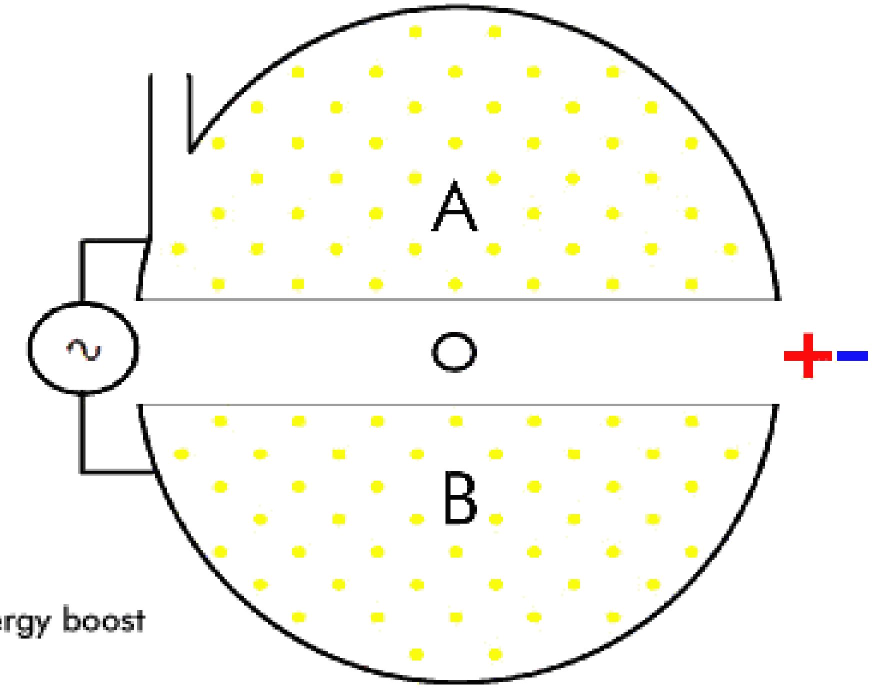
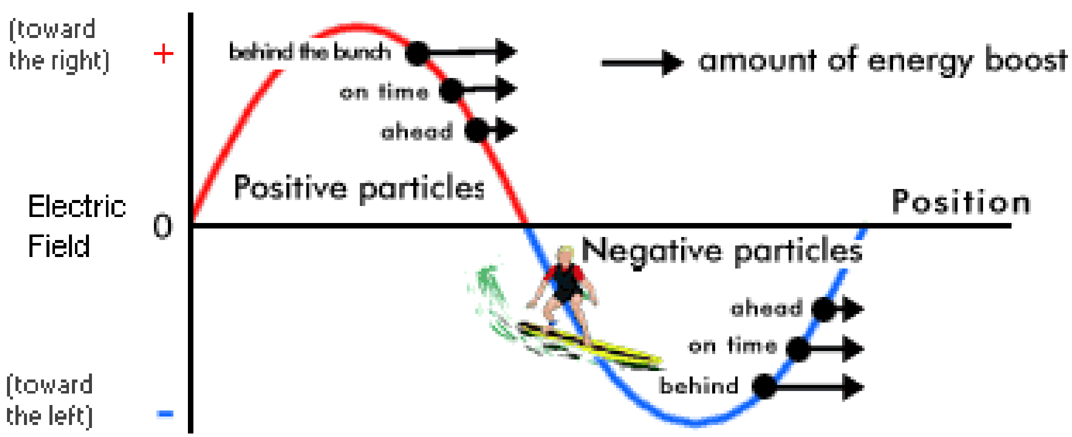
Three historic lines of accelerators

Resonant Accelerators

Direct Voltage Accelerators

Transformer Accelerator

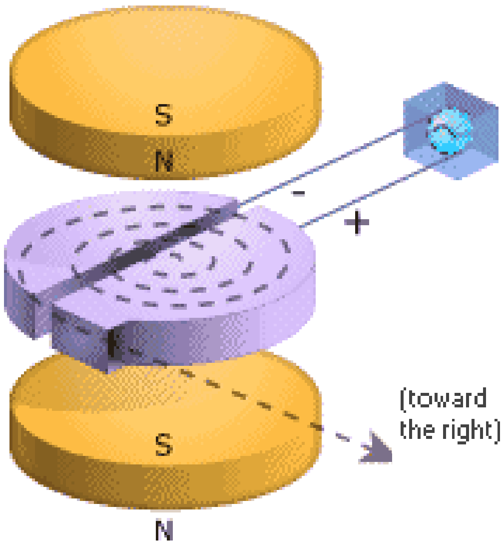




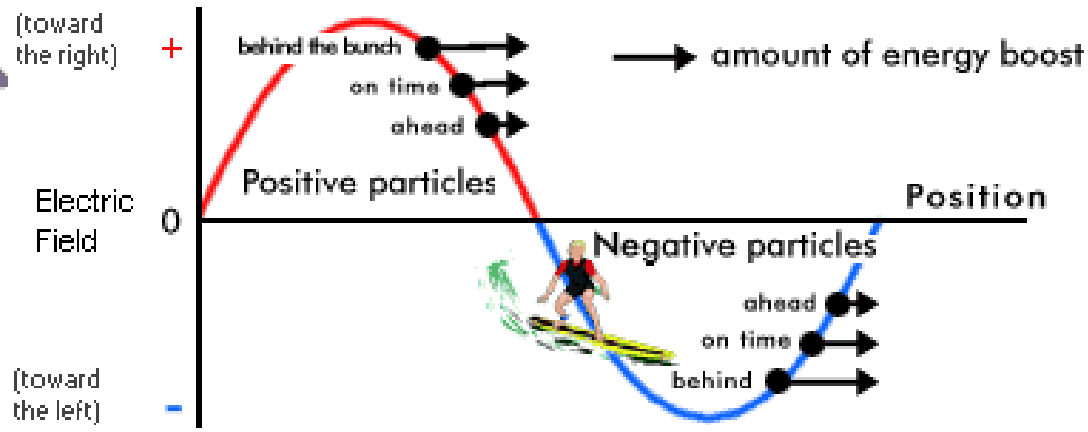
Particles must have the correct phase relation to the accelerating voltage.

