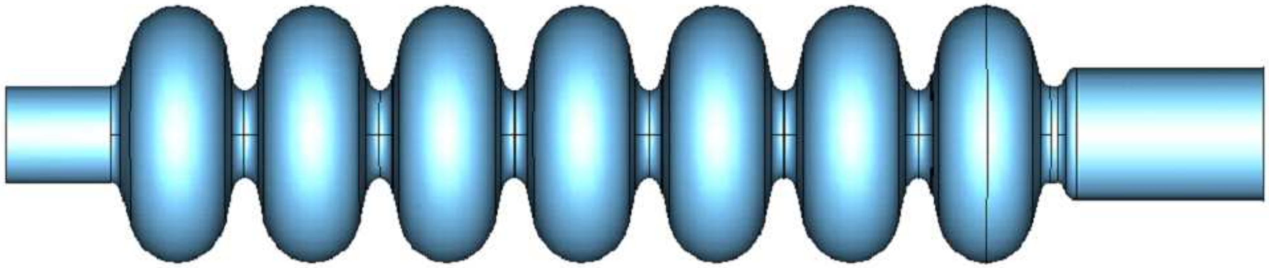
Accelerating (S)RF Cavities

Matthias Liepe



- **Accelerating cavities**
  - DC accelerators
  - RF cavities
  - Examples
- **RF Cavity Fundamentals**
  - Pill box cavity
  - Figures of merit
  - Accelerating mode
  - Multicell cavities and circuit models
- **RF Cavity Design**
  - NC vs. SC
  - Objectives and cell shapes
  - RF codes

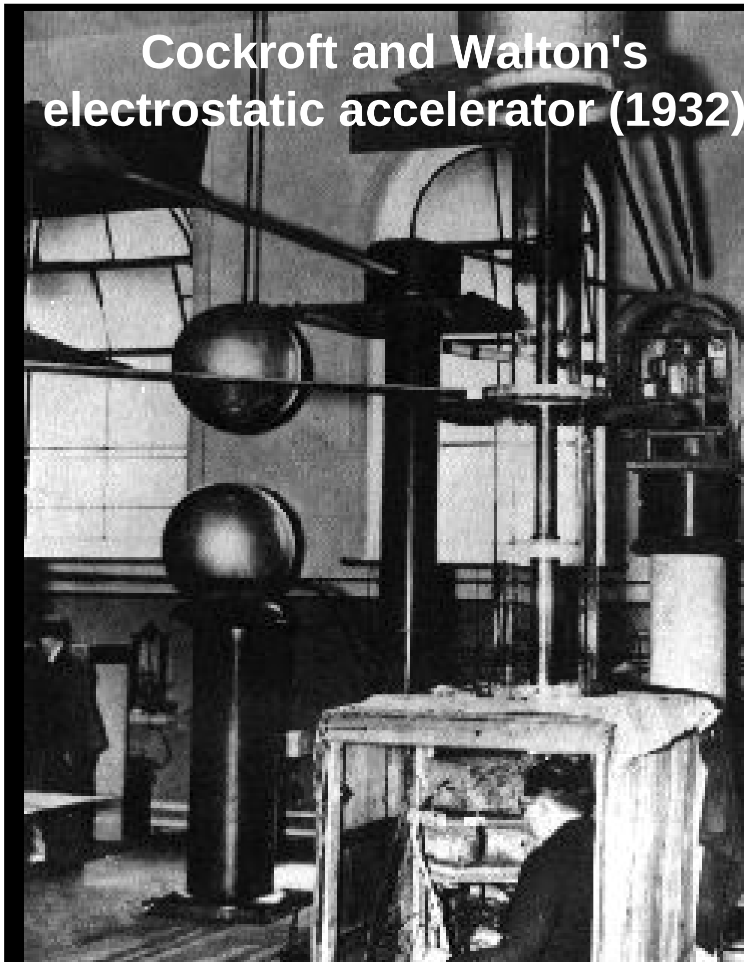
- **Accelerating cavities**
  - DC accelerators
  - RF cavities
  - Examples

DC Accelerators

- **Use high DC voltage to accelerate particles**

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- **No work done by magnetic fields**



Protons were accelerated and slammed into lithium atoms producing helium and energy.

DC Accelerators: Limitations

1) DC ($\frac{\partial}{\partial t} \equiv 0$): $\nabla \times \vec{E} = 0$ which is solved by $\vec{E} = -\nabla \Phi$

Limit: If you want to gain 1 MeV, you need a potential of 1 MV!

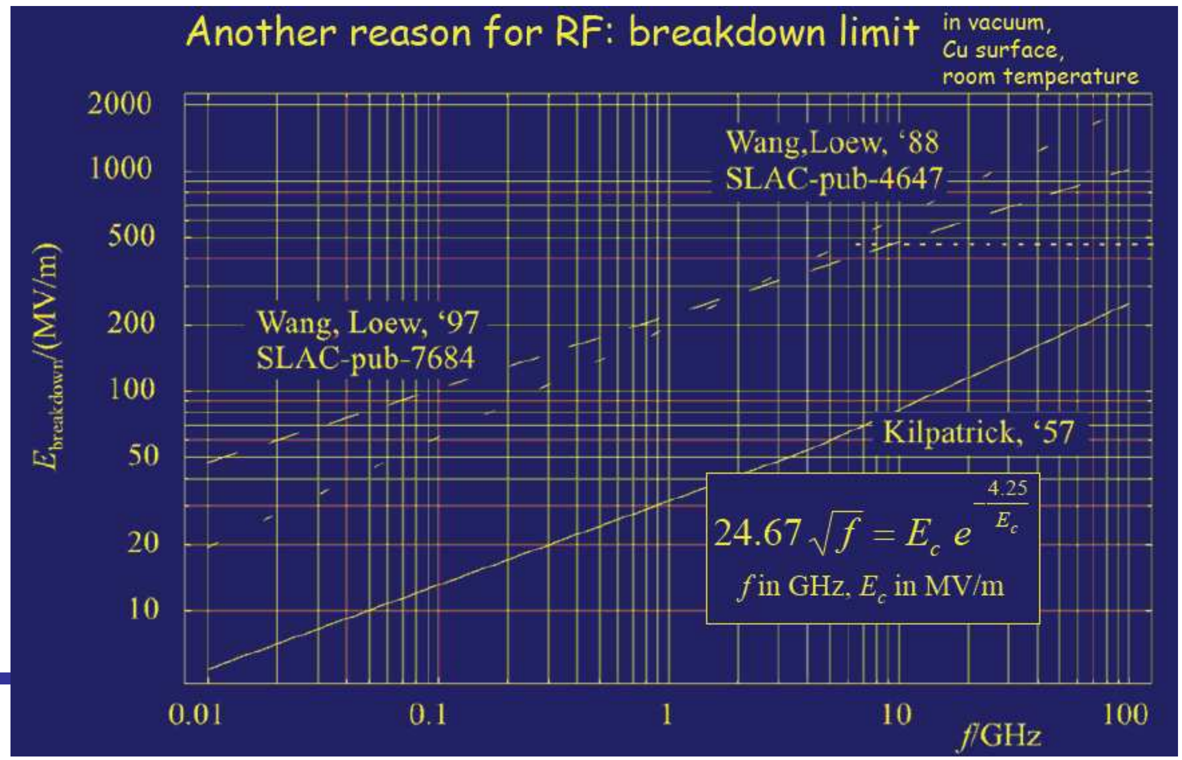
2) Circular machine: DC acceleration impossible since $\oint \vec{E} \cdot d\vec{s} = 0$

⇒ Use time-varying fields!

Maxwell's equation in vacuum (contd.)

$$\nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E} = 0 \quad \nabla \cdot \vec{B} = 0$$

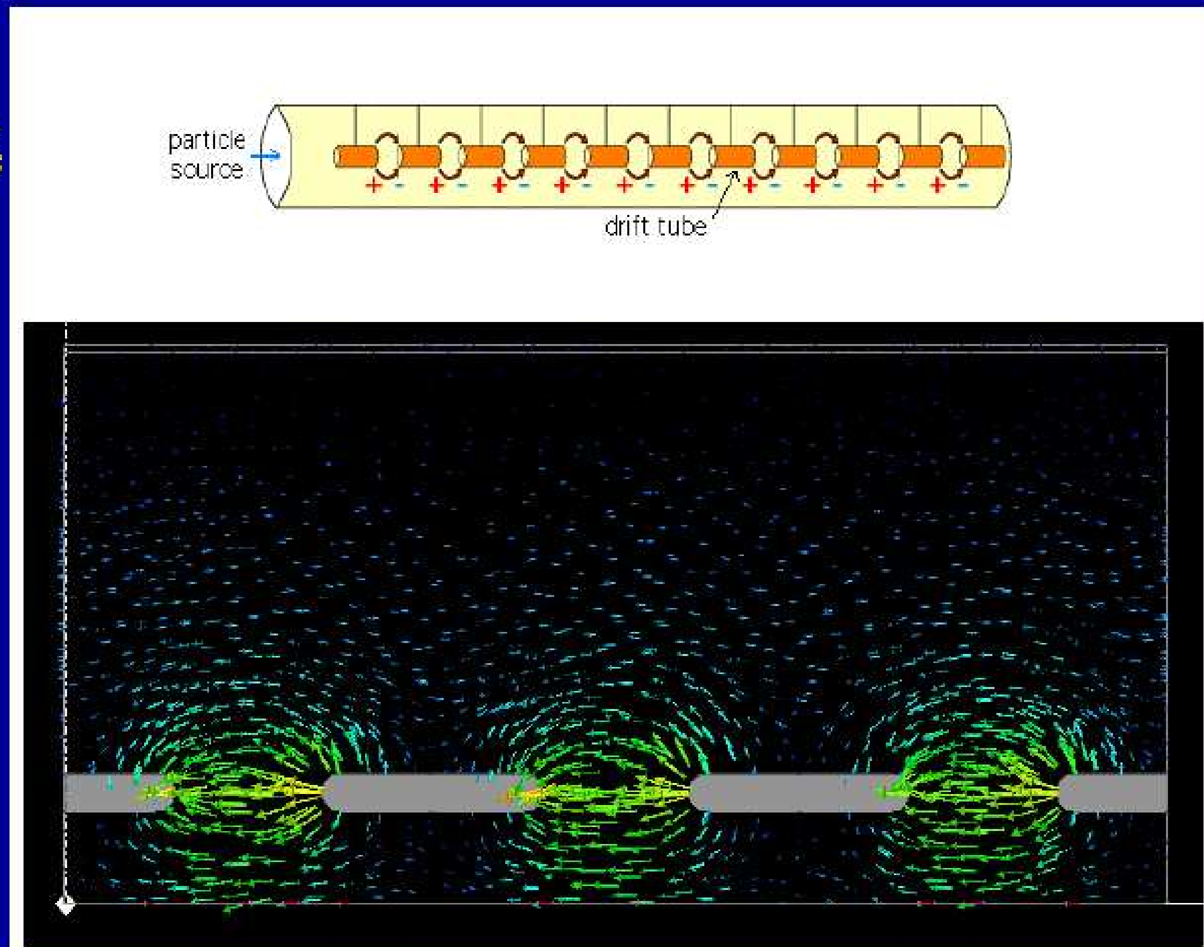
$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \quad \nabla \cdot \vec{E} = 0$$



