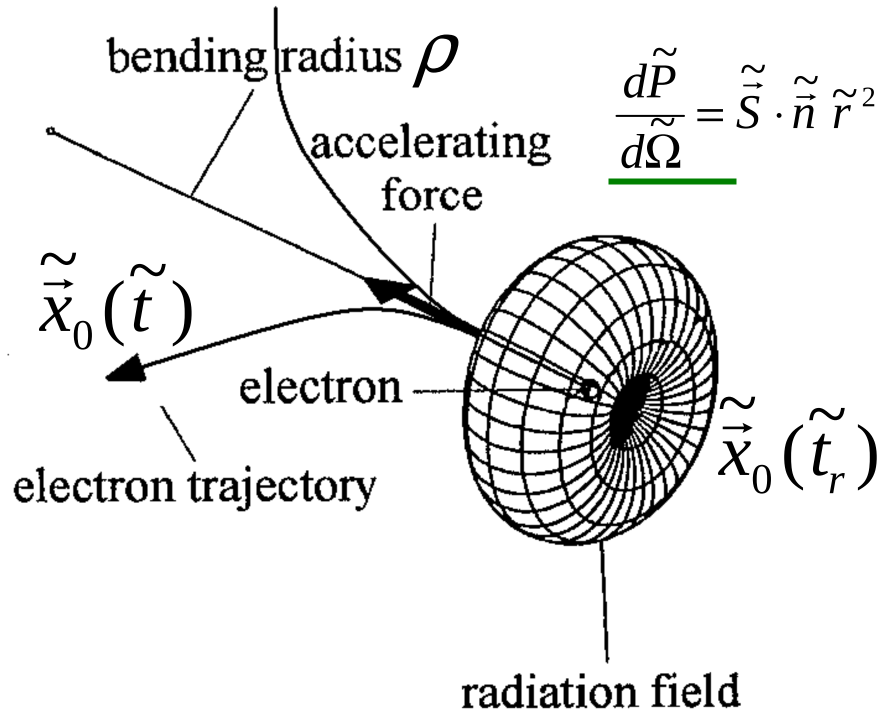
Rest-Frame-Radiation Pattern

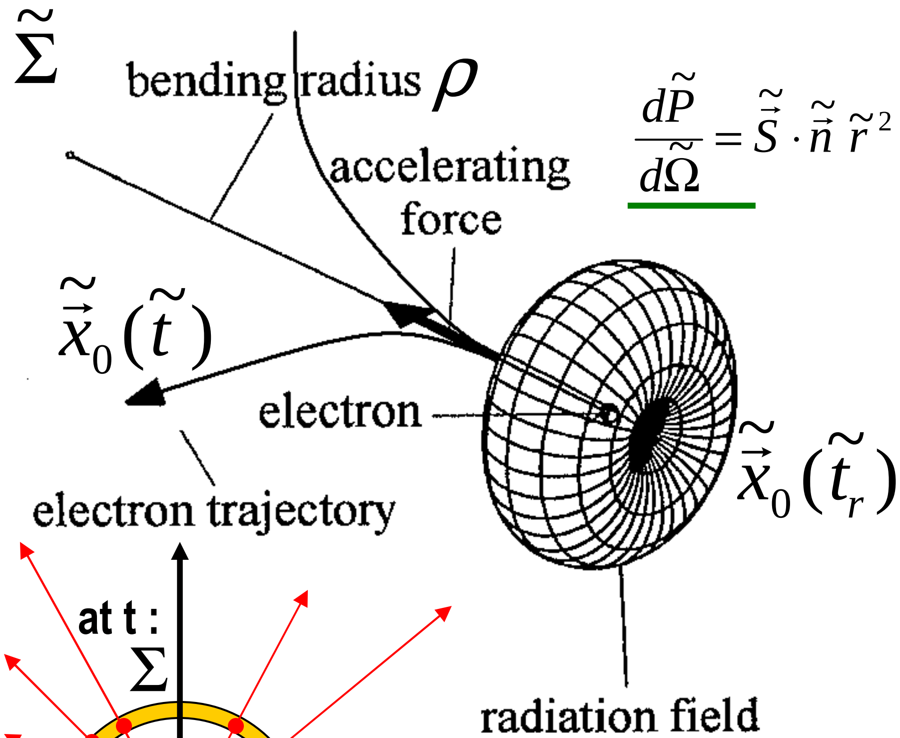
CHESS & LEPP

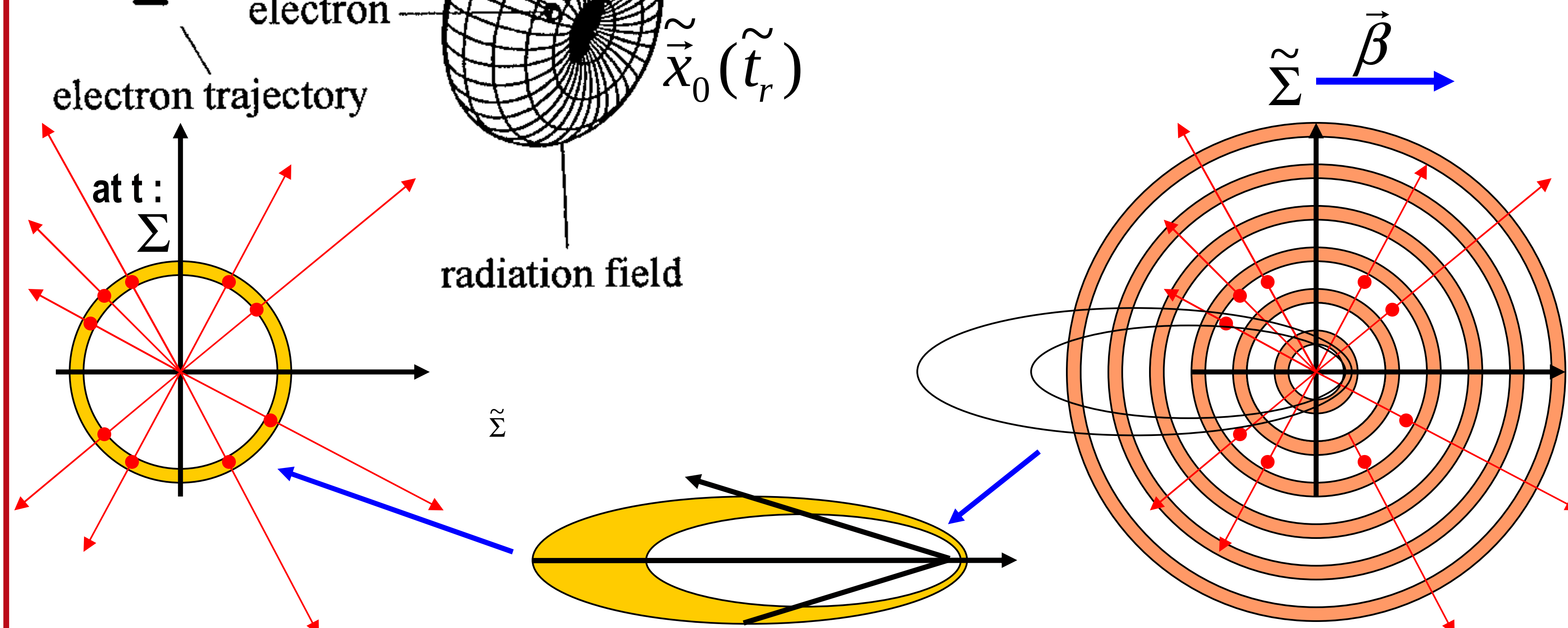


$$\frac{d\tilde{P}}{d\tilde{\Omega}} = \tilde{\vec{S}} \cdot \tilde{\vec{n}}\, \tilde{r}^2 = \frac{1}{\mu_0 c^3} \tilde{\Phi}^2 \tilde{\dot{\vec{\beta}}}_{r\perp}^2\, \tilde{r}^2 = \frac{r_0 mc}{4\pi} \tilde{\dot{\vec{\beta}}}_{r\perp}^2 = \frac{r_0 mc}{4\pi} [\tilde{\dot{\vec{\beta}}}_r^2 - (\tilde{\dot{\vec{\beta}}} \cdot \tilde{\vec{n}})^2]$$