The Cyclotron

CHESS & LEPP



- 1930: Lawrence proposes the Cyclotron
  (before he develops a workable color TV screen)
- 1932: Lawrence and Livingston use a cyclotron for
  1.25MeV protons and mention longitudinal
  (phase) focusing



NP 1939
Ernest O Lawrence
USA 1901-1958

- 1934: Livingston builds the first Cyclotron
  away from Berkely (2MeV protons)
  at Cornell (in room B54)

M Stanley Livingston
USA 1905-1986

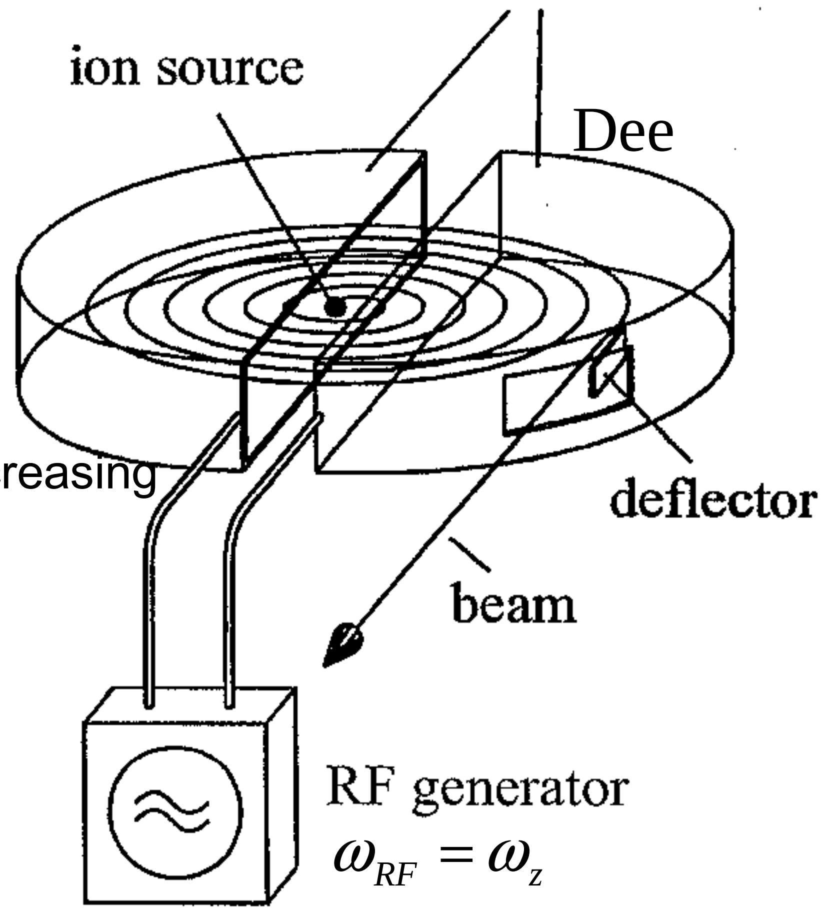
The cyclotron frequency

$$F_r = m_0 \gamma \omega_z v = qvB_z$$

$$\omega_z = \frac{q}{m_0 \gamma} B_z = \text{const}$$

Condition: Non-relativistic particles.

Therefore not for electrons.



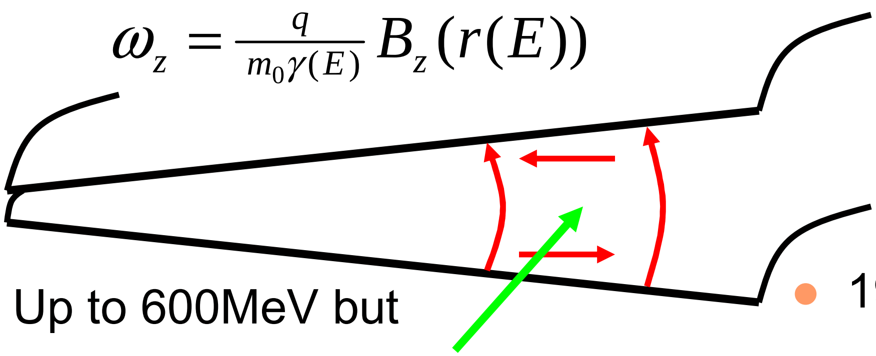
- The synchrocyclotron:

  Acceleration of bunches with decreasing

$$\omega_z(E) = \frac{q}{m_0 \gamma(E)} B_z$$

- The isocyclotron with constant

$$\omega_z = \frac{q}{m_0 \gamma(E)} B_z(r(E))$$



Up to 600MeV but

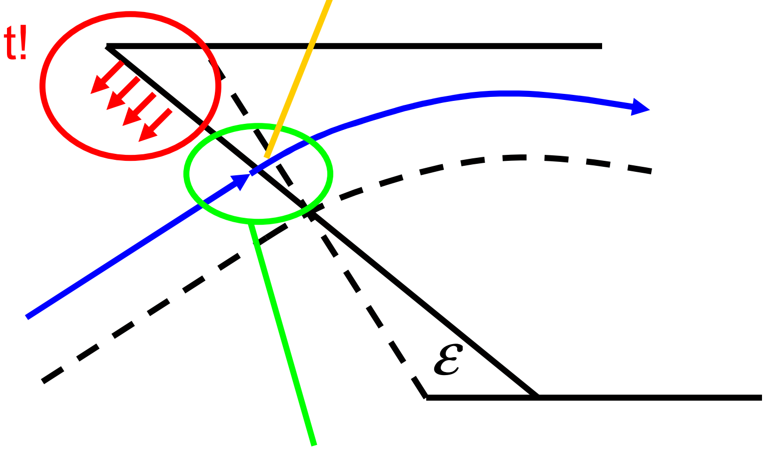
this vertically defocuses the beam

- 1938: Thomas proposes strong

  (transverse) focusing for a cyclotron

Edge Focusing

Horizontal focusing with $\Delta x' = -x\frac{\tan(\varepsilon)}{\rho}$

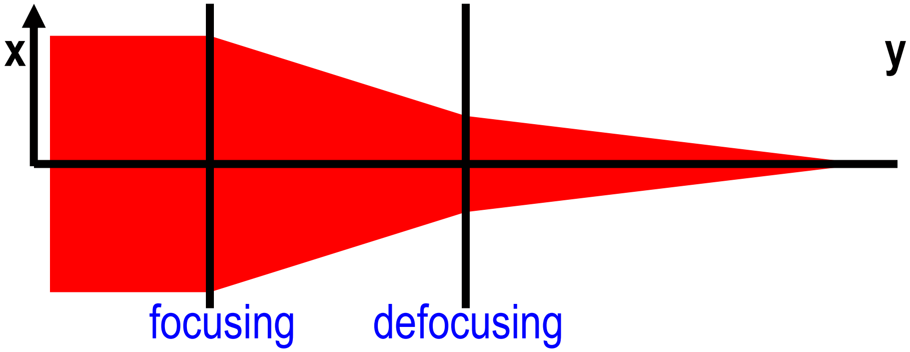


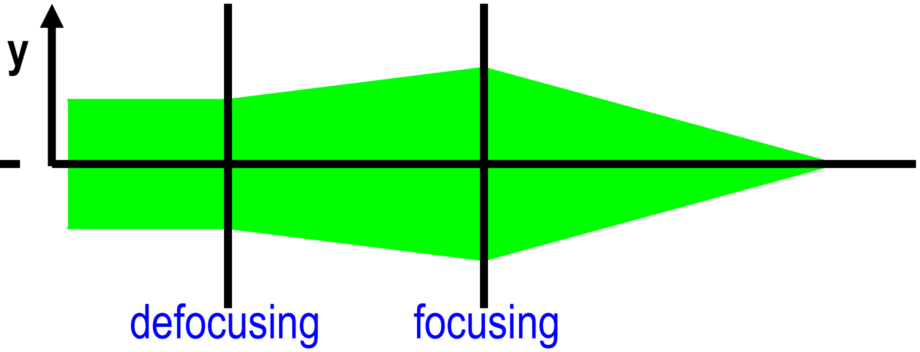
The longitudinal field above the enter plain defocuses, turns out to: $\Delta y' = y\frac{\tan(\varepsilon)}{\rho}$

Quadrupole effect: focusing in x and defocusing in y or defocusing in x and focusing in y.

Alternating gradient focusing, e.g. with edges

Transverse fields defocus in one plane if they focus in the other plane.
But two successive elements, one focusing the other defocusing,
can focus in both planes:

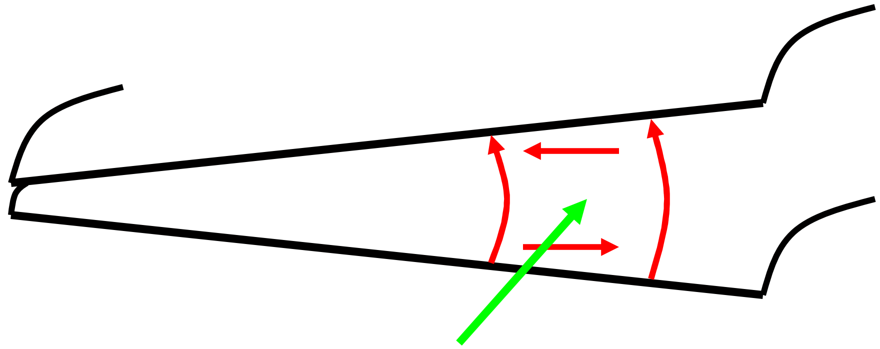




Cyclotrons with edge focusing

- The isocyclotron with constant

$$\omega_z = \frac{q}{m_0\gamma(E)} B_z(r(E))$$



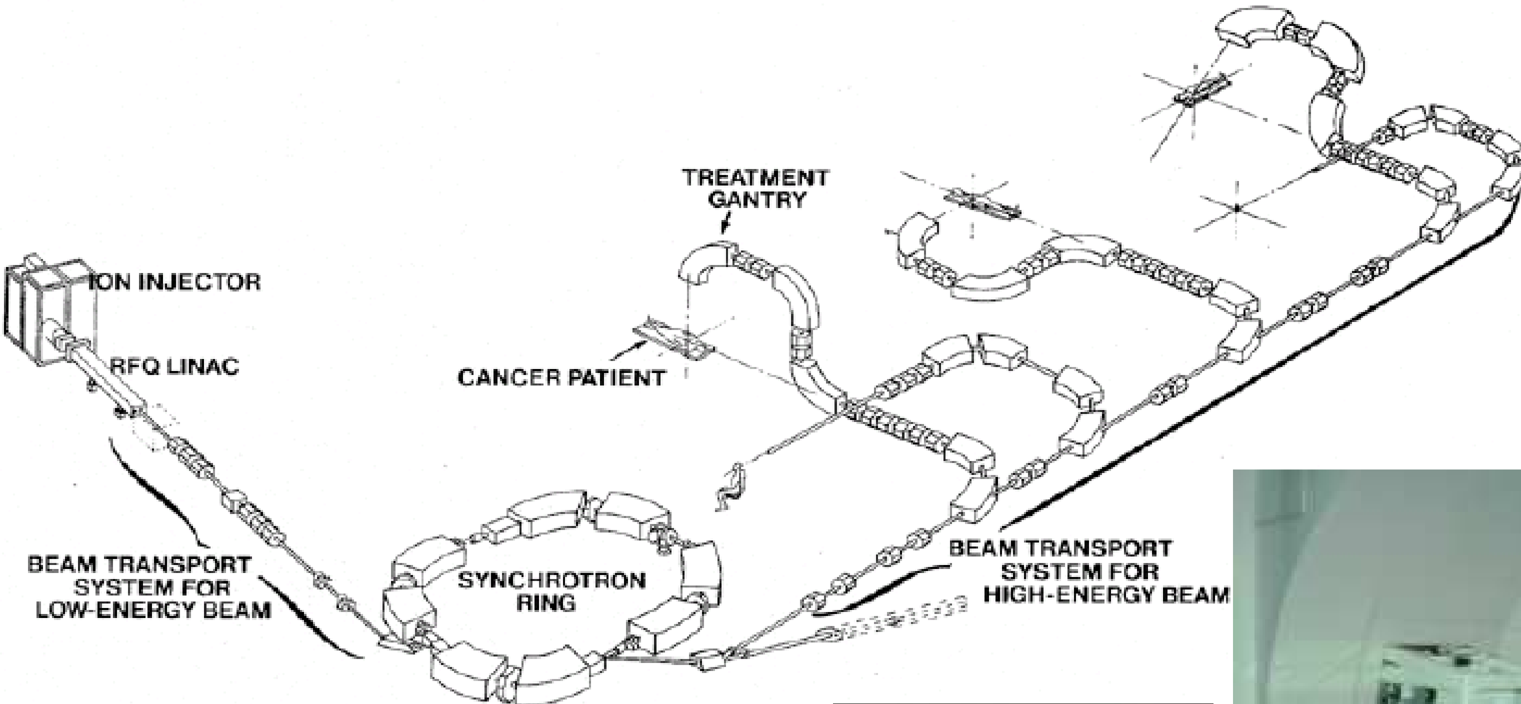
Up to 600MeV but
this vertically defocuses the beam.
Edge focusing is therefore used.