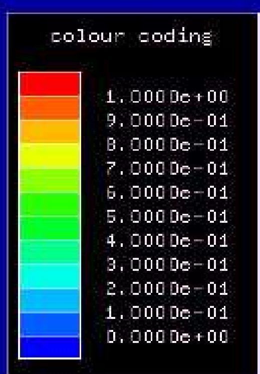
Example of Time Varying Fields: Drift Tube Linac

Drift Tube Linac (DTL) - how it works

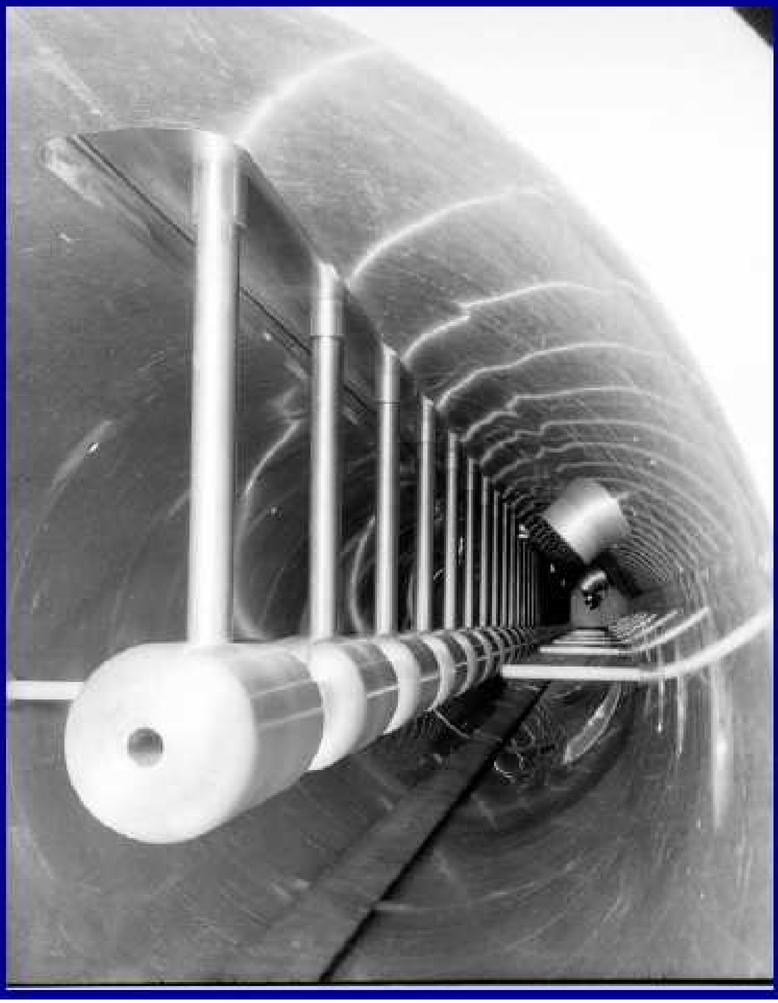
For slow particles – protons @ few MeV e.g. – the drift tube lengths can easily be adapted.

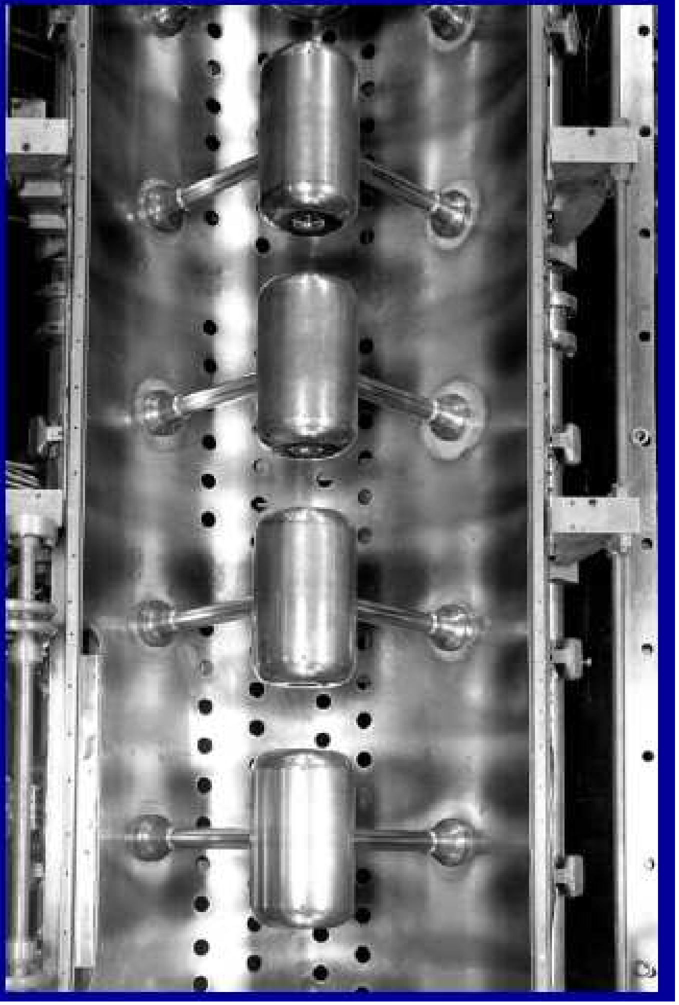


electric field



Example of Time Varying Fields: Drift Tube Linac

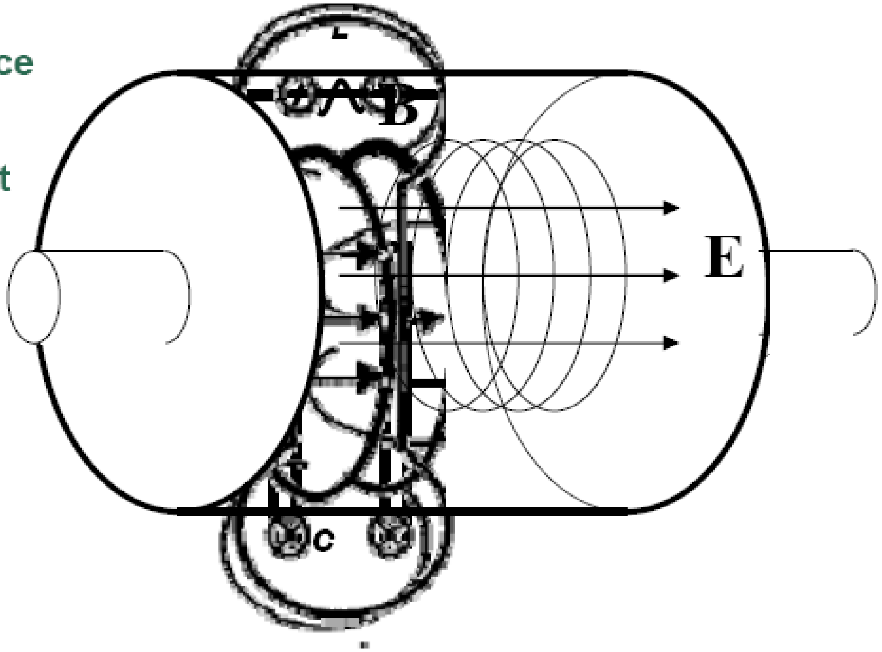




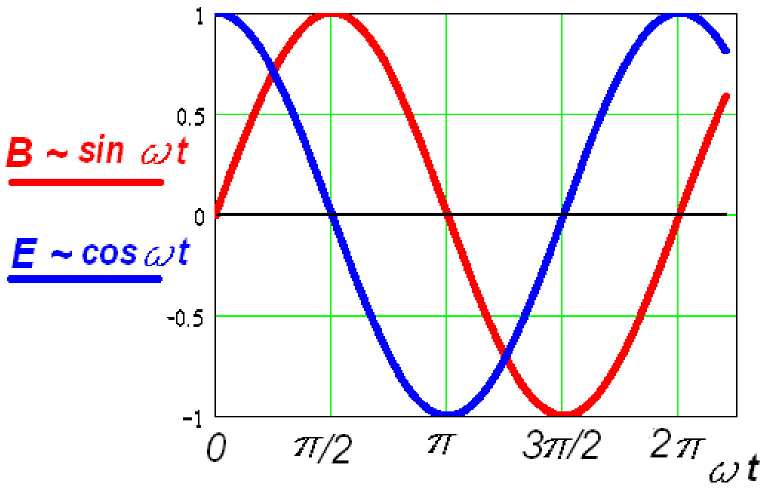
- For acceleration we require an oscillating RF field
- Simplest form is an LC circuit
- Let L = 0.1 mH, C = 0.01 μF → f = 160 kHz
- To increase the frequency, lower L, eventually only have a single wire
- To reach even lower values must add inductances in parallel
- Eventually have we have a solid wall
- Shorten „wires" even further to reduce inductance
- → Pillbox cavity, „simplest form"
- Add beam tubes to let the particles enter and exit

