Accelerating (S)RF Cavities

Matthias Liepe
• **Accelerating cavities**
  - DC accelerators
  - RF cavities
  - Examples

• **RF Cavity Fundamentals**
  - Pill box cavity
  - Figures of merit
  - Accelerating mode
  - Multicell cavities and circuit models

• **RF Cavity Design**
  - NC vs. SC
  - Objectives and cell shapes
  - RF codes
• **Accelerating cavities**
  – DC accelerators
  – RF cavities
  – Examples
• Use high DC voltage to accelerate particles

\[
\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)
\]

• No work done by magnetic fields

DC Accelerators

Cockroft and Walton’s electrostatic accelerator (1932)

Protons were accelerated and slammed into lithium atoms producing helium and energy.
DC Accelerators: Limitations

1) **DC** \( \left( \frac{\partial}{\partial t} = 0 \right) : \nabla \times \vec{E} = 0 \) which is solved by \( \vec{E} = -\nabla \Phi \)

Limit: If you want to gain 1 MeV, you need a potential of 1 MV!

2) **Circular machine**: DC acceleration impossible since \( \oint \vec{E} \cdot d\vec{s} = 0 \)

⇒ **Use time-varying fields!**

**Maxwell’s equation in vacuum (contd.)**

\[
\begin{align*}
\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} &= 0 \\
\n\nabla \cdot \vec{B} &= 0 \\
\n\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} &= 0 \\
\n\n\nabla \cdot \vec{E} &= 0
\end{align*}
\]

Matthias Liepe; 03/24/2008
Drift Tube Linac (DTL) - how it works

For slow particles - protons @ few MeV e.g. - the drift tube lengths can easily be adapted.

electric field

colour coding

<table>
<thead>
<tr>
<th>Value</th>
<th>Colour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0e+00</td>
<td>Red</td>
</tr>
<tr>
<td>0.9e+00</td>
<td>Orange</td>
</tr>
<tr>
<td>0.8e+00</td>
<td>Yellow</td>
</tr>
<tr>
<td>0.7e+00</td>
<td>Green</td>
</tr>
<tr>
<td>0.6e-01</td>
<td>Blue</td>
</tr>
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<td>0.5e-01</td>
<td>Cyan</td>
</tr>
<tr>
<td>0.4e-01</td>
<td>White</td>
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<tr>
<td>0.3e-01</td>
<td>Grey</td>
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<td>Black</td>
</tr>
<tr>
<td>0.0e+00</td>
<td>Black</td>
</tr>
</tbody>
</table>
Example of Time Varying Fields: Drift Tube Linac
- For acceleration we require an oscillating RF field
- Simplest form is an LC circuit
- Let $L = 0.1 \text{ mH}$, $C = 0.01 \text{ µF} \Rightarrow f = 160 \text{ kHz}$
- To increase the frequency, lower $L$, eventually only have a single wire
- To reach even lower values must add inductances in parallel
- Eventually have we have a solid wall
- Shorten „wires“ even further to reduce inductance
  - Pillbox cavity, „simplest form“
- Add beam tubes to let the particles enter and exit
- Magnetic field concentrated in the cavity wall, losses will be here.
• Time dependent electromagnetic field inside metal box

\[ B \sim \sin \omega t \]
\[ E \sim \cos \omega t \]
• The main purpose of using RF cavities in accelerators is to provide energy gain to charged-particle beams.

• The highest achievable gradient, however, is not always optimal for an accelerator. There are other factors (both machine-dependent and technology-dependent) that determine operating gradient of RF cavities and influence the cavity design, such as accelerator cost optimization, maximum power through an input coupler, necessity to extract HOM power, etc.