First Medical Applications

- 1939: Lawrence uses 60' cyclotron for 9MeV protons, 19MeV deuterons, and 35MeV 4He. First tests of tumor therapy with neutrons via d + t $\mapsto$ n + α With 200-800keV d to get 10MeV neutrons.

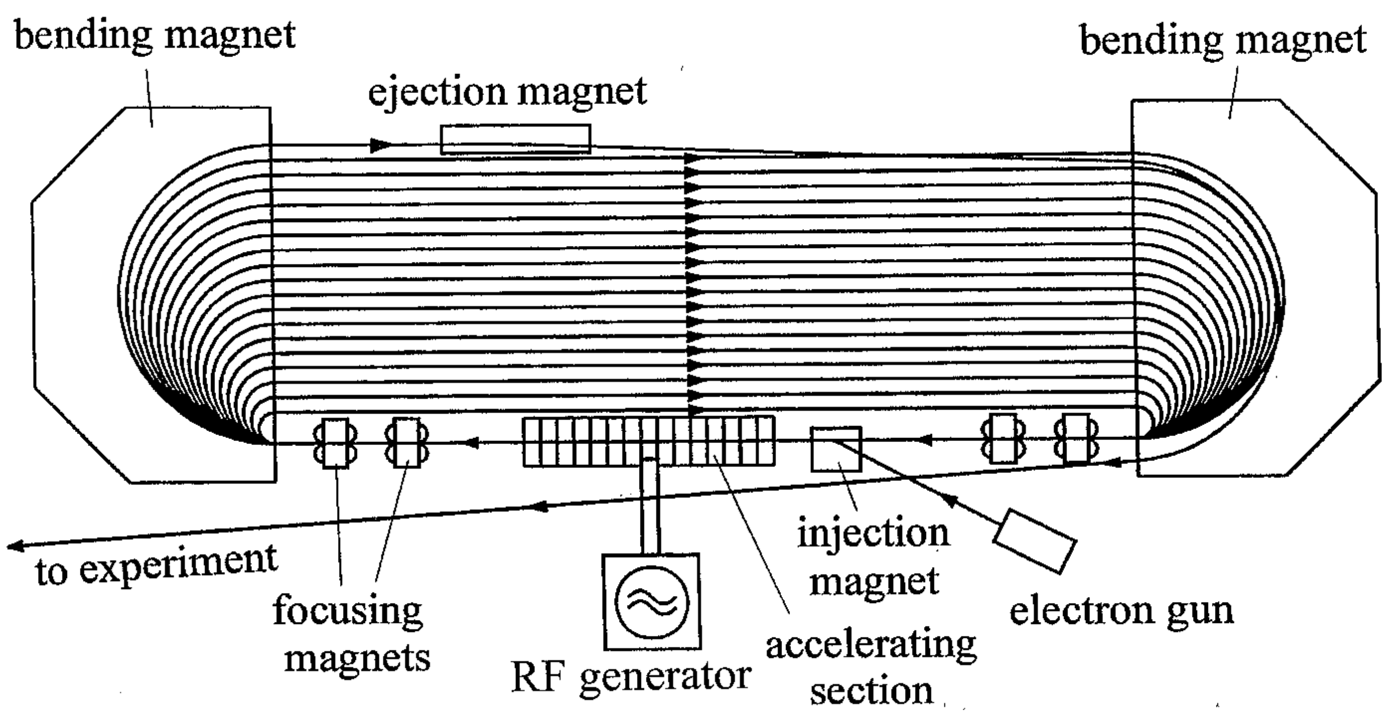
Modern Nuclear Therapy

The Loma Linda proton therapy facility



The microtron

- Electrons are quickly relativistic and cannot be accelerated in a cyclotron.
- In a microtron the revolution frequency changes, but each electron misses an integer number of RF waves.

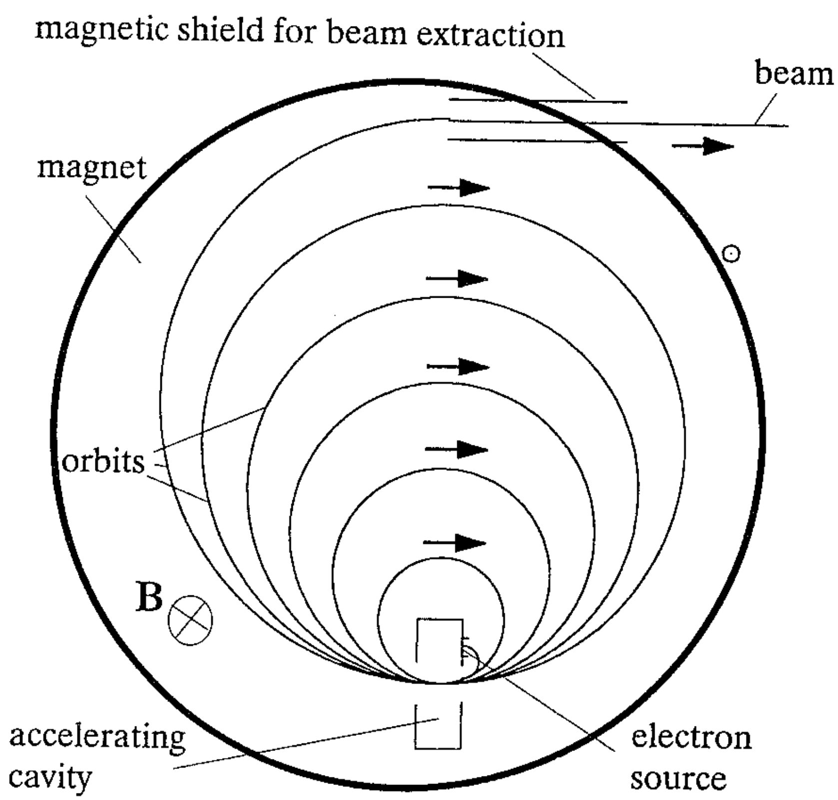


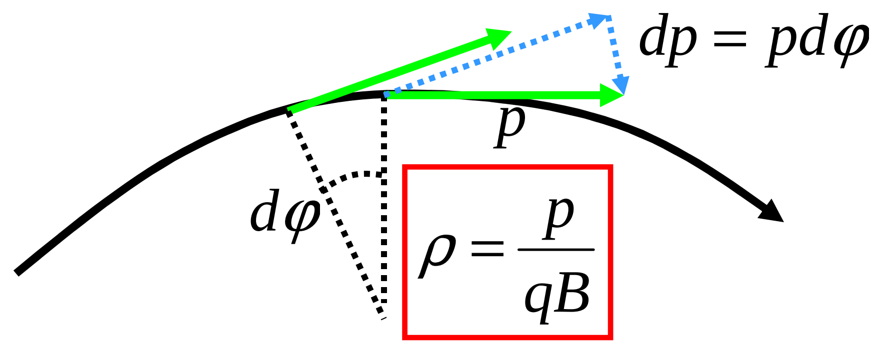
- Today: Used for medical applications with one magnet and 20MeV.
- Nuclear physics: MAMI designed for 820MeV as race track microtron.