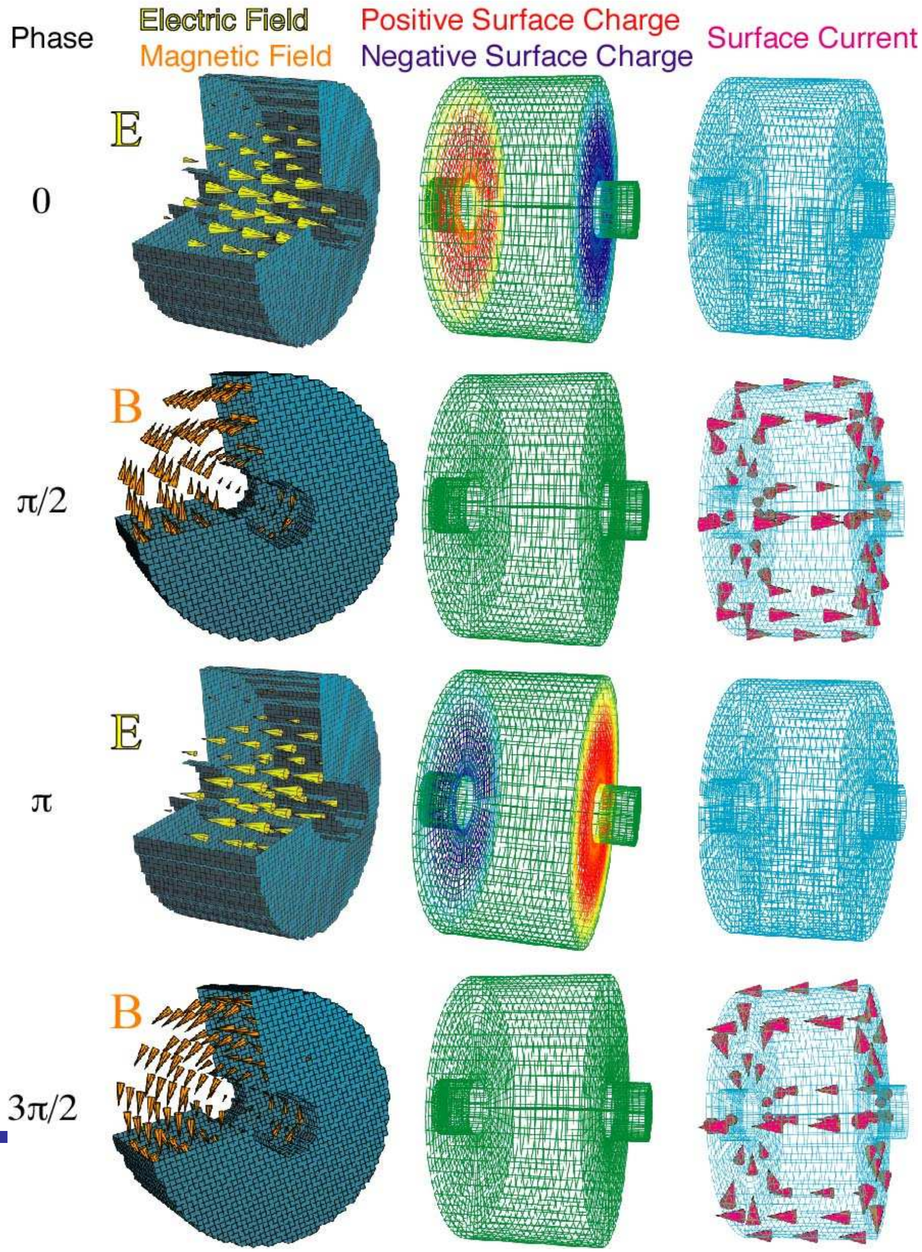
- Magnetic field concentrated in the cavity wall, losses will be here.

Based on Feynman's Lect. on Physics.



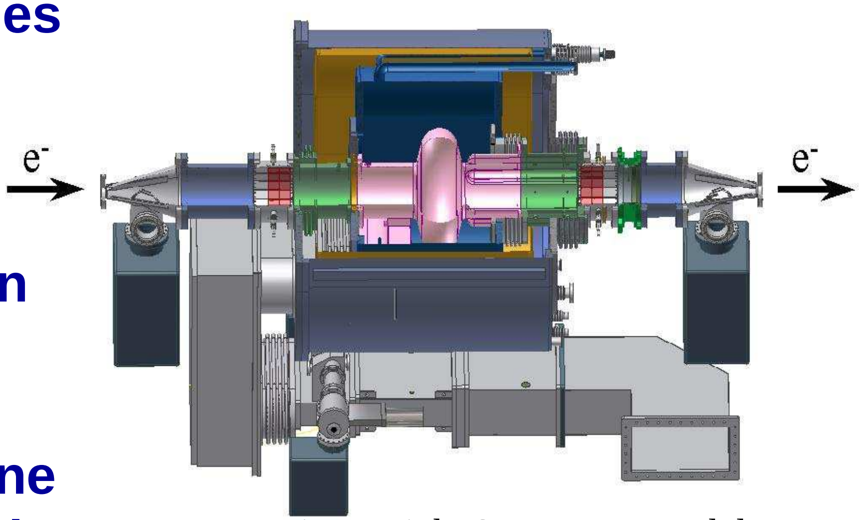
- **Time dependent electromagnetic field inside metal box**





Introduction to RF Cavities

- The main purpose of using RF cavities in accelerators is to provide energy gain to charged-particle beams
- The highest achievable gradient, however, is not always optimal for an accelerator. There are other factors (both machine-dependent and technology-dependent) that determine operating gradient of RF cavities and influence the cavity design, such as accelerator cost optimization, maximum power through an input coupler, necessity to extract HOM power, etc.
- In many cases requirements are competing.

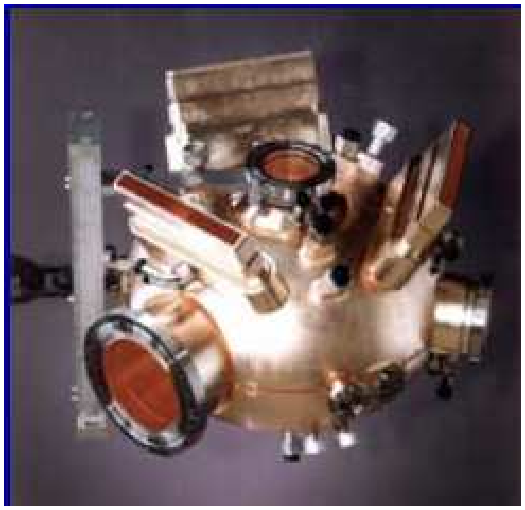


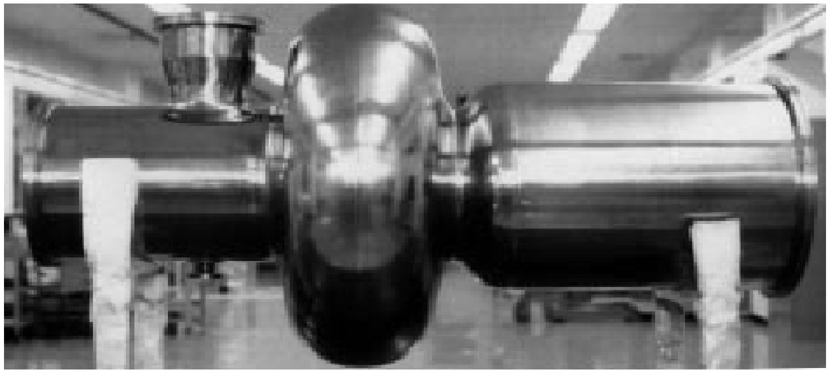
Taiwan Light Source cryomodule

- NC or SC
- Relatively low gradient
  (1…9 MV/m)
- Strong HOM damping
  (Q ~ 10^2)
- High average RF power
  (hundreds of kW)

CESR cavities

PEP II Cavity

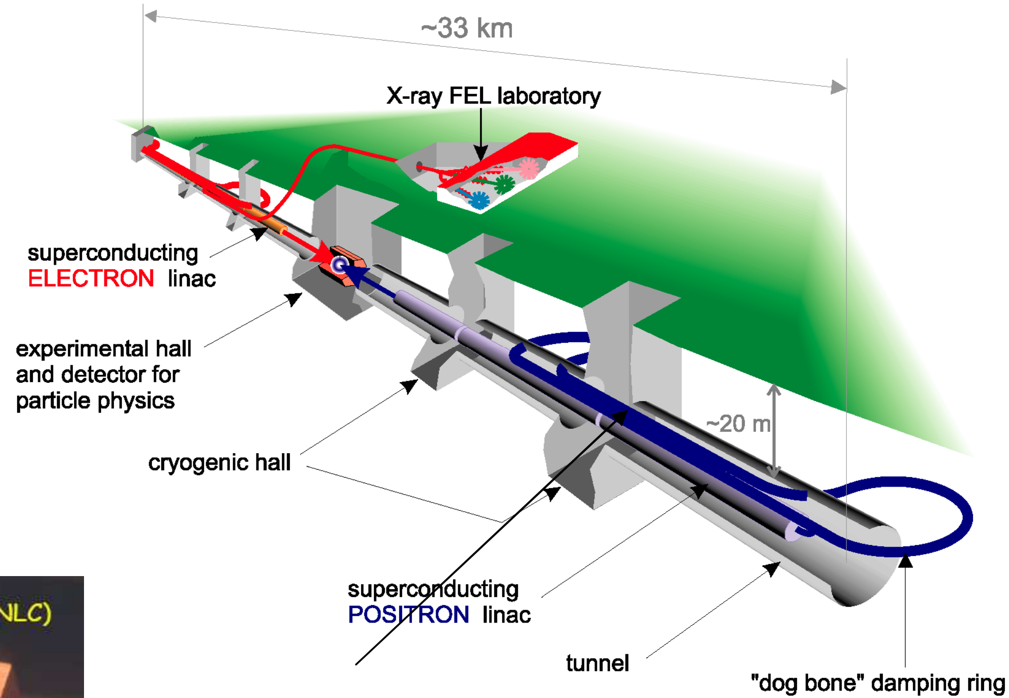


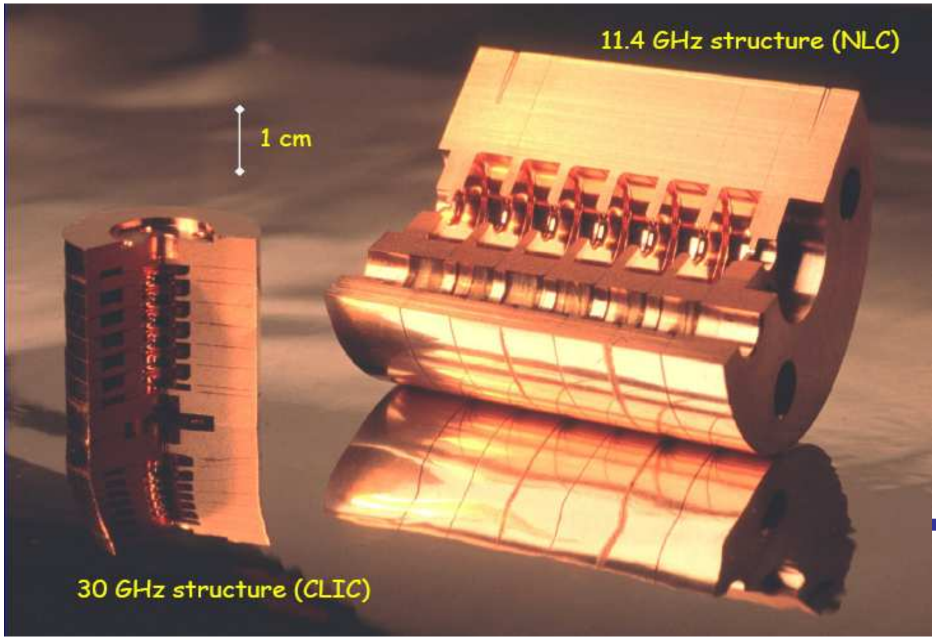


KEK cavity

Pulsed Linacs (ILC, XFEL, ...)