Rest-Frame-Radiation Pattern

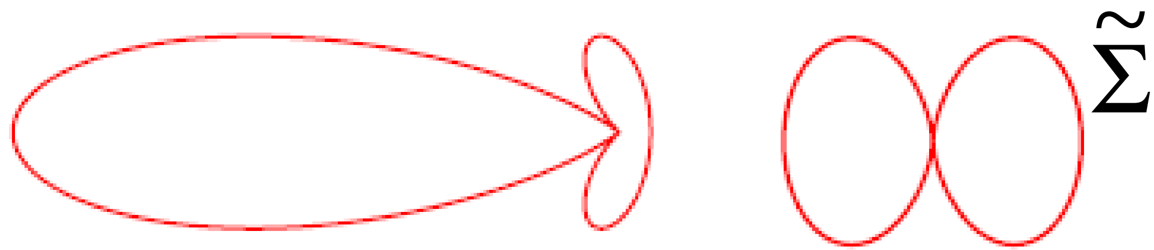
$$\frac{d\tilde{P}}{d\tilde{\Omega}} = \vec{\tilde{S}} \cdot \vec{\tilde{n}}\, \tilde{r}^2 = \frac{1}{\mu_0 c^3} \tilde{\Phi}^2 \tilde{\dot{\vec{\beta}}}{}^2_{r\perp}\, \tilde{r}^2 = \frac{r_0 mc}{4\pi} \tilde{\dot{\vec{\beta}}}{}^2_{r\perp} = \frac{r_0 mc}{4\pi}[\tilde{\dot{\vec{\beta}}}{}^2_r - (\tilde{\dot{\vec{\beta}}}_r \cdot \vec{\tilde{n}})^2]$$