The microtron condition

- The extra time that each turn takes must be a multiple of the RF period.

$$\frac{dp}{dt} = qvB \Rightarrow \rho = \frac{dl}{d\varphi} = \frac{vdt}{dp/p} = \frac{p}{qB}$$





$$\rho = \frac{p}{qB}$$

$$\Delta t = 2\pi\left(\frac{\rho_{n+1}}{v_{n+1}} - \frac{\rho_n}{v_n}\right)$$

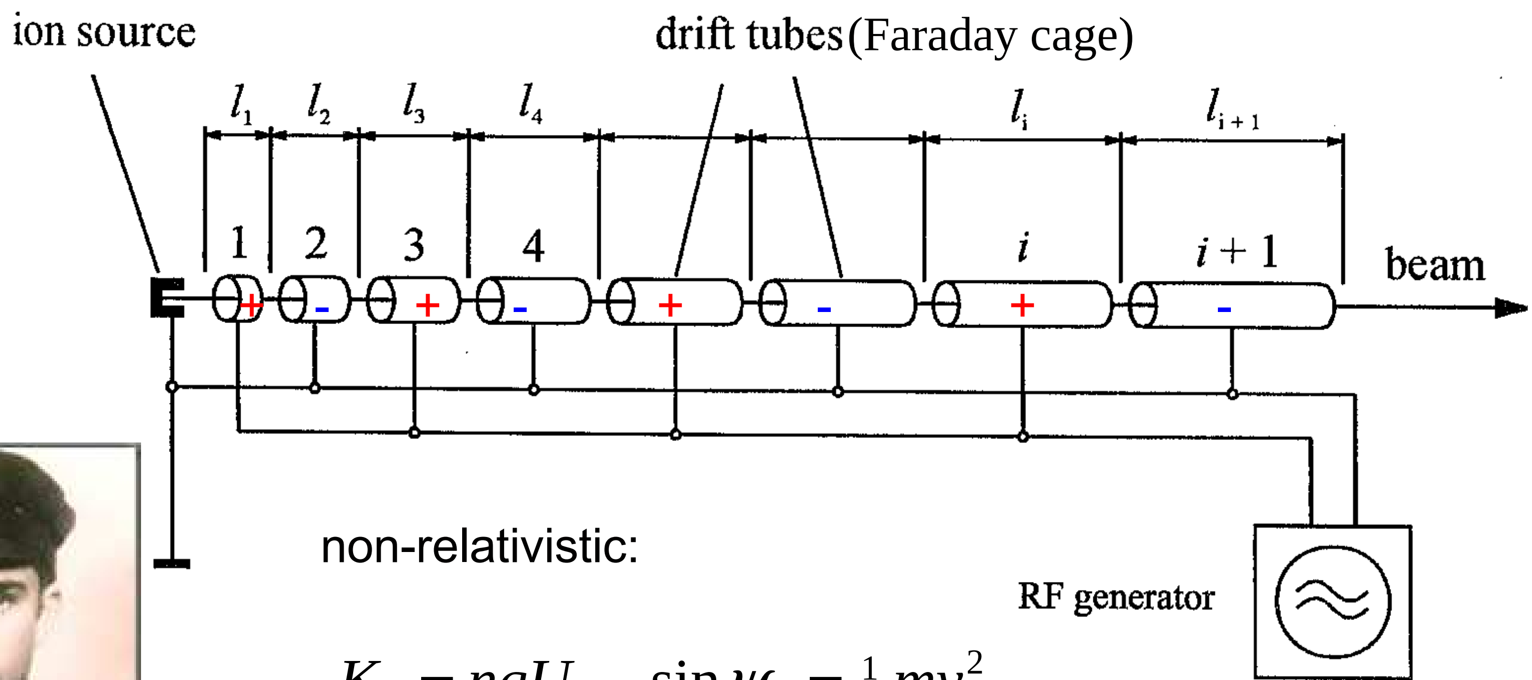
$$= \frac{2\pi}{qB}(m_0\gamma_{n+1} - m_0\gamma_n) = \frac{2\pi}{qBc^2}\Delta K$$

$$\Delta K = n\frac{qBc^2}{\omega_{RF}}$$ for an integer n

B=1T, n=1, and f_{RF}=3GHz leads to 4.78MeV

This requires a small linear accelerator.

Wideroe linear accelerator




Wideroe

non-relativistic:

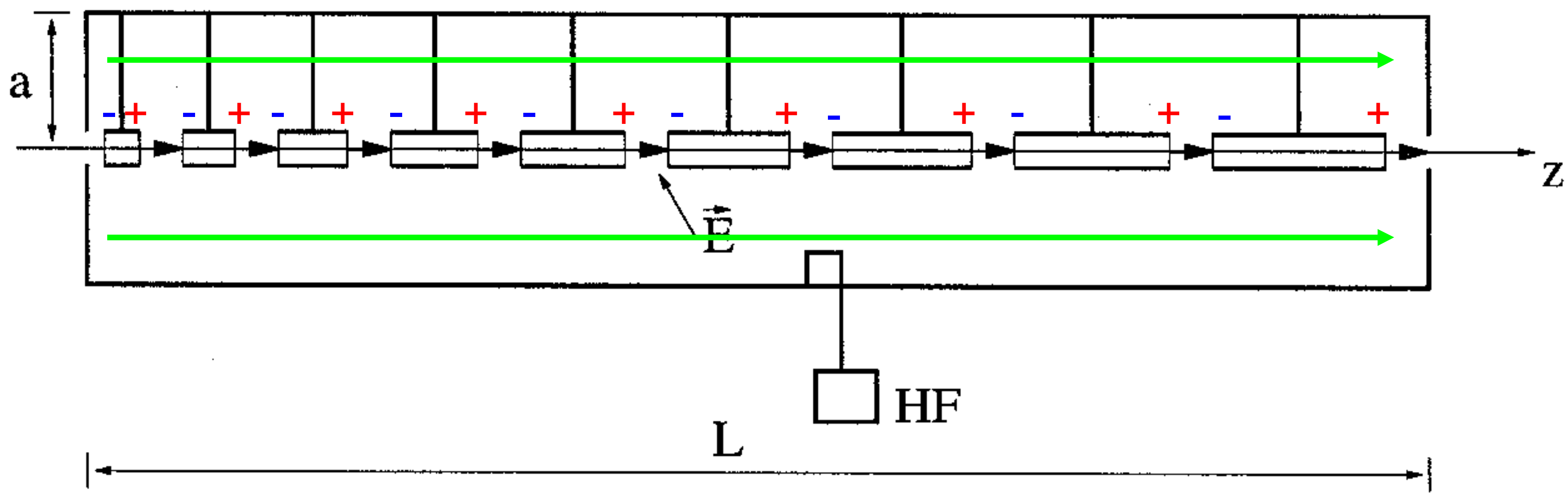
$$K_n = nqU_{\max} \sin\psi_0 = \tfrac{1}{2} m v_n^2$$