• In many cases requirements are competing.
• NC or SC
• Relatively low gradient (1…9 MV/m)
• Strong HOM damping (Q ~ 10^2)
• High average RF power (hundreds of kW)
• High gradients
• Moderate HOM damping reqs.
• High peak RF power

ILC: 21,000 cavities!
ILC / XFEL cavities
Traveling Wave Cavities

- Input coupler
- Output coupler
- shown: $\text{Re} \{ \text{Poynting vector} \}$ (power density)

RF accelerator

Travelling wave structure
(CTF3 drive beam, 3 GHz)
CW low-current linacs (CEBAF, ELBE)

- SRF cavities
- Moderate to low gradient (8…20 MV/m)
- Relaxed HOM damping requirements
- Low average RF power (5…13 kW)

CEBAF cavities

ELBE cryomodule
• SRF cavities
• Moderate gradient
  (15…20 MV/m)
• Strong HOM damping
  \((Q = 10^2…10^4)\)
• Low average RF power (few kW)
RF Cavity Fundamentals (Standing wave cavities)

- Cavity eigenmodes
- Figures of merit
- Accelerating mode
- Multicell cavities and circuit model
Cavity is an arbitrary volume, partially closed by the metal wall, capable to store the E-H energy

\[ \approx 3.95 \text{ GHz is the lowest frequency} \]

First assumption:
1. Stored E-H fields are harmonic in time.

Maxwell equations for the harmonic, lossless case with no free charge in the volume:

\[ \begin{align*}
\nabla \times H &= i\omega \varepsilon E \\
\nabla \times E &= -i\omega \mu H \\
\n\nabla \cdot E &= 0 \\
\n\nabla \cdot H &= 0
\end{align*} \]
Second assumption (good approximation for the elliptical cavities):

2. The volume is cylindrically symmetric. We commonly use the \((r, \phi, z)\) coordinates.

\[ \nabla_c \times H = i \omega \varepsilon E \]
\[ \nabla_c \times E = -i \omega \mu H \]
\[ \nabla_c \cdot E = 0 \]
\[ \nabla_c \cdot H = 0 \]

\[ \nabla_c \times A = \hat{r}\left( \frac{1}{r} \frac{\partial A_r}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi}\left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{z}\left( \frac{1}{r} \frac{\partial (rA_\phi)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right) \]
\[ \nabla_c \cdot A = \frac{1}{r} \frac{\partial (rA_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \]
3. For the acceleration are suitable field patterns with strong $E$ along the beam trajectory. This ensures, by the proper phasing, maximal energy exchange between the cavity and beam.

**TM0xx-like monopole** modes have “very strong” $E_z$ component on the symmetry axis.

Fields of the monopole modes are independent on $\varphi$.

$$\frac{\partial E}{\partial \varphi} = 0 \quad \frac{\partial H}{\partial \varphi} = 0$$

**Non monopole (HOM)** modes have component $E_z = 0$ on the symmetry axis.

Their fields dependent on $\varphi$. 
Maxwell equations + boundary conditions for \( \mathbf{E} \) and \( \mathbf{H} \) fields lead to the Helmholtz equation, which is an eigenvalue problem.

For \( H(r,z) \) field of a monopole mode the equation is:

\[
(\nabla_c^2 + \omega^2 \varepsilon \mu) H = 0
\]

\[
\nabla \cdot H = 0 \quad \text{on metal wall}
\]

\[
\begin{cases}
H = 0 \\
\n\nabla \cdot H = 0
\end{cases}
\quad \text{optionally on non metal boundary}
\]

\[
\nabla_c^2 A = \nabla_c (\nabla_c \cdot A) - \nabla_c \times \nabla_c \times A
\]

There is infinity number of TM0xx solutions (modes) to the Helmholtz equation. All modes are determine by:

\[
H_n(r,z) = [0, H_{\phi,n}(r,z), 0],
\]

\[
E_n(r,z) = [E_{r,n}(r,z), 0, E_{z,n}(r,z)]
\]

and frequency \( \omega_n \).
• Time dependent electromagnetic field inside metal box

\[ B \sim \sin \omega t \]
\[ E \sim \cos \omega t \]
• Frequency
• For maximum acceleration we need
\[ T_{\text{cav}} = \frac{d}{c} = \frac{T_{\text{RF}}}{2} \]
so that the field always points in the same direction as the bunch traverses the cavity.
• Accelerating voltage then is
\[ V_{\text{cav}} = \text{Re} \left[ \int_0^d E_z (\phi = 0, z) e^{i \alpha_b z/c} \, dz \right] = d \cdot \frac{\sin \alpha_b d}{\alpha_b d} \cdot \frac{c}{c} = d \cdot E_0 \cdot T \]
• Accelerating field is
\[ E_{\text{acc}} = \frac{V_{\text{cav}}}{d} = \frac{2 E_0}{\pi} \]