Lab-Frame-Radiation Pattern

CHESS & LEPP

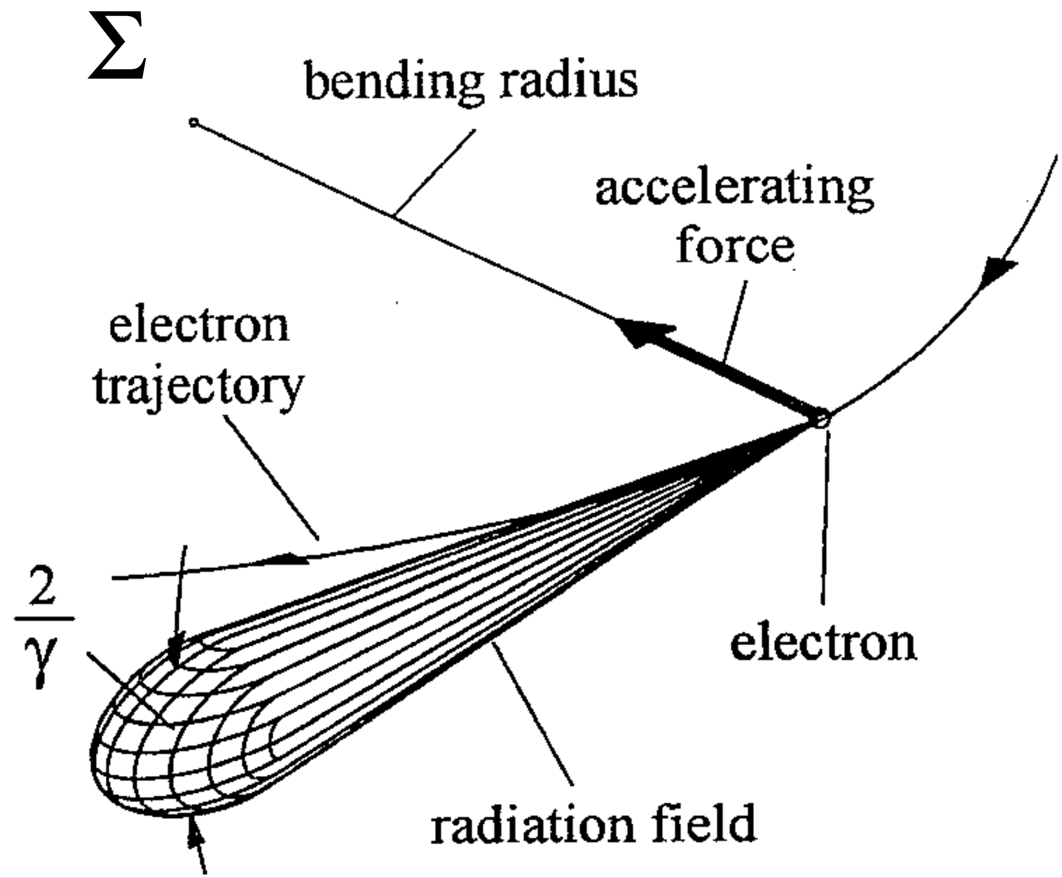


$$r^2 d\Omega = \tilde{r}^2 d\tilde{\Omega} \quad \Rightarrow \quad \frac{dP}{d\Omega} = \frac{d\tilde{P}}{d\tilde{\Omega}} \frac{r^2}{\tilde{r}^2}$$

$$\begin{pmatrix} ct \\ \vec{x} \end{pmatrix} = \begin{pmatrix} r \\ r\,\vec{n} \end{pmatrix} \quad \Rightarrow \quad \tilde{r} = \gamma(r - \vec{\beta} \cdot \vec{r})$$

$$\Rightarrow \quad \frac{r^2}{\tilde{r}^2} = \frac{1}{\gamma^2 (1 - \beta \cos \vartheta)^2}$$

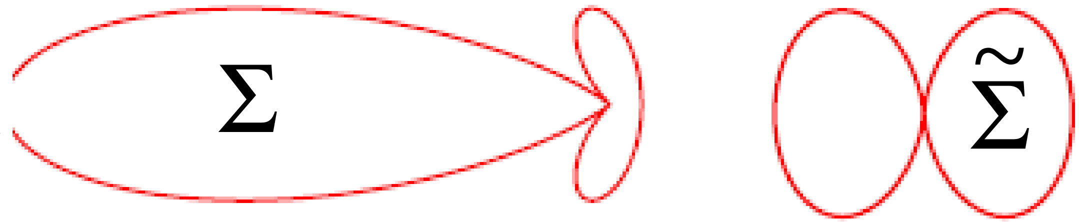
$$\frac{dP}{d\Omega} = \frac{r_0 mc}{4\pi} \frac{\tilde{\ddot{\vec{\beta}}}_r^2 - (\tilde{\ddot{\vec{\beta}}}_r \cdot \tilde{\vec{n}})^2}{\gamma^2 (1 - \beta \cos \vartheta)^2}$$



Lab-Frame-Radiation Pattern

CHESS & LEPP

$$\begin{pmatrix} ct \\ \vec{x} \end{pmatrix} = \begin{pmatrix} r \\ r\,\vec{n} \end{pmatrix} \Rightarrow \begin{matrix} \tilde{r} = \gamma(r - \vec{\beta}\cdot\vec{r}) \\ \tilde{r}\,\tilde{\vec{n}} = \gamma(r\,\vec{n}_{\parallel} - \vec{\beta}\,r) + r\,\vec{n}_{\perp} \end{matrix}$$



$$\begin{pmatrix} \frac{1}{c}\gamma\dot{E} \\ \gamma\dot{\vec{p}} \end{pmatrix} = \begin{pmatrix} 0 \\ mc\gamma^2\dot{\vec{\beta}} \end{pmatrix} \Rightarrow \begin{matrix} 0 = 0 \\ \dot{\tilde{\vec{\beta}}} = \gamma^2\dot{\vec{\beta}} \end{matrix}$$

