- pairs.
- A following linac has the correct phase to accelerate e+ and decelerate e-.
- Due to multiple collisions in the target, the energy spread is up to 30MeV and
- The beam is very wide. A following damping ring is needed to produce narrow beams.