, here, \( T = \frac{2}{\pi} \) is flight time factor
(for pill-box with this length \( d \) only, other shapes can have a different value of \( T \)).
• Surface currents ($\propto H$) result in dissipation proportional to the surface resistance ($R_s$):

$$\frac{dP_c}{ds} = \frac{1}{2} R_s |H|^2.$$

• Dissipation in the cavity wall given by surface integral:

$$P_c = \frac{1}{2} R_s \int_S |H|^2 \, ds$$

• Stored energy is:

$$U = \frac{1}{2} \mu_0 \int_V |H|^2 \, dv$$

• Define Quality ($Q$) as

$$Q_0 = \frac{\omega_0 U}{P_c} = 2 \pi \frac{U}{T_{rf} P_c}$$

which is $\sim 2 \pi$ number of cycles it takes to dissipate the energy stored in the cavity $\rightarrow$ Easy way to measure $Q$

• $Q_{nc} \approx 10^4$, $Q_{sc} \approx 10^{10}$. 

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Since the time averaged energy in the electric field equals that in magnetic field, the total energy in the cavity is given by

$$U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2 \, dv = \frac{1}{2}\varepsilon_0 \int_V |\mathbf{E}|^2 \, dv,$$

where the integral is taken over the volume of the cavity.

Power dissipated in the cavity walls is

$$P_c = \frac{1}{2}R_s \int_S |\mathbf{H}|^2 \, ds,$$

where the integration is taken over the interior cavity surface.

$$Q_0 = \frac{\omega_0 U}{P_c}, \quad Q_0 = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 \, dv}{R_s \int_S |\mathbf{H}|^2 \, ds}.$$ 

The $Q_0$ is frequently written as

$$Q_0 = \frac{G}{R_s},$$

where

$$G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 \, dv}{\int_S |\mathbf{H}|^2 \, ds}.$$ 

$G$ is known as the geometry constant. From the last eq. we can see that it depends on the cavity shape but not its size.
Accelerating π-mode:

Accelerating voltage:

\[ V_{\text{acc}} = \frac{\text{maximum energy gain}}{\text{charge}} = \int_{-L/2}^{+L/2} E_z e^{i \omega (z/c)} dz \]

Shunt-impedance:

\[ R_a = \frac{(V_{\text{acc}})^2}{P_{\text{dis}}} \]

Quality factor:

\[ Q_L = \frac{\omega U}{P_{\text{dis}}} \]

\[ \frac{R_a}{Q_L} = \frac{R}{Q} = \frac{(V_{\text{acc}})^2}{\omega U} \]
• Shunt impedance \( R_a \) determines how much acceleration one gets for a given dissipation (analogous to Ohm’s Law)

\[
R_a = \frac{V_c^2}{P_c}
\]

→ To maximize acceleration (\( P_c \) given), must maximize shunt impedance.

Another important figure of merit is

\[
\frac{R_a}{Q_0} = \frac{V_c^2}{\omega_0 U},
\]

• \( Ra/Q \) only depends on the cavity geometry

→ This quantity is also used for determining the level of mode excitation by charges passing through the cavity.

→ To minimize losses (\( P_c \)) in the cavity, we must maximize \( G*Ra/Q_0 \):

\[
P_c = \frac{V_c^2}{R_a} = \frac{V_c^2}{Q_0 \cdot (R_a / Q_0)} = \frac{V_c^2}{(R_s \cdot Q_0)(R_a / Q_0) / R_s} = \frac{V_c^2 \cdot R_s}{G \cdot (R_a / Q_0)}
\]
Typical Values for Single Cells

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Cornell SC 500 MHz</th>
<th>Pillbox</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>270 Ω</td>
<td>257 Ω</td>
</tr>
<tr>
<td>$R_a/Q_0$</td>
<td>88 Ω/cell</td>
<td>196 Ω/cell</td>
</tr>
<tr>
<td>$E_{pk}/E_{acc}$</td>
<td>2.5</td>
<td>1.6</td>
</tr>
<tr>
<td>$H_{pk}/E_{acc}$</td>
<td>52 Oe/MV/m</td>
<td>30.5 Oe/(MV/m)</td>
</tr>
</tbody>
</table>

Current is high, it excites a lot of Higher Order Modes, so the hole is made big to propagate HOMs, and this is why $H_{pk}$ and $E_{pk}$ grew, and $R/Q$ drops.