$$\frac{dP}{d\Omega} = \frac{r_0 mc}{4\pi}\gamma^2 \frac{\dot{\vec{\beta}}_r^2 - (\dot{\vec{\beta}}_r \cdot \frac{r}{\tilde{r}}\vec{n}_{\perp})^2}{(1-\beta\cos\vartheta)^2}$$

$$= \frac{r_0 mc}{4\pi}\gamma^2\dot{\vec{\beta}}_r^2 \frac{(1-\beta\cos\vartheta)^2 - (\frac{\sin\vartheta\cos\varphi}{\gamma})^2}{(1-\beta\cos\vartheta)^4} \approx \frac{r_0 mc}{4\pi}\gamma^2\dot{\vec{\beta}}_r^2 \frac{(\frac{1}{2\gamma^2}+\frac{\vartheta^2}{2})^2 - \frac{\vartheta^2}{\gamma^2}\cos^2\varphi}{(\frac{1}{2\gamma^2}+\frac{\vartheta^2}{2})^4}$$

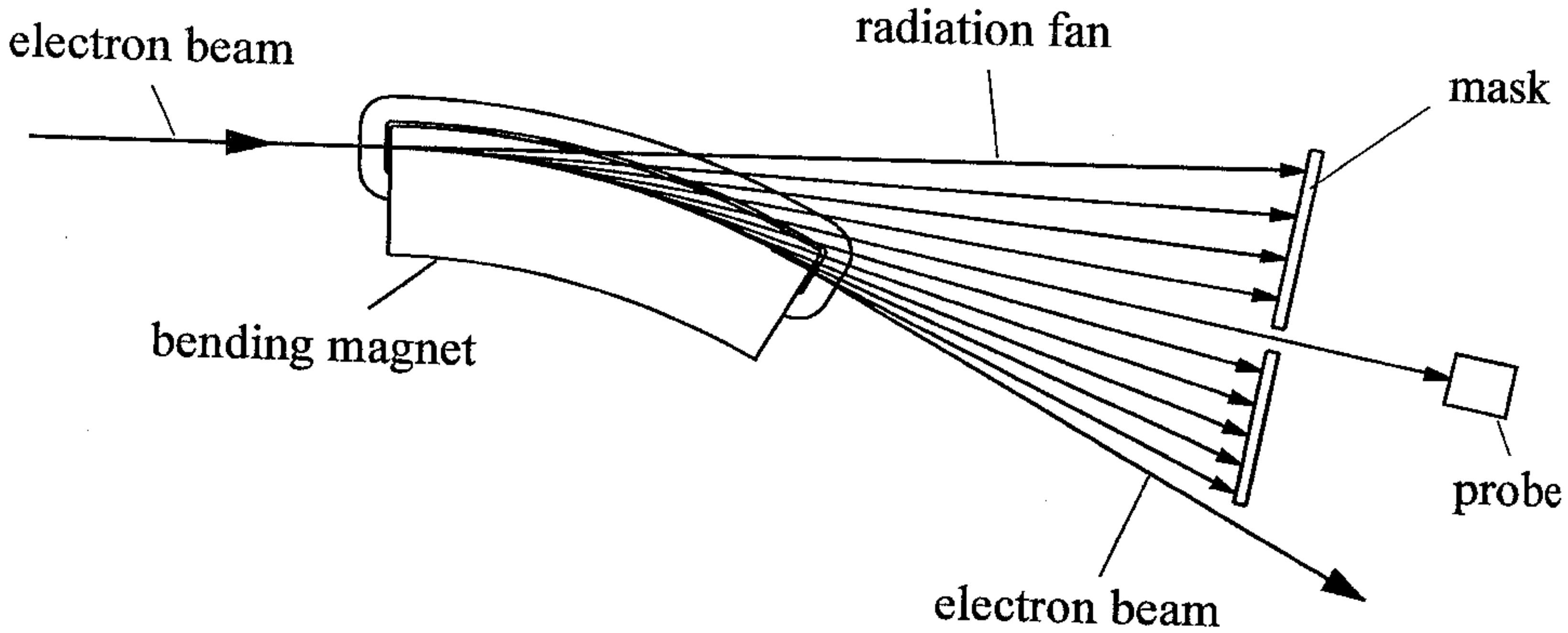
$$= \begin{cases} \frac{3\pi}{2}\gamma^2 P \frac{(1-\xi^2)^2}{(1+\xi^2)^4} & \text{for horizontal} \\ & [\xi = \vartheta\gamma] \\ \frac{3\pi}{2}\gamma^2 P \frac{(1+\xi^2)^2}{(1+\xi^2)^4} & \text{for vertical} \end{cases}$$