$$l_n = \tfrac{1}{2} v_n T_{RF} = \tfrac{1}{2} \beta_n \lambda_{RF} \propto \sqrt{n}$$

Called the π or the $1/2\beta\lambda$ mode

The Alvarez Linear Accelerator

2π or the $\beta\lambda$ mode

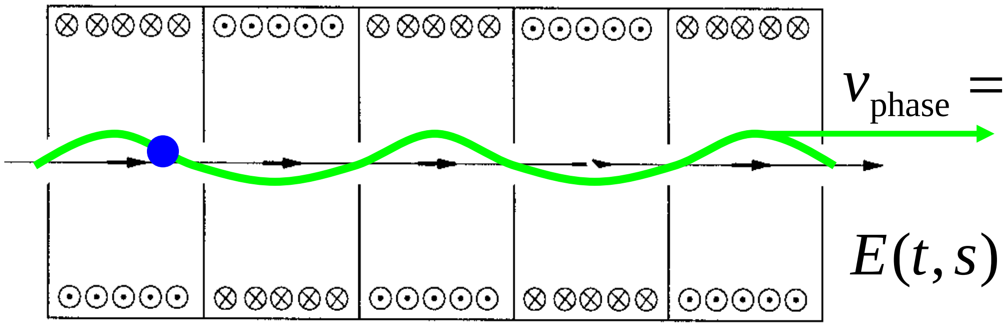


Needs only one power input coupler and fewer walls dissipate less energy.

Accelerating cavities

- 1933: J.W. Beams uses resonant cavities for acceleration

Traveling wave cavity:

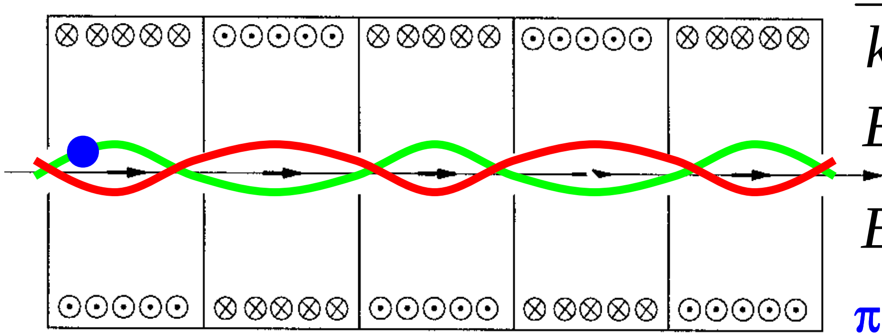


$$v_{\text{phase}} = \frac{\omega}{k} = v_{\text{particle}}$$

Here v=c for electrons

$$E(t,s) \approx E_{\max} \sin(\omega t - ks)$$

Standing wave cavity:



$$\frac{\omega}{k} = v_{\text{particle}}$$

$$E(t,s) \approx E_{\max} \sin(\omega t)\sin(ks)$$

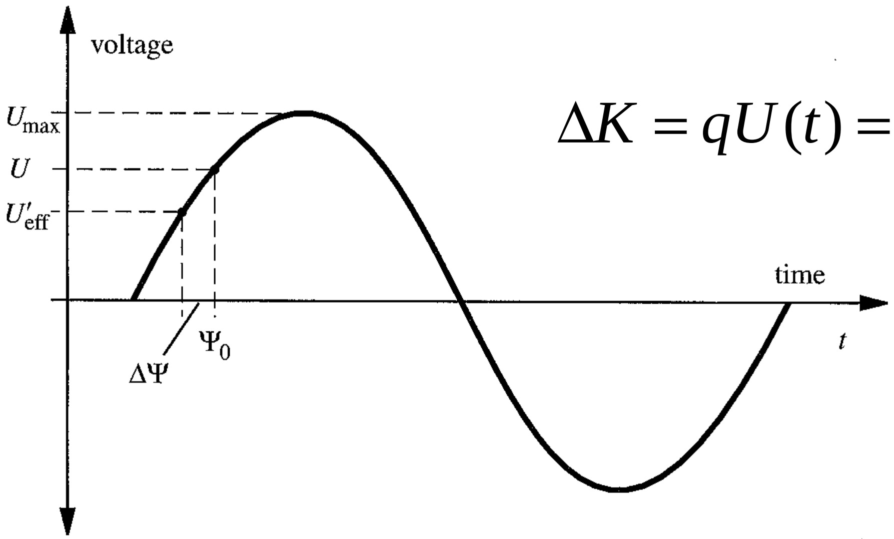
$$E(\tfrac{s}{v_{\text{particle}}}, s) \approx E_{\max} \sin^2(ks)$$

π or the 1/2βλ mode

Transit factor (for this example): $\langle E \rangle = \frac{1}{\lambda_{RF}} \int_0^{\lambda_{RF}} E(\tfrac{s}{v_{\text{particle}}}, s)\, ds \approx \tfrac{1}{2} E_{\max}$

Phase focusing

- 1945: Veksler (UDSSR) and McMillan (USA) realize the importance of phase focusing



$$\Delta K = qU(t) = qU_{\max}\sin(\omega(t-t_0)+\psi_0)$$

$$\Delta K(0) > 0 \quad \text{(Acceleration)}$$

$$\Delta K(t) > \Delta K(0) \text{ for } t>0 \Rightarrow \tfrac{d}{dt}\Delta K(t) > 0 \quad \text{(Phase focusing)}$$