- High gradients
- Moderate HOM damping reqs.
- High peak RF power

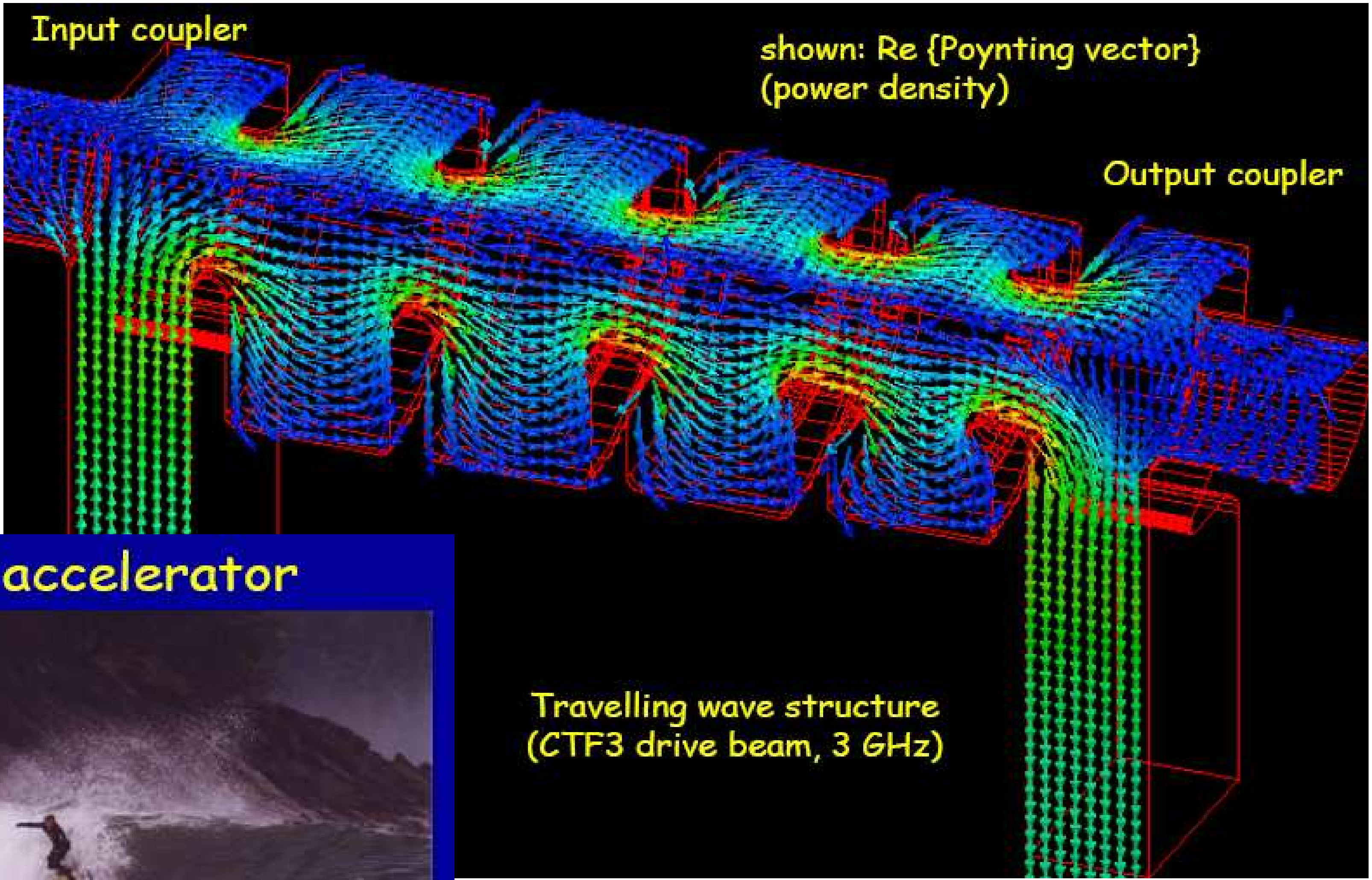




ILC: 21,000 cavities!

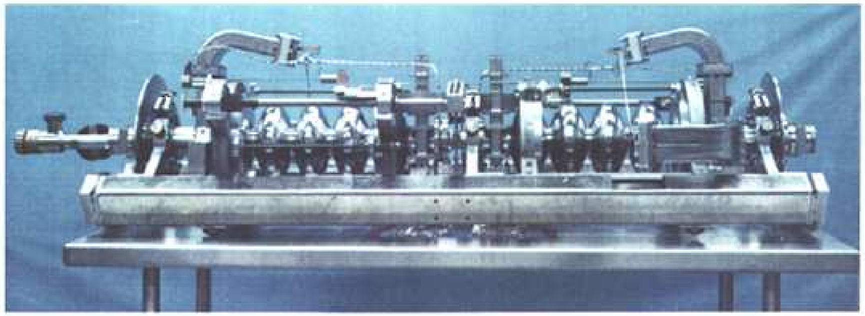
ILC / XFEL cavities

Traveling Wave Cavities

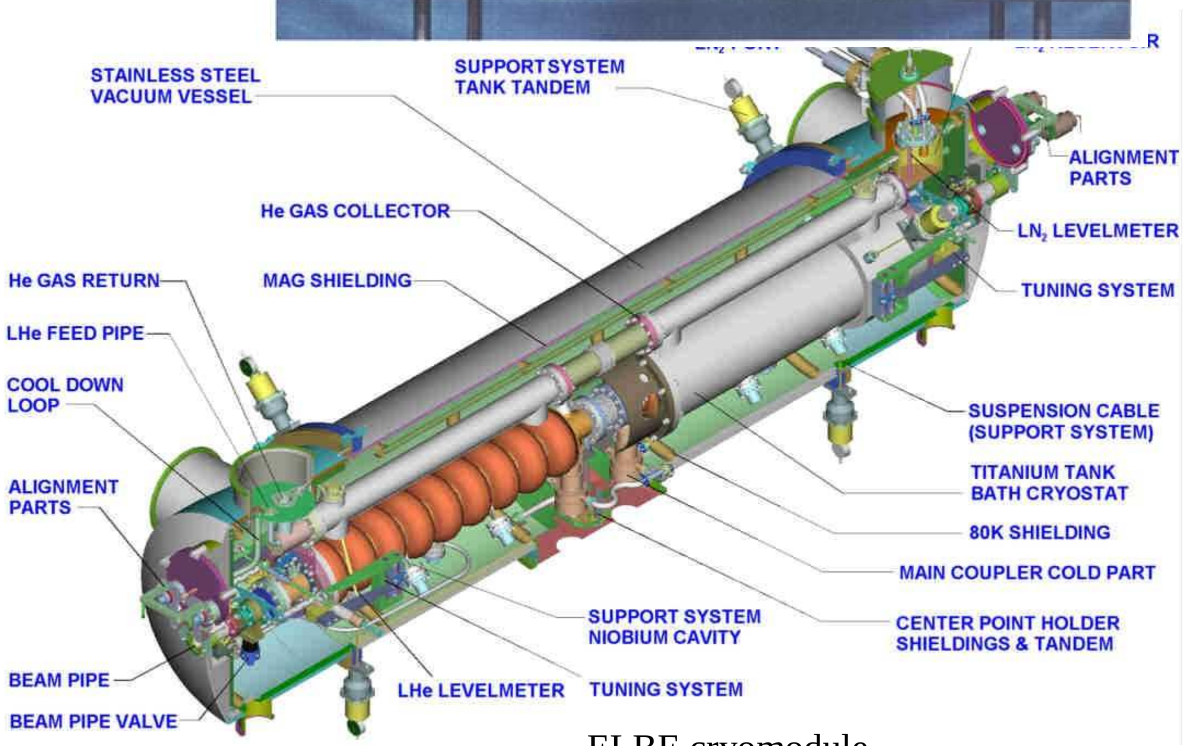


RF accelerator

CEBAF cavities

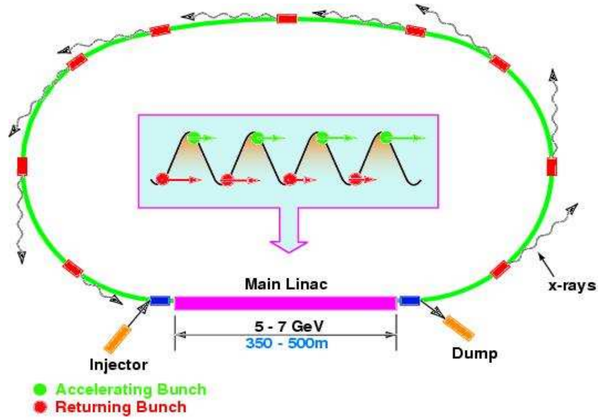


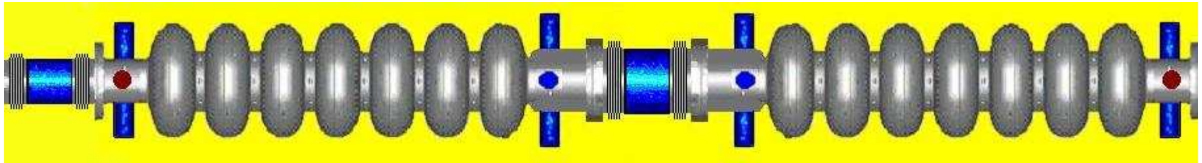
- SRF cavities
- Moderate to low gradient
  (8…20 MV/m)
- Relaxed HOM damping requirements
- Low average RF power
  (5…13 kW)



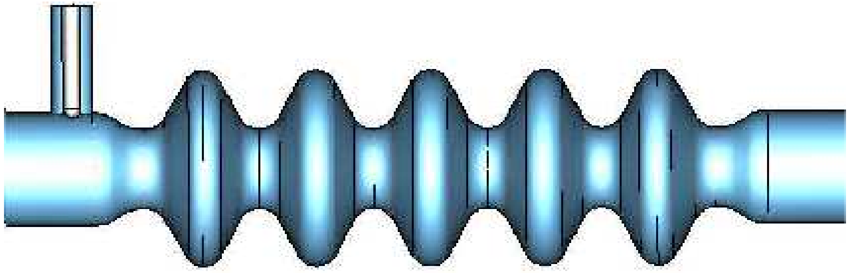
ELBE cryomodule

- SRF cavities
- Moderate gradient
  (15…20 MV/m)
- Strong HOM damping
  ($Q = 10^2 \ldots 10^4$)
- Low average RF power (few kW)





Cornell ERL cavities



BNL ERL cavity

- **RF Cavity Fundamentals (Standing wave cavities)**
  - Cavity eigenmodes
  - Figures of merit
  - Accelerating mode
  - Multicell cavities and circuit model

RF Cavities and their Eigenmodes I

Cavity≡ an arbitrary volume, partially closed by the metal wall, capable to store the E-H energy

~ 3.95 GHz is the lowest frequency

First assumption:

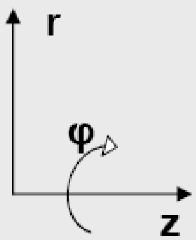
1. Stored E-H fields are harmonic in time.