Conclusion: Angular power density falls off with ϑ^{-4}

Peak angular power density growth with γ^6 / ρ^2

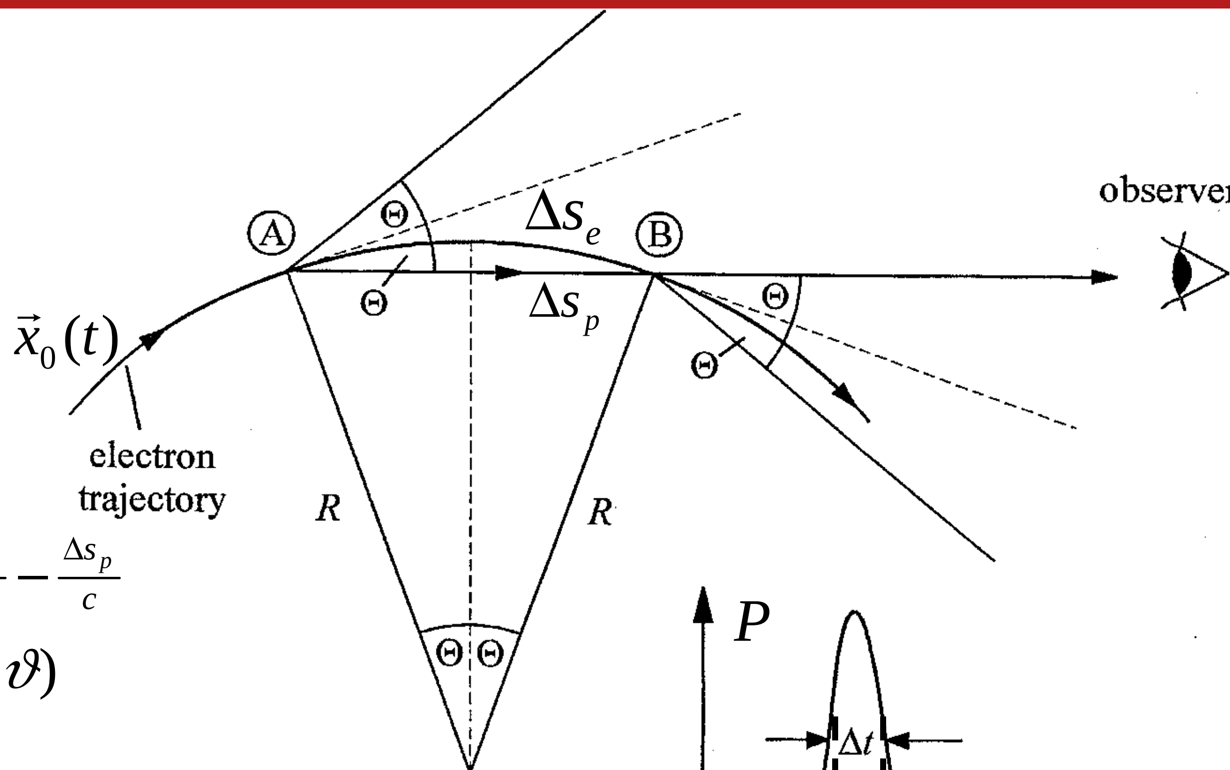
Long and Short Magnet Radiation

CHESS & LEPP



The exposure time at the probe is either given by the opening angle of radiation, or by the bend angle of the magnet, whichever is smaller.

Spectrum of bend radiation

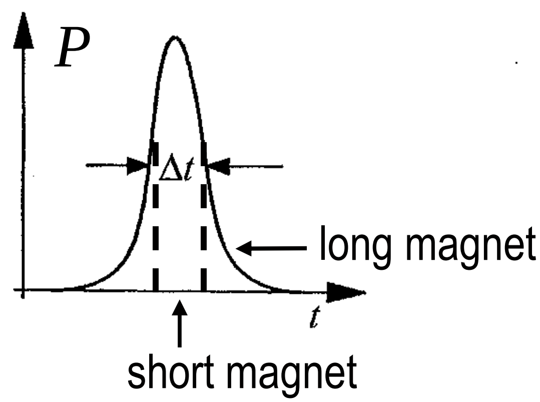


$$\Delta t = t_{electron} - t_{photon} = \frac{\Delta s_e}{v} - \frac{\Delta s_p}{c}$$

$$= \frac{1}{\beta c}(2\rho\vartheta - 2\beta\rho\sin\vartheta)$$

$$\approx \frac{2\rho}{c}\frac{1-\beta}{\beta}\vartheta \approx \frac{\rho\vartheta}{c\gamma^2}$$

$$\Delta\omega \approx 1/\Delta t \approx \frac{c\gamma^2}{\rho\vartheta} \approx \begin{cases} \frac{c\gamma^3}{\rho} \text{ for a long magnet} \\ \frac{c\gamma^2}{L} \text{ for a short magnet} \end{cases}$$



Characteristic frequency: $\omega_c \approx \frac{3}{2}\frac{c\gamma^3}{\rho}$

Radiation Formulas to Remember

CHESS & LEPP

Characteristic frequency: $\omega_c \approx \frac{3}{2}\frac{c\gamma^3}{\rho}$

Maximum spectral radiation power $\frac{dP}{d\Omega}$ at $\omega = \frac{1}{3}\omega_c$

Half radiation power above and half below $\omega = \omega_c$

For a current I of electrons with: $P = \frac{2r_e c}{3} mc^2 \frac{(\beta\gamma)^4}{\rho^2} \quad in \quad \Delta\psi = \gamma^{-1}$

Linear power density along a vacuum pipe (needs to be cooled): $\frac{dP}{dl} = \frac{I}{q}\frac{2r_e}{3} mc^2 \frac{(\beta\gamma)^4}{\rho^2}$