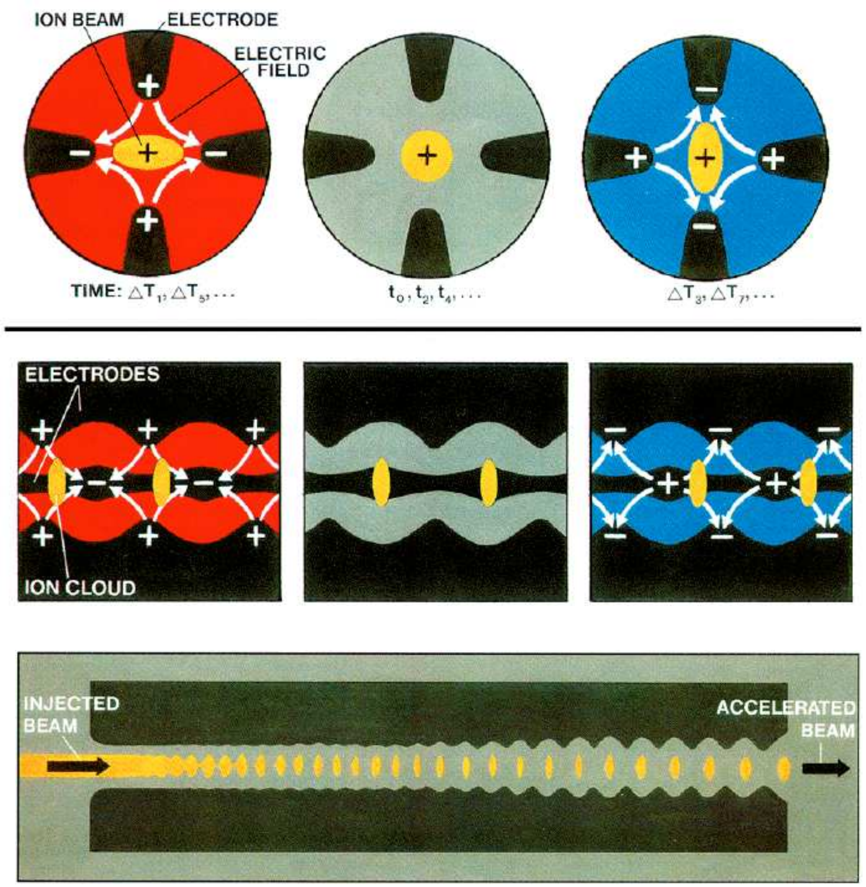
$$\left.\begin{aligned} qU(t) &> 0 \\ q\tfrac{d}{dt}U(t) &> 0 \end{aligned}\right\} \quad \psi_0 \in (0,\frac{\pi}{2})$$

Phase focusing is required in any RF accelerator.

The RF quadrupole (RFQ)

CHESS & LEPP

- 1970: Kapchinskii and Teplyakov invent the RFQ



Three historic lines of accelerators

Transformer Accelerator

Direct Voltage Accelerators Resonant Accelerators

- 1924: Wideroe invents the betatron
- 1940: Kerst and Serber build a betatron for 2.3MeV electrons and understand betatron (transverse) focusing (in 1942: 20MeV)

Betatron:

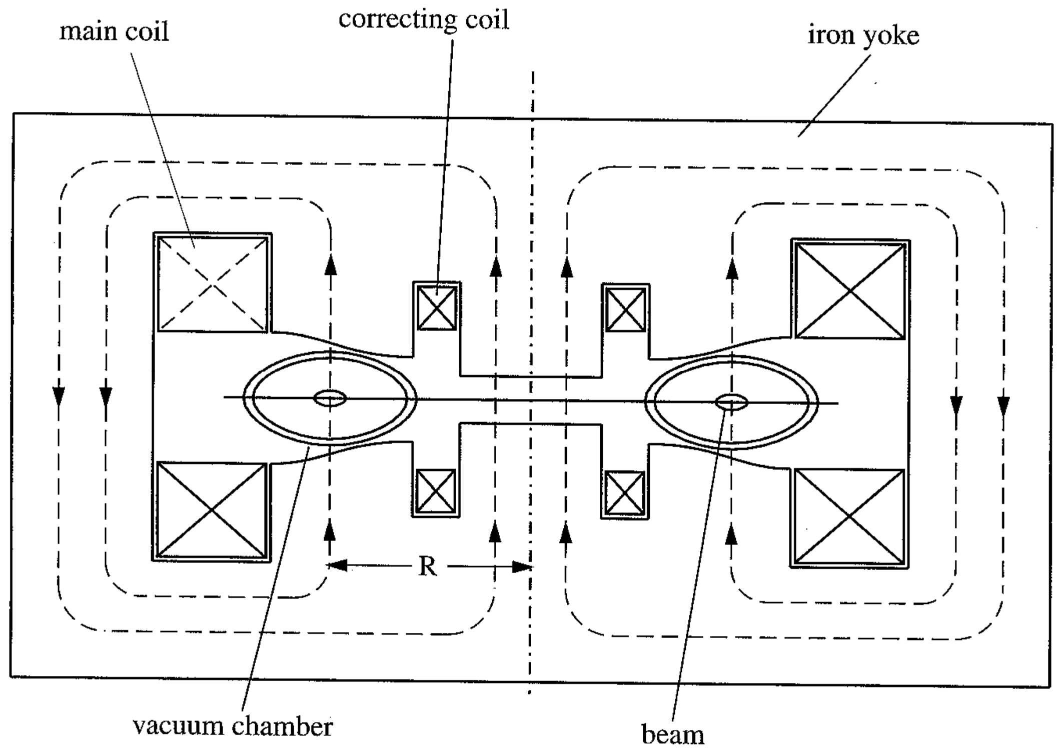
R=const, B=B(t)

Whereas for a cyclotron:

R(t), B=const

No acceleration section is needed since

$$\oint_{\partial A} \vec{E} \cdot d\vec{s} = -\iint_A \frac{d}{dt} \vec{B} \cdot d\vec{a}$$



The Betatron Condition

Condition: $R = \frac{-p_\varphi(t)}{qB_z(R,t)} = \text{const.}$ given $\oint_{\partial A} \vec{E}\cdot d\vec{s} = -\iint_A \frac{d}{dt}\vec{B}\cdot d\vec{a}$

$$E_\varphi(R,t) = -\tfrac{1}{2\pi R}\int \tfrac{d}{dt}B_z(r,t)\,r\,dr\,d\varphi = -\tfrac{R}{2}\left\langle \tfrac{d}{dt}B_z\right\rangle$$

$$\tfrac{d}{dt}p_\varphi(t) = qE_\varphi(R,t) = -q\tfrac{R}{2}\left\langle \tfrac{d}{dt}B_z\right\rangle$$

$$p_\varphi(t) = p_\varphi(0) - q\tfrac{R}{2}[\langle B_z\rangle(t) - \langle B_z\rangle(0)] = -RqB_z(R,t)$$

$$\boxed{B_z(R,t) - B_z(R,0) = \frac{1}{2}[\langle B_z\rangle(t) - \langle B_z\rangle(0)]}$$

Small deviations from this condition lead to transverse beam oscillations called betatron oscillations in all accelerators.