Maxwell equations for the harmonic, lossless case with no free charge in the volume

$$\begin{cases} \nabla \times H = i\omega\varepsilon E \\ \nabla \times E = -i\omega\mu H \\ \nabla \cdot E = 0 \\ \nabla \cdot H = 0 \end{cases}$$

Second assumption (good approximation for the elliptical cavities):

2. The volume is cylindrically symmetric. We commonly use the (r, φ, z) coordinates.



z is conventional direction of the acceleration and symmetry axis

$$\begin{cases} \nabla_c \times H = i\omega\varepsilon E \\ \nabla_c \times E = -i\omega\mu H \\ \nabla_c \cdot E = 0 \\ \nabla_c \cdot H = 0 \end{cases}$$

$$\nabla_c \times A = \vec{i}_r\left(\frac{1}{r}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \vec{i}_\varphi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \vec{i}_z\left(\frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial\varphi}\right)$$

$$\nabla_c \cdot A = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

RF Cavities and their Eigenmodes III

3. For the acceleration are suitable field patterns with strong E along the beam trajectory. This ensures, by the proper phasing, maximal energy exchange between the cavity and beam.

TM0xx-like monopole modes have "very strong" E_z component on the symmetry axis. Fields of the monopole modes are independent on φ.

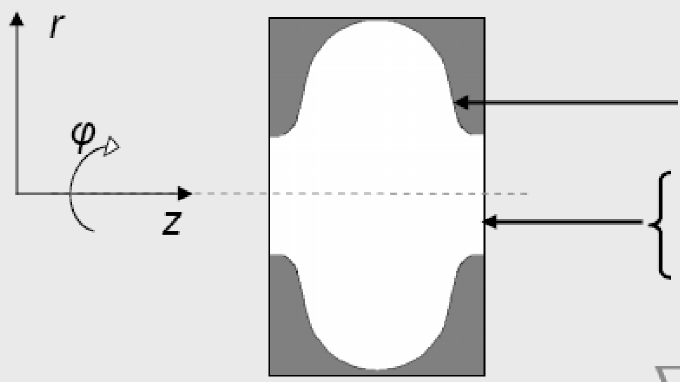
$$\frac{\partial E}{\partial \varphi} = 0 \qquad \frac{\partial H}{\partial \varphi} = 0$$

Non monopole (HOM) modes have component $E_z = 0$ on the symmetry axis. Their fields dependent on φ.

RF Cavities and their Eigenmodes IV

Maxwell equations + boundary conditions for E and H fields lead to the Helmholtz equation, which is an eigenvalue problem.

For H(r,z) field of a monopole mode the equation is:



$$(\nabla_c^2 + \omega^2 \varepsilon\mu)H = 0$$

$n \cdot H = 0$ on metal wall

$\begin{cases} H = 0 \\ n \cdot H = 0 \end{cases}$ optionally on non metal boundary

$$\nabla_c^2 A = \nabla_c(\nabla_c \cdot A) - \nabla_c \times \nabla_c \times A$$

There is infinity number of TM0xx solutions (modes) to the Helmholtz equation.

All modes are determine by:

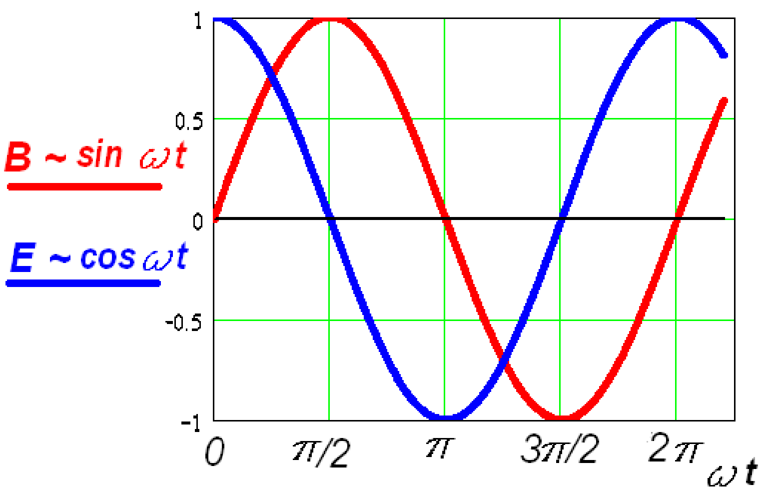
$\mathbf{H}_n(r,z)=[0, H_{\varphi,n}(r,z),0]$,

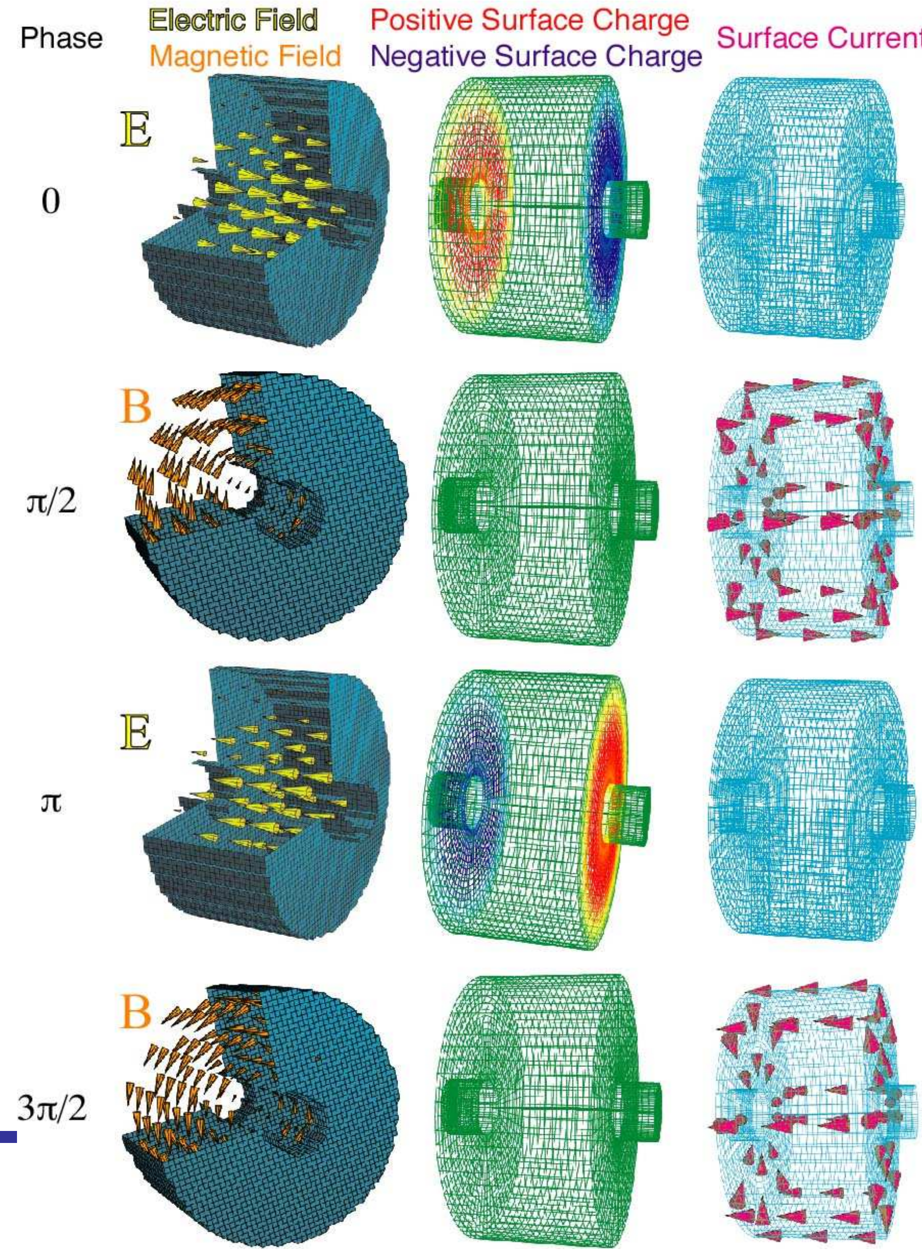
$\mathbf{E}_n(r,z)=[E_{r,n}(r,z),0, E_{z,n}(r,z)]$

and frequency ω_n .

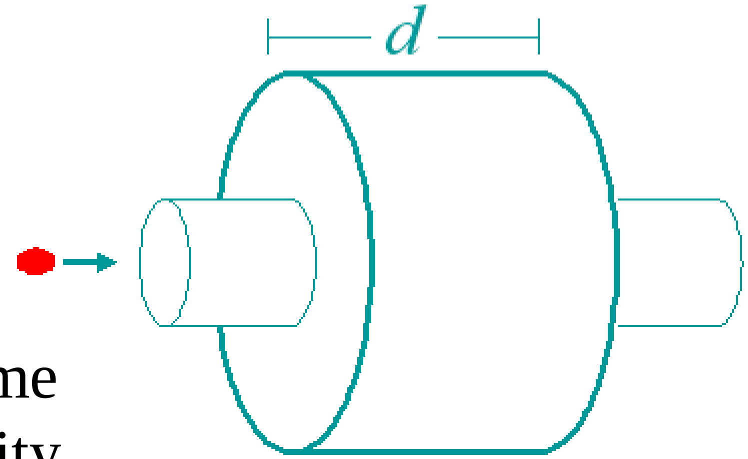
Accelerating Mode

- **Time dependent electromagnetic field inside metal box**





Accelerating Voltage & Accelerating Field (v = c for Particles)

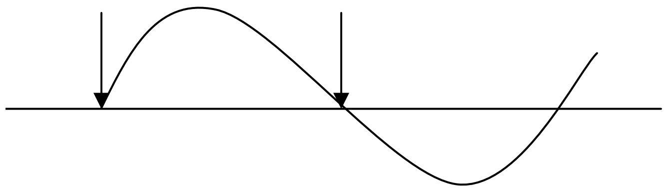


- Frequency
- For maximum acceleration we need

$$T_{cav} = \frac{d}{c} = \frac{T_{RF}}{2}$$

so that the field always points in the same direction as the bunch traverses the cavity

•Accelerating voltage then is



$$V_{cav} = \mathrm{Re}\left[\int_0^d E_z(\rho=0,z)e^{i\omega_0 z/c}dz\right] = d \cdot E_0 \frac{\sin\frac{\omega_0 d}{c}}{\frac{\omega_0 d}{c}} = d \cdot E_0 T$$

, here, $T = 2/\pi$ is flight time factor (for pill-box with this length *d* only, other shapes can have a different value of *T*).