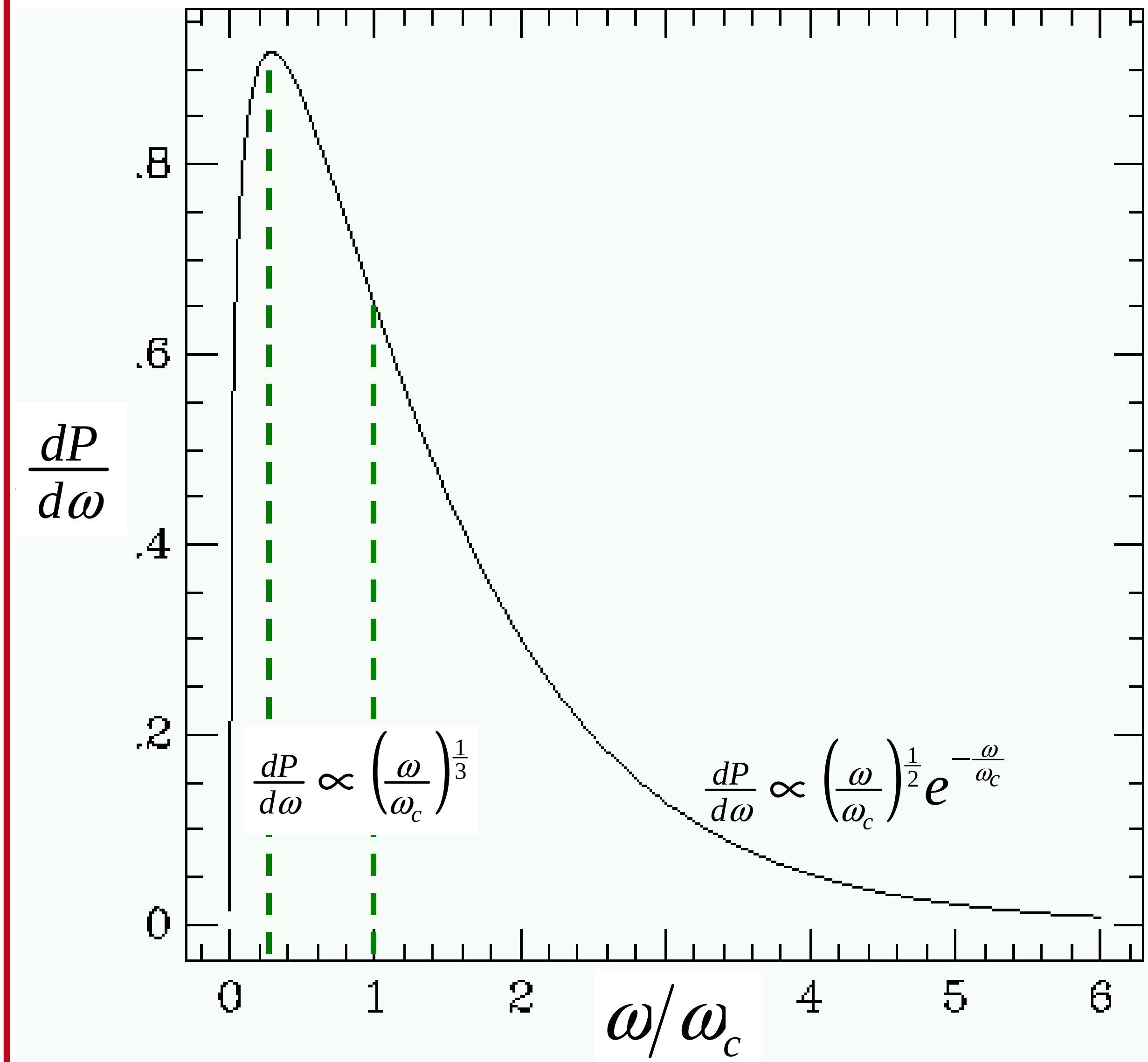
Power density along a vacuum pipe of radius R (needs to be cooled): $\frac{dP}{dl} = \frac{I}{q}\frac{2r_e}{3D} m\gamma c^2 \frac{(\beta\gamma)^4}{\rho^2}$

Linear photon flux density (causes gas desorption): $\frac{d\dot{N}_p}{dl} \approx \frac{2r_e I}{3q}\frac{mc^2}{\hbar\omega_c}\frac{\gamma^4}{\rho^2} = \frac{4}{9}\frac{I}{q}\frac{r_e mc}{\hbar}\frac{\gamma}{\rho} = \frac{4}{9}\frac{I}{q}\frac{\alpha}{\rho\frac{1}{\gamma}}$

Note: Each electron radiates only α=1/137 photons per opening angle !