Diff. applications – diff. trade-offs.
Multicell Cavities: Why?

- **higher fill-factor:**
  - active length
  \[ F = \frac{\text{active length}}{\text{total length}} \]

- \Rightarrow lower costs
- \Rightarrow better beam

- fewer
  - input couplers
  - waveguide elements
  - RF control systems
  - ...

\[ F \approx 75\% \]
Example: 500 GeV Linear Collider

21,024 9-cell cavities: 27.8 km (17.3 miles)

189,216 1-cell cavities: 75.4 km (46.8 miles)
The last parameter, relevant for multi-cell accelerating structures, is the coupling $k_{cc}$ between cells for the accelerating mode passband (Fundamental Mode passband).

Single-cell structures are attractive from the RF-point of view:
- Easier to manage HOM damping
- No field flatness problem.
- Input coupler transfers less power
- Easy for cleaning and preparation
- But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.

Multi-cell structures are less expensive and offers higher real-estate gradient but:
- Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells
- Other problems arise: HOM trapping...
**Single Cell:**

$\text{TM}_{010}$ mode

**Coupled Cells**
Two Coupled Cells: TM010 Modes

0 - Mode

π - Mode

2 coupled cells

n coupled cells

2 TM\(_{010}\) modes

n TM\(_{010}\) modes
Two Coupled Cells: TM010 Modes

Resonators closed by metal wall:

Symmetry planes for the H field

Symmetry plane for the E field which is an additional solution
Cell-to-Cell Coupling

Coupling factor:

\[ k_{cc} = \frac{c}{\omega_0} \int_s \left( \bar{e} \times \bar{h} \right) \cdot \bar{u}_z ds \]

\[ k_{cc} = \frac{\omega_\pi - \omega_0}{\omega_\pi + \omega_0} \]

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Solve Maxwell’s equations for given boundary conditions

\[ \bar{E}^{(m)}(\bar{r},t) = \bar{E}^{(m)}(\bar{r})e^{i\omega^{(m)}t} \]
\[ \bar{H}^{(m)}(\bar{r},t) = \bar{H}^{(m)}(\bar{r})e^{i\omega^{(m)}t+i\pi/2} , \quad m=1,2,... \]

\[ \bar{e}^{(m)}(\bar{r}) = \sqrt{\frac{\varepsilon_0}{2U^{(m)}}} \bar{E}^{(m)}(\bar{r}) \]
\[ \int_{V} \bar{e}^{(m)}(\bar{r}) \bar{e}^{(n)}(\bar{r}) dV = \delta_{mn} \]

\[ \bar{h}^{(m)}(\bar{r}) = \sqrt{\frac{\mu_0}{2U^{(m)}}} \bar{H}^{(m)}(\bar{r}) \]
\[ \int_{V} \bar{h}^{(m)}(\bar{r}) \bar{h}^{(n)}(\bar{r}) dV = \delta_{mn} \]
With Maxwell’s equations in vacuum:
\[ \nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \quad \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \]
and the eigenmodes
\[ \mathbf{E}^{(m)}(\mathbf{r},t) = \bar{\mathbf{E}}^{(m)}(\mathbf{r}) e^{i \omega^{(m)} t} \]
\[ \mathbf{H}^{(m)}(\mathbf{r},t) = \bar{\mathbf{H}}^{(m)}(\mathbf{r}) e^{i \omega^{(m)} t + i \pi / 2} \]
and the normalized eigenfunctions
\[ \bar{\mathbf{h}}^{(m)}(\mathbf{r}) = \sqrt{\frac{\mu_0}{2U^{(m)}}} \bar{\mathbf{H}}^{(m)}(\mathbf{r}) \]
\[ \bar{\mathbf{e}}^{(m)}(\mathbf{r}) = \sqrt{\frac{\varepsilon_0}{2U^{(m)}}} \bar{\mathbf{E}}^{(m)}(\mathbf{r}) \]
we get the relations
\[ \nabla \times \bar{\mathbf{h}}^{(m)} = \frac{\omega^{(m)}}{c} \bar{\mathbf{e}}^{(m)} \]
\[ \nabla \times \bar{\mathbf{e}}^{(m)} = \frac{\omega^{(m)}}{c} \bar{\mathbf{h}}^{(m)} \]
A Circuit Model: Step 1