•Accelerating field is $E_{acc} = \frac{V_{cav}}{d} = 2E_0/\pi$

- Surface currents ($\propto H$) result in dissipation proportional to the surface resistance (R_s):

$$\frac{dP_c}{ds} = \frac{1}{2} R_s |\mathbf{H}|^2$$

- Dissipation in the cavity wall given by surface integral:

$$P_c = \frac{1}{2} R_s \int_S |\mathbf{H}|^2 \, ds$$

- Stored energy is: ⟶

$$U = \frac{1}{2} \mu_0 \int_V |\mathbf{H}|^2 \, dv$$

- Define Quality (Q) as $Q_0 = \frac{\omega_0 U}{P_c} = 2\pi \frac{U}{T_{rf} P_c}$

which is ~ 2 π number of cycles it takes to dissipate the energy stored in the cavity → Easy way to measure Q

- $Qnc \approx 10^4$, $Qsc \approx 10^{10}$

Since the time averaged energy in the electric field equals that in magnetic field, the total energy in the cavity is given by

$$U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2\, dv = \frac{1}{2}\epsilon_0 \int_V |\mathbf{E}|^2\, dv,$$

where the integral is taken over the volume of the cavity.

Power dissipated in the cavity walls is $P_c = \frac{1}{2} R_s \int_S |\mathbf{H}|^2\, ds,$

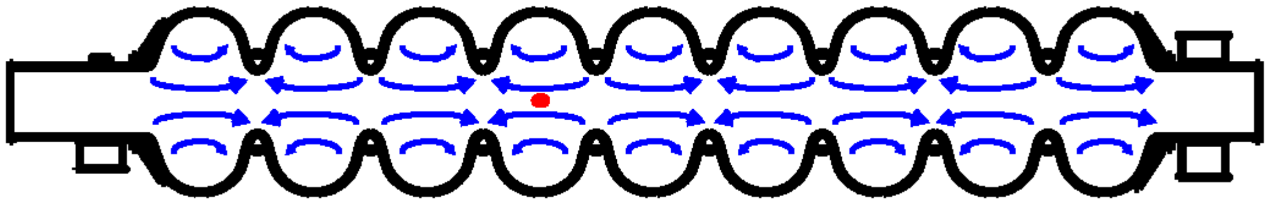
where the integration is taken over the interior cavity surface.

$$Q_0 = \frac{\omega_0 U}{P_c}, \qquad Q_0 = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2\, dv}{R_s \int_S |\mathbf{H}|^2\, ds}.$$

The Q_0 is frequently written as $Q_0 = \frac{G}{R_s},$

where $G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2\, dv}{\int_S |\mathbf{H}|^2\, ds}$. G is known as the geometry constant. From the last eq. we can see that it depends on the cavity shape but not its size.

Accelerating π-mode:



Accelerating voltage:

$$V_{acc} = \frac{\text{maximum energy gain}}{\text{charge}} = \left| \int_{-L/2}^{+L/2} E_z e^{i\omega(z/c)} dz \right|$$

Shunt-impedance:

$$R_a = \frac{(V_{acc})^2}{P_{dis}}$$

Quality factor:

$$Q_L = \frac{\omega U}{P_{dis}}$$

$$\frac{R_a}{Q_L} = \frac{R}{Q} = \frac{(V_{acc})^2}{\omega U}$$

- Shunt impedance (R_a) determines how much acceleration one gets for a given dissipation (analogous to Ohm's Law)

$$R_{\mathrm{a}} = \frac{V_{\mathrm{c}}^2}{P_{\mathrm{c}}}$$

→ To maximize acceleration (P_c given), must maximize shunt impedance.

Another important figure of merit is $$\frac{R_{\mathrm{a}}}{Q_0} = \frac{V_{\mathrm{c}}^2}{\omega_0 U},$$

•*Ra/Q* only depends on the cavity geometry

→ This quantity is also used for determining the level of mode excitation by charges passing through the cavity.

→ To minimize losses (P_c) in the cavity, we must maximize $G*R_a/Q_0$:

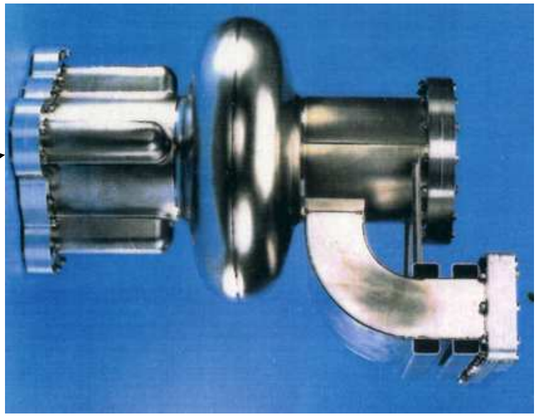
$$P_c = \frac{V_c^2}{R_a} = \frac{V_c^2}{Q_0 \cdot (R_a / Q_0)} = \frac{V_c^2}{(R_s \cdot Q_0)(R_a / Q_0) / R_s} = \frac{V_c^2 \cdot R_s}{G \cdot (R_a / Q_0)}$$

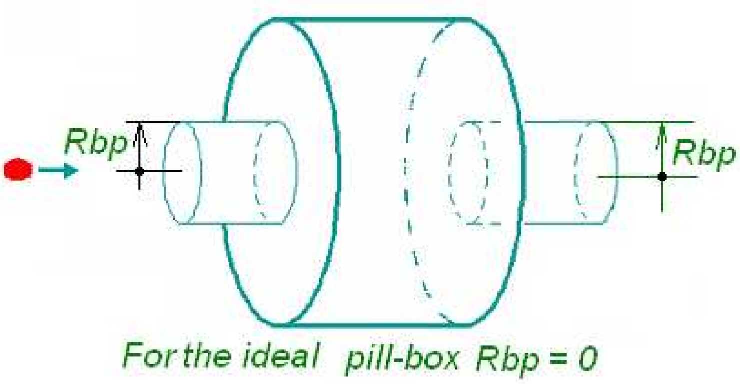
Typical Values for Single Cells

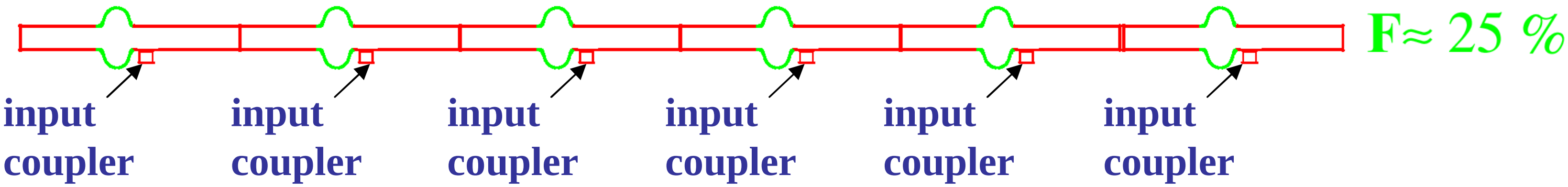
Quantity	Cornell SC 500 MHz	Pillbox
G	270 Ω	257 Ω
R_a/Q_0	88 Ω /cell	196 Ω/cell
E_{pk}/E_{acc}	2.5	1.6
H_{pk}/E_{acc}	52 Oe/MV/m	30.5 Oe/(MV/m)

Current is high, it excites a lot of Higher Order Modes, so the hole is made big to propagate HOMs, and this is why *Hpk* and *Epk* grew, and *R/Q* drops.

Diff. applications – diff. trade-offs.







higher fill-factor:

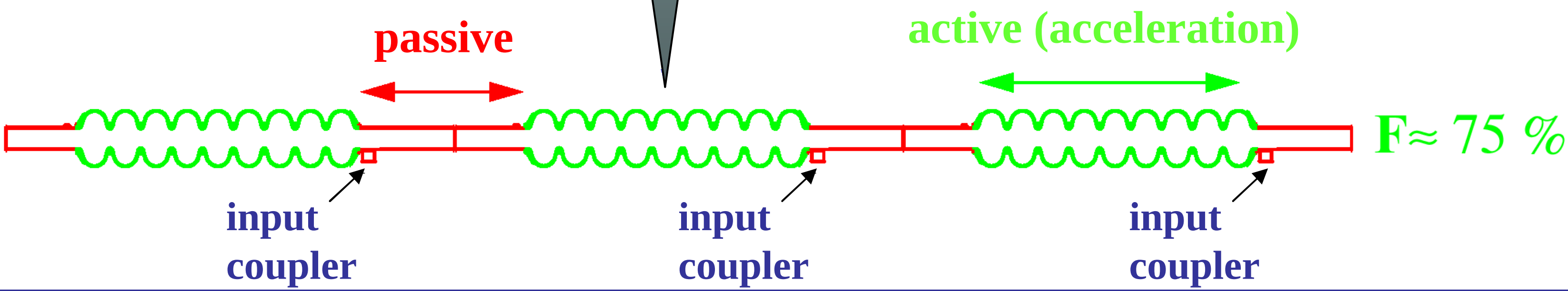
$$F = \frac{\text{active length}}{\text{total length}}$$