Synchrotron-Radiation Spectrum

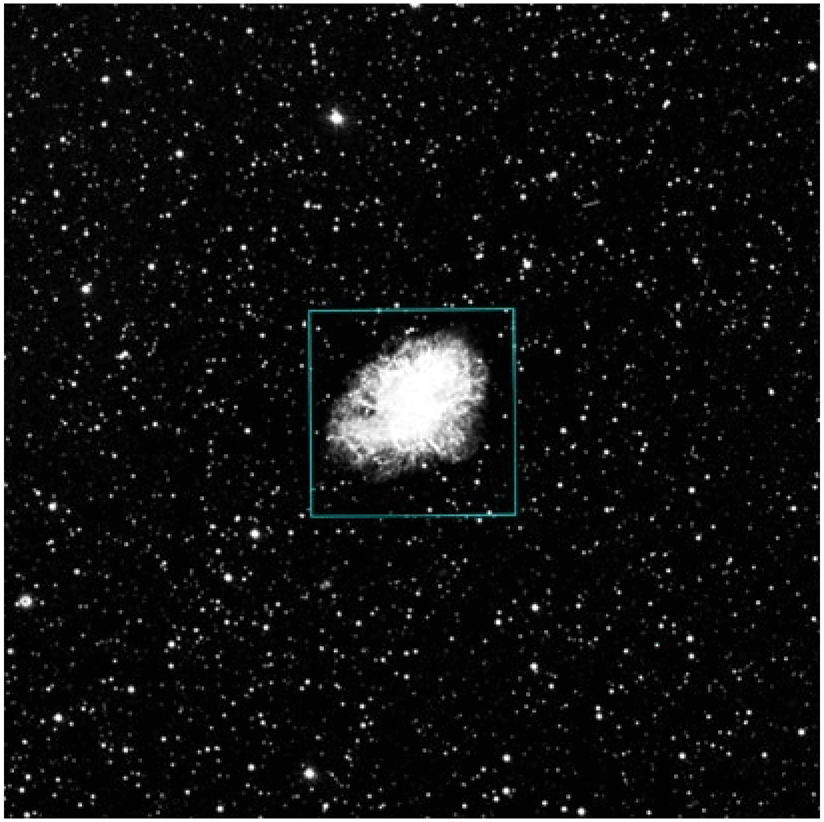
CHESS & LEPP



The First Observed Synchrotron

with images from http://www.astro.utu.fi/~cflynn/astroII/l4.html

CHESS & LEPP

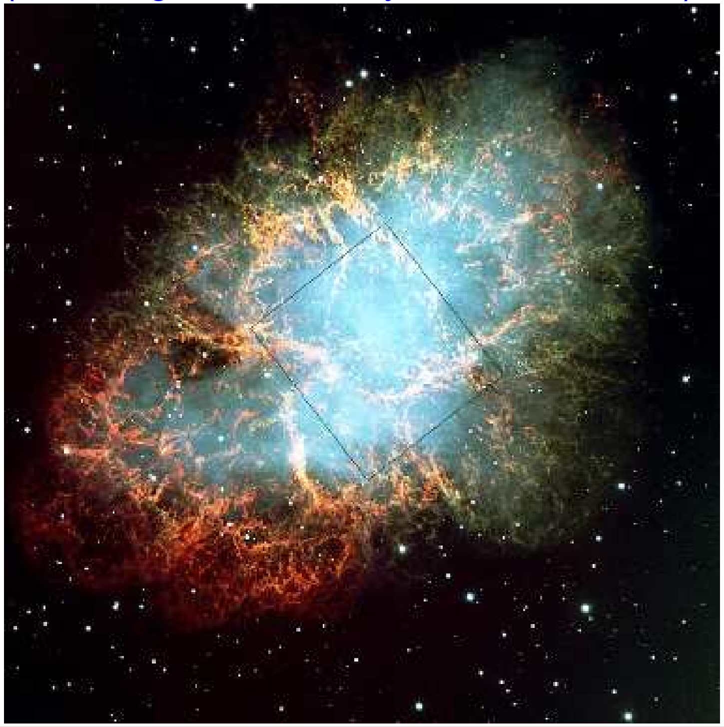


The Crab nebula:

Remnant of the first observed supernova, 4th of July, China 1054

Visible by eye during daytime for a month, during night time for 2 years.

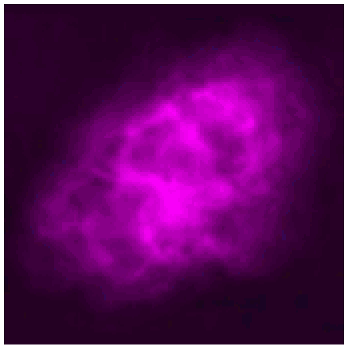
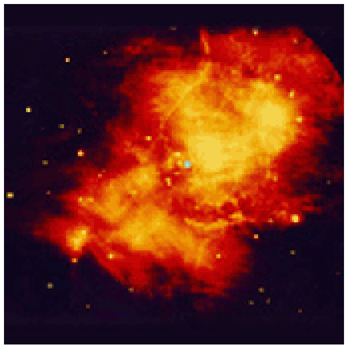
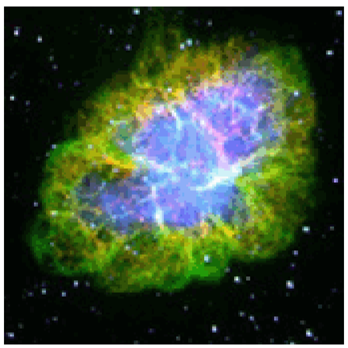
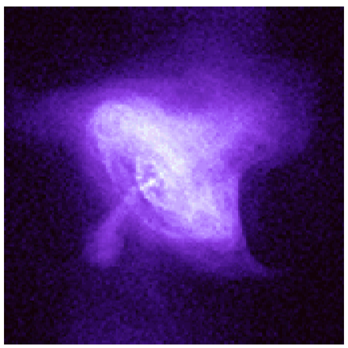
(The largest 4th of July firework, ever !)