Eigenmodes: Example: Pillbox-Cavity

**lowest frequency mode:**  
\[ \text{TM010} \]

transverse magnetic

\[ 0 \Rightarrow \text{monopole mode} \]

**electric field**

\[ \vec{E}(r, t) = E_0 J_0 \left( \frac{\omega_0}{c} r \right) \cos(\omega_0 t) \vec{e}_z \]

⇒ acceleration

**magnetic field**
**Accelerating mode:**

\[ TM_{010} : \text{electric field} \quad \text{magnetic field} \]

\[ f_0 = 1.3 \text{ GHz für TESLA} \]
Each time dependent field in a cavity can be written as a sum of the cavity eigenfunctions with time dependent amplitudes:

\[ \vec{E}(\vec{r},t) = \sum_{m} \hat{E}^{(m)}(t) \vec{e}^{(m)}(\vec{r}) \]

\[ \vec{H}(\vec{r},t) = \sum_{m} \hat{H}^{(m)}(t) \vec{h}^{(m)}(\vec{r}) \]
Insert expansion of fields into Maxwell’s equations in vacuum:

\[
\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}
\]

insert eigenmode-expansion of fields… \(\downarrow\) …and the equations from page 37

\[
\sum_m \hat{H}^{(m)}(t) \frac{\omega^{(m)}}{c} \hat{e}^{(m)}(\vec{r}) = \sum_m \varepsilon_0 \frac{d\hat{E}^{(m)}(t)}{dt} \hat{e}^{(m)}(\vec{r})
\]

use orthogonality of eigenmodes: \(\downarrow\) multiply by \(\hat{e}^{(m)}\) and integrate over cavity volume

\[
\frac{\omega^{(m)}}{c} \hat{H}^{(m)}(t) - \varepsilon_0 \frac{d}{dt} \hat{E}^{(m)}(t) = 0 \quad , \quad m = 1, 2, \ldots
\]
Insert expansion of fields into Maxwell’s equations:

\[ \tilde{\nabla} \times \tilde{E} = -\mu_0 \frac{\partial \tilde{H}}{\partial t} \]

insert eigenmode-

expansion of fields… \[\downarrow\] …and the equations

from page 37

\[ \sum_m \tilde{E}^{(m)}(t) \frac{\omega^{(m)}}{c} \tilde{h}^{(m)}(\tilde{r}) = -\sum_m \mu_0 \frac{d \hat{H}^{(m)}(t)}{dt} \tilde{h}^{(m)}(\tilde{r}) \]

use orthogonality of

eigenmodes: \[\downarrow\] multiply by \( h^{(m)} \) and

integrate over cavity

volume

\[ \frac{\omega^{(m)}}{c} \tilde{E}^{(m)}(t) + \mu_0 \frac{d}{dt} \hat{H}^{(m)}(t) = 0 \quad , m = 1,2... \]
Circuit Model: Step 3.3

Differential Equation for the Eigenmode Amplitudes

\[
\frac{\omega^{(m)}}{c} \hat{H}^{(m)}(t) - \varepsilon_0 \frac{d}{dt} \hat{E}^{(m)}(t) = 0 \quad , \quad m = 1, 2, \ldots
\]

\[
\frac{\omega^{(m)}}{c} \hat{E}^{(m)}(t) + \mu_0 \frac{d}{dt} \hat{H}^{(m)}(t) = 0 \quad , \quad m = 1, 2, \ldots
\]

\[
\frac{d^2}{dt^2} \hat{E}^{(m)}(t) + \left( \frac{\omega^{(m)}}{} \right)^2 \hat{E}^{(m)}(t) = 0 \quad , \quad m = 1, 2, \ldots
\]

amplitude of eigenmode \#m

⇒ Differential equation of an oscillator (we will add losses and a generator later)!
TM010 modes in couples cells:

N coupled cells ⇒ N coupled oscillators

⇒ N coupled differential equations:

\[
\frac{d^2}{dt^2} \hat{E}_n + \omega_n^2 \hat{E}_n + \omega_n^2 \frac{K}{2} (\hat{E}_n - \hat{E}_{n-1}) + \omega_n^2 \frac{K}{2} (\hat{E}_n - \hat{E}_{n+1}) = 0, \quad n = 1, 2, \ldots, N
\]

TM010 eigenfrequency of cell \#n

TM010 field amplitude in cell \#n

N-cell structure

⇒ N coupled differential equations

cell-to-cell coupling factor

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\[ \frac{d^2}{dt^2} \hat{E}_n + \omega_n^2 \hat{E}_n + \omega_n^2 \frac{K}{2} (\hat{E}_n - \hat{E}_{n-1}) + \omega_n^2 \frac{K}{2} (\hat{E}_n - \hat{E}_{n+1}) = 0 \quad , \ n = 1, 2, ..., N \]

**substitute:**

\[ \omega_n^2 = \frac{1}{LC_n} \]

\[ K_{n,n+1} = 2 \frac{C_n}{C_{n,n+1}} = 2k_{n,n+1} \]
Circuit Model: Step 6
Eigenmode Equation for the TM010 Modes

\[
\frac{d^2}{dt^2} \hat{E}_n + \omega_0^2 \hat{E}_n + \alpha_0^2 \frac{K}{2} (\hat{E}_n - \hat{E}_{n-1}) + \omega_0^2 \frac{K}{2} (\hat{E}_n - \hat{E}_{n+1}) = 0 \quad , \quad n = 1, 2, ..., N
\]

steady state ansatz

\[
\begin{bmatrix}
1 + k_{1,2} & -k_{1,2} \\
-k_{1,2} & 1 + k_{1,2} + k_{2,3} & -k_{2,3} \\
& \ddots & \ddots & \ddots \\
& k_{N-2,N-1} & 1 + k_{1,2} + k_{2,3} & k_{N-1,N} \\
& & k_{N-1,N} & 1 + k_{N-1,N}
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2 \\
\vdots \\
A_N
\end{bmatrix}
 = \left( \frac{\omega}{\omega_0} \right)^2 
\begin{bmatrix}
A_1 \\
A_2 \\
\vdots \\
A_N
\end{bmatrix}
\]

Eigenvalue: \( \lambda \) cell amplitudes of a \( \text{TM}_{010} \) eigenmode of a multicell cavity.

**eigenvector \#j**

**eigenfrequency of mode \#j**
TM010 Eigenmodes
Example: 9-Cell Cavity (1)

- Mode #1
- Mode #2
- Mode #3
- Mode #4
- Mode #5
- Mode #6

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accelerating mode: $\pi$ cell-to-cell phase advance
Circuit Model: Step 7
Add Losses and a Generator

\[ Q_n = \frac{\omega L}{R_0} = \frac{\omega U_n}{P_{dis,n}} \]

generator current

losses:
Simulation Example:
TM010 Eigenmode-Spectrum of a 9-Cell Cavity

Amplitude (cell #1) [dBA]

Frequency [MHz]

\( \frac{1}{9} \pi, \frac{2}{9} \pi, \frac{3}{9} \pi, \frac{4}{9} \pi, \frac{5}{9} \pi, \frac{6}{9} \pi, \frac{7}{9} \pi, \frac{8}{9} \pi, \pi \)
The working point. If it is too close to the neighbor point this neighbor mode can also be excited. To avoid this, more cell-to-cell coupling is needed: broader aperture.
Modeling of the transient state (mode beating)
Example: 7-cells, $k_{cc}=1.85\%$, $Q_L=3.4 \times 10^6$