- Today: Betatrons with typically about 20MeV for medical applications

The Synchrotron

CHESS & LEPP

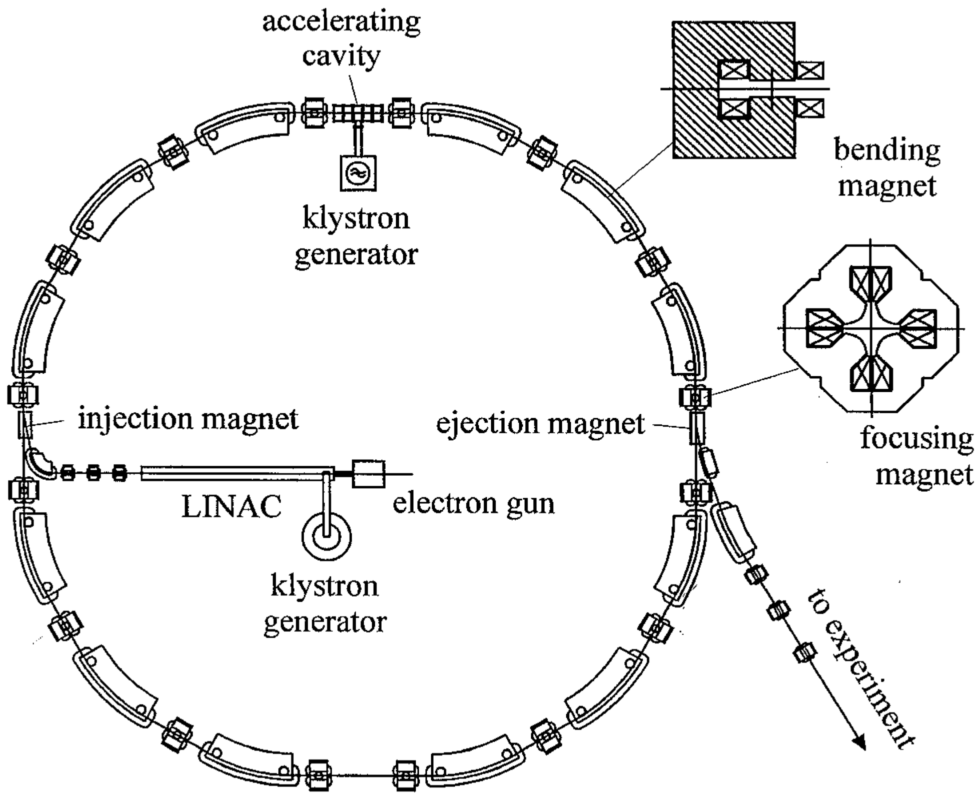
- 1945: Veksler (UDSSR) and McMillan (USA) invent the synchrotron
- 1946: Goward and Barnes build the first synchrotron (using a betatron magnet)
- 1949: Wilson et al. at Cornell are first to store beam in a synchrotron (later 300MeV, magnet of 80 Tons)
- 1949: McMillan builds a 320MeV electron synchrotron

- Many smaller magnets instead of one large magnet
- Only one acceleration section is needed, with

$$R = \frac{p(t)}{qB(R,t)} = \text{const.}$$

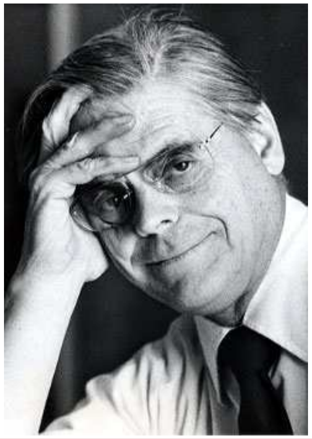
$$\omega = 2\pi \frac{v_{\text{particle}}}{L} n$$

for an integer n called the harmonic number



Rober R Wilson, Architecture

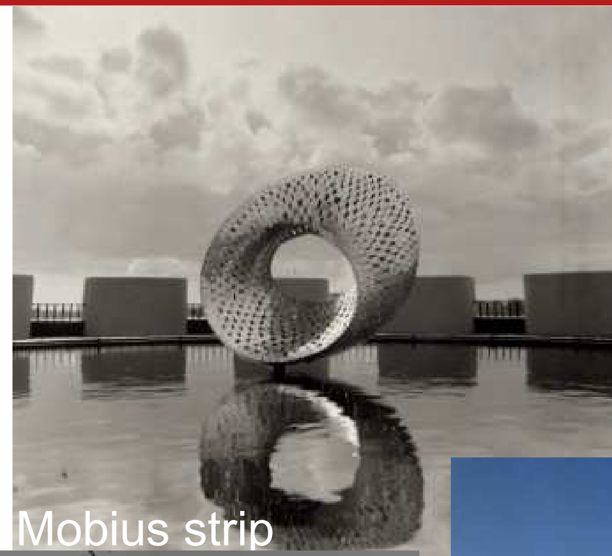

Science Ed Center, FNAL (1990)


Robert R Wilson
USA 1914-2000

Wilson Hall, FNAL

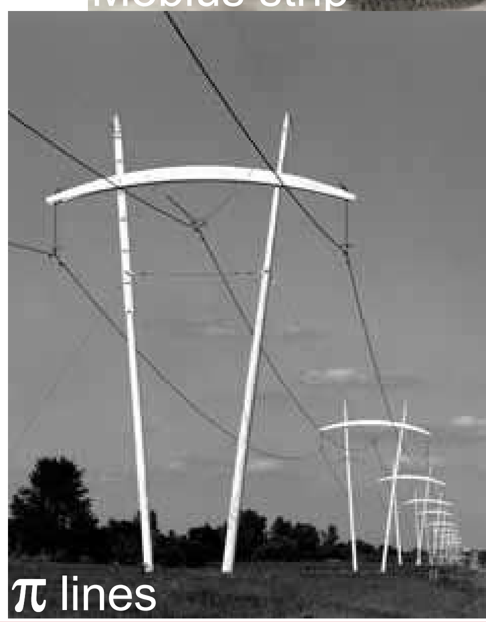
Rober R Wilson, Cornell & FNAL

Broken symmetry



Mobius strip

Tractricious



π lines

Hyperbolic obelisk