Emission from the filaments is mostly in Hydrogen and Oxygen lines, although the total optical emission of the remnant has a broad spectrum: Where is it from?

Synchrotron Radiation in Space

CHESS & LEPP

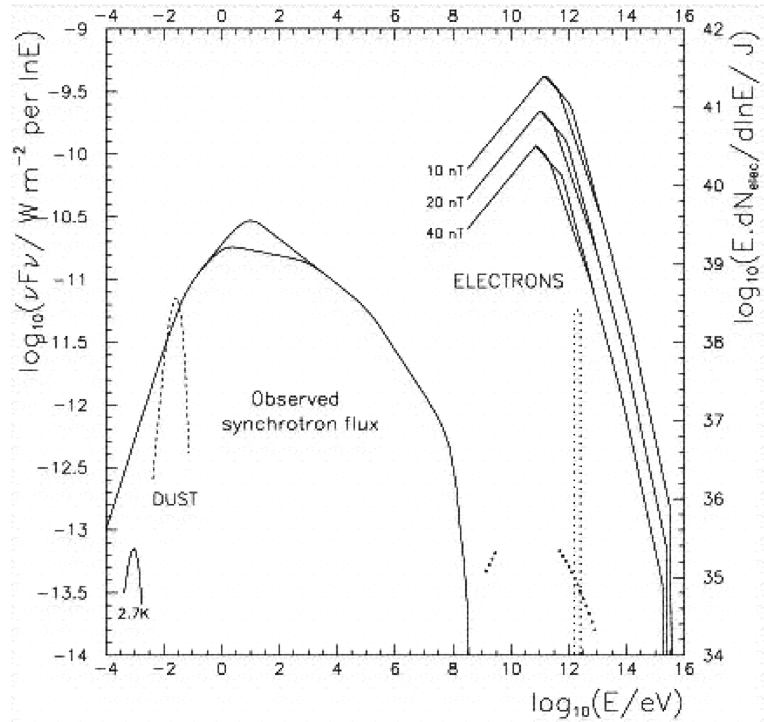
The Crab nebula:

The spectrum has a law in the radio region.

The radiation is to about 7% polarized.

This was an early hint for synchrotron radiation.

The full spectrum clearly verified this conclusion.



But where do all the electrons come from ?

Synchrotron-Radiation Rings in Space

CHESS & LEPP

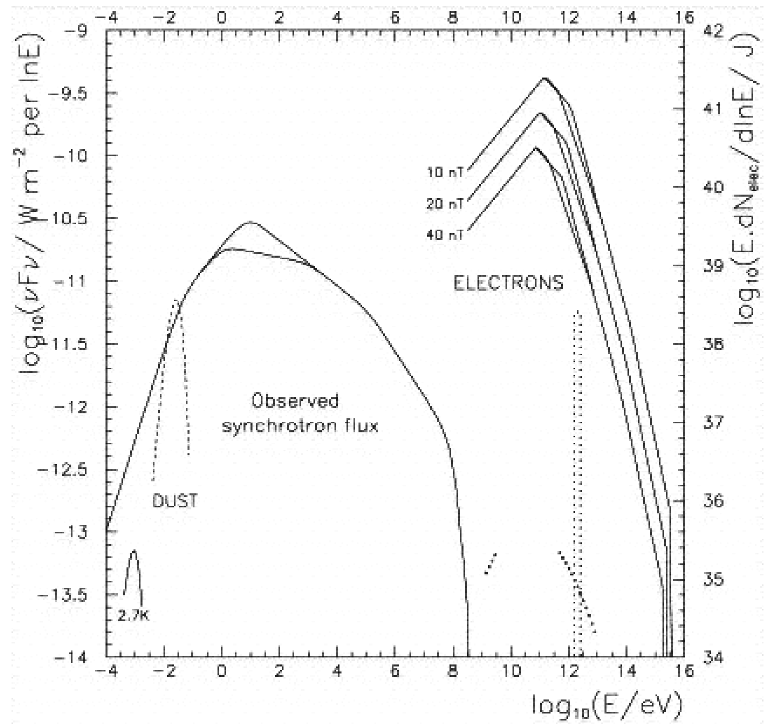
The Crab nebula:

The spectrum has a law in the radio region.

The radiation is to about 7% polarized.

This was an early hint for synchrotron radiation.

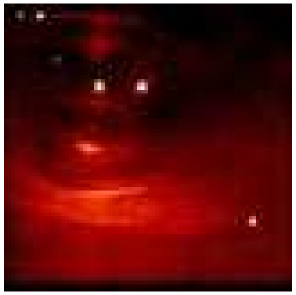
The full spectrum clearly verified this conclusion.



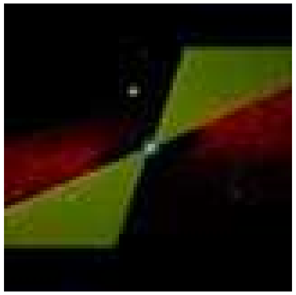
The source of the electrons is probably the central pulsar, or neutron star (seen here as one of the two bright stars near the center).

But where do all the electrons come from ?

Pulsars and Radiation Rings



Moving radiation rings
In the Crab nebula.



Animation explaining
radiation rings.