⇒ *lower costs*

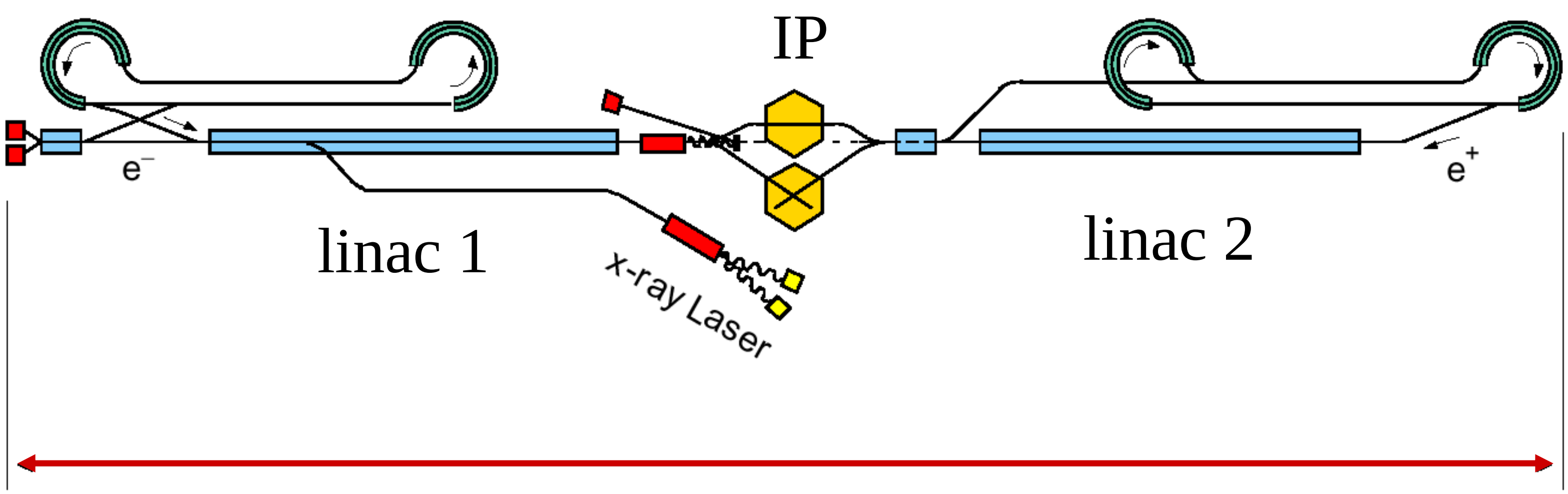
⇒ *better beam*

fewer

- **input couplers**
- **waveguide elements**
- **RF control systems**
- **…**



Example: 500 GeV Linear Collider

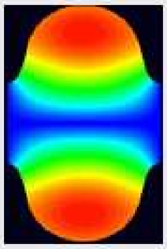


21,024 9-cell cavities: 27.8 km (17.3 miles)

189,216 1-cell cavities: 75.4 km (46.8 miles)

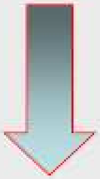
Multicell Cavities and Cell-to-Cell Coupling

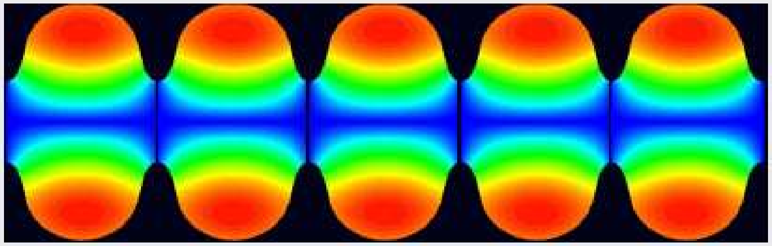
The last parameter, relevant for multi-cell accelerating structures, is the coupling k_{cc} between cells for the accelerating mode passband (Fundamental Mode passband).



Single-cell structures are attractive from the RF-point of view:

- Easier to manage HOM damping
- No field flatness problem.
- Input coupler transfers less power
- Easy for cleaning and preparation
- But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.



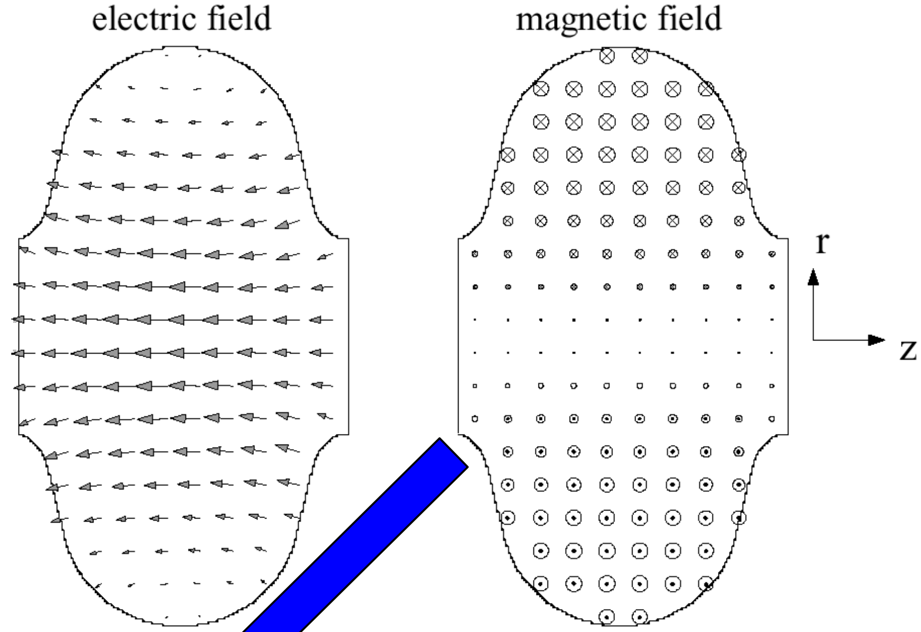


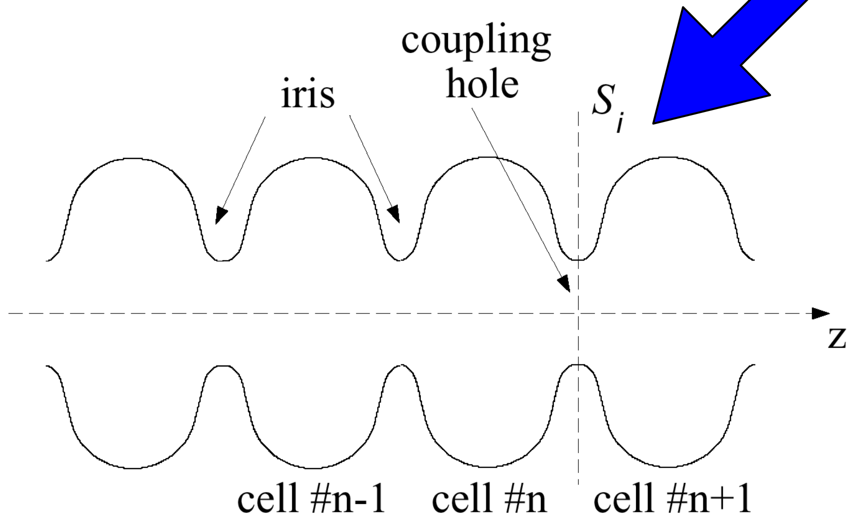
Multi-cell structures are less expensive and offers higher real-estate gradient but:

- Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells
- Other problems arise: HOM trapping…

Single Cell:

TM_{010} mode

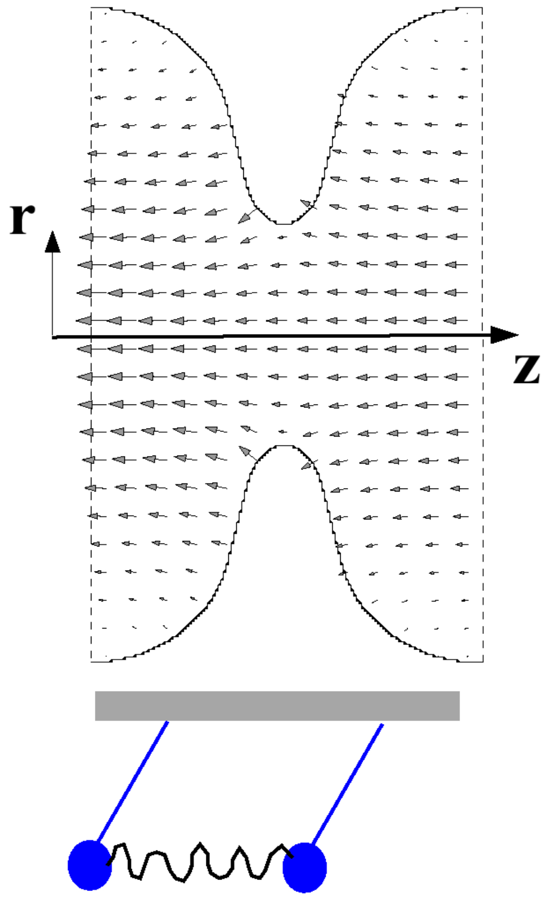




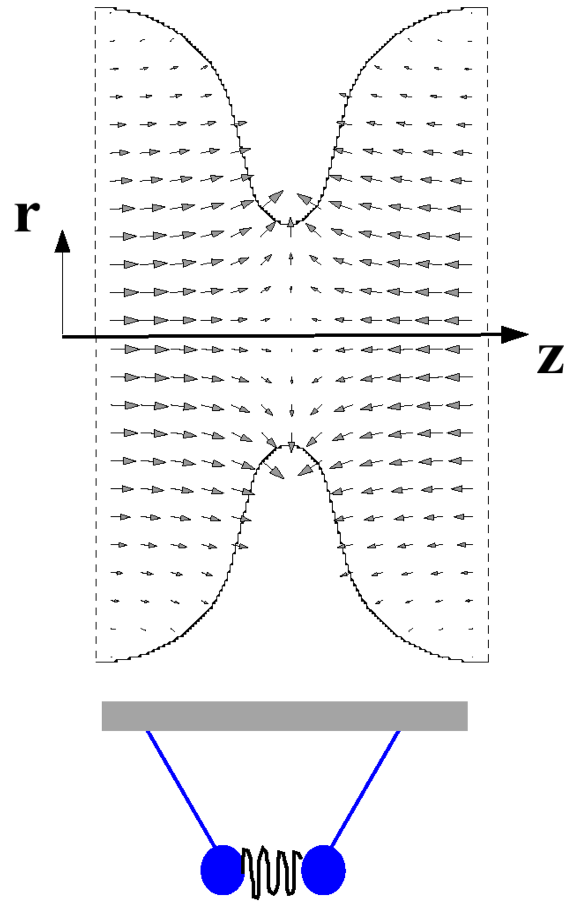
Coupled Cells

Two Coupled Cells: TM010 Modes

0 - Mode



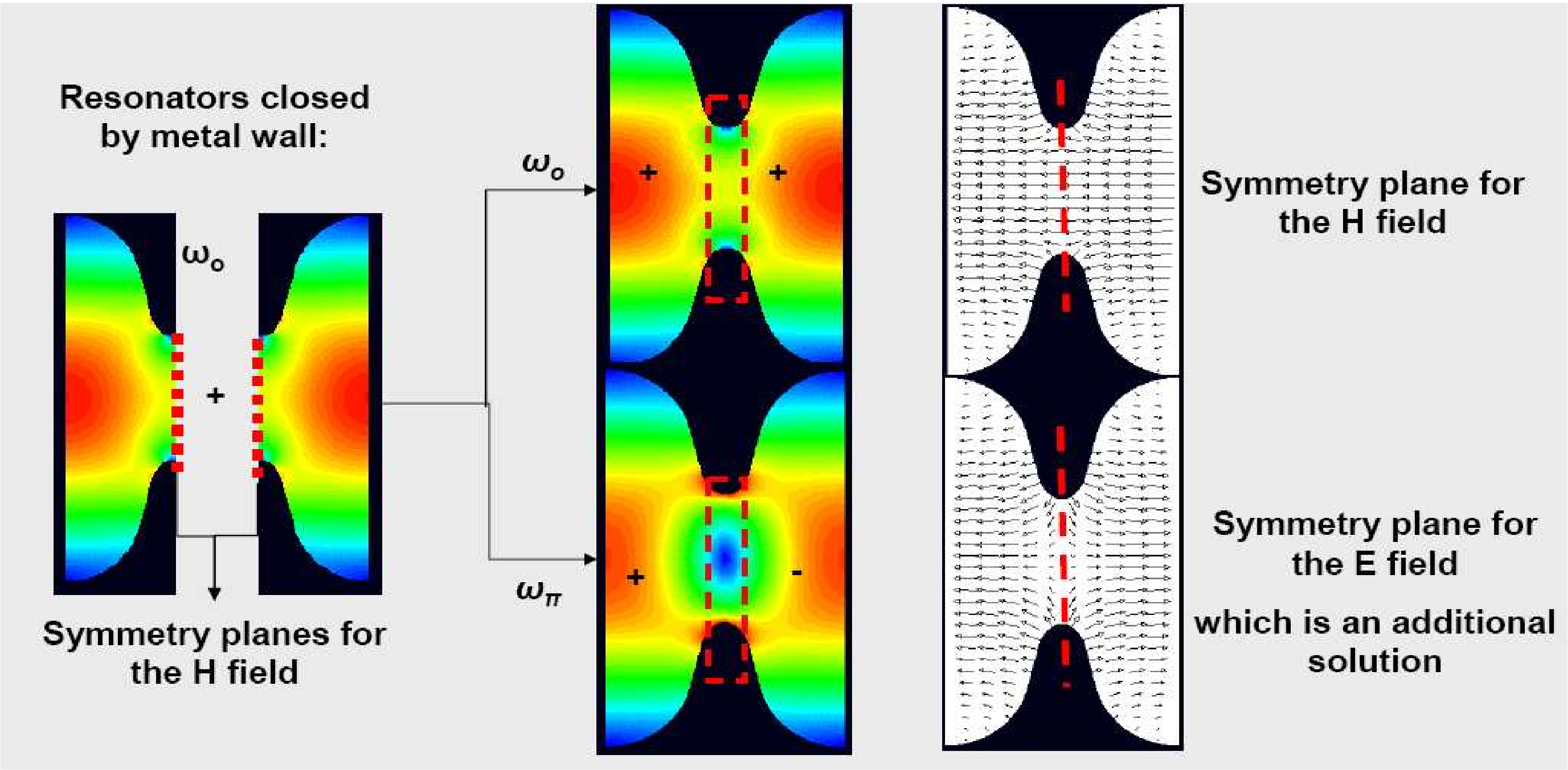
π - Mode

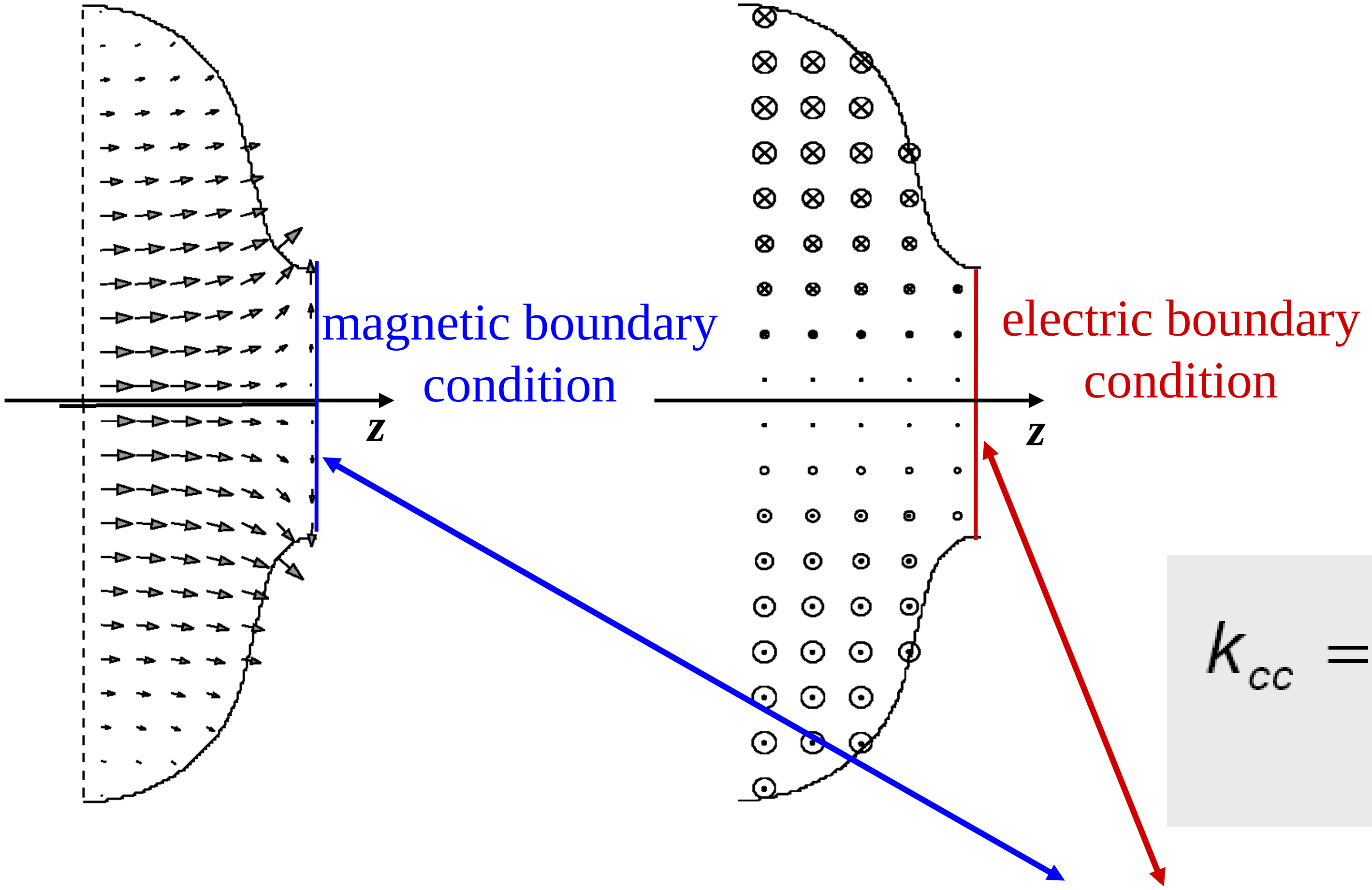


2 coupled cells → ***2 TM_{010} modes***

n coupled cells → ***n TM_{010} modes***

Two Coupled Cells: TM010 Modes





$$k_{cc} = \frac{\omega_\pi - \omega_0}{\frac{\omega_\pi + \omega_0}{2}}$$

Coupling factor:

$$k_{cc} = \frac{c}{\omega_0} \int_{S_i} \left(\vec{e} \times \vec{h} \right) \cdot \vec{u}_z ds$$

A Circuit Model: Let's start at the Beginning: Step 1.1

Solve Maxwell's equations for given boundary conditions

$$\Downarrow$$

orthogonal eigenmodes

$$\vec{E}^{(m)}(\vec{r},t) = \vec{E}^{(m)}(\vec{r})e^{i\omega^{(m)}t}$$

$$\vec{H}^{(m)}(\vec{r},t) = \vec{H}^{(m)}(\vec{r})e^{i\omega^{(m)}t+i\pi/2} \quad , \mathrm{m} = 1,2,...$$

$$\Downarrow$$

orthononal eigenfunctions

$$\vec{\mathrm{e}}^{(\mathrm{m})}(\vec{r}) = \sqrt{\frac{\varepsilon_0}{2U^{(\mathrm{m})}}}\vec{E}^{(\mathrm{m})}(\vec{r})$$

$$\int_V \vec{\mathrm{e}}^{(\mathrm{m})}(\vec{r})\vec{\mathrm{e}}^{(\mathrm{n})}(\vec{r})dv = \delta_{mn}$$

$$\vec{\mathrm{h}}^{(\mathrm{m})}(\vec{r}) = \sqrt{\frac{\mu_0}{2U^{(\mathrm{m})}}}\vec{H}^{(\mathrm{m})}(\vec{r})$$

$$\int_V \vec{\mathrm{h}}^{(\mathrm{m})}(\vec{r})\vec{\mathrm{h}}^{(\mathrm{n})}(\vec{r})dv = \delta_{mn}$$

A Circuit Model: Step 1.2

With Maxwell's equations in vacuum:

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

and the eigenmodes

$$\vec{E}^{(m)}(\vec{r},t) = \vec{E}^{(m)}(\vec{r}) e^{i\omega^{(m)} t}$$

$$\vec{H}^{(m)}(\vec{r},t) = \vec{H}^{(m)}(\vec{r}) e^{i\omega^{(m)} t + i\pi/2}$$

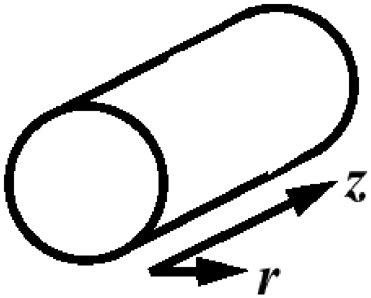
and the normalized eigenfunctions

$$\vec{\mathrm{h}}^{(\mathrm{m})}(\vec{r}) = \sqrt{\frac{\mu_0}{2U^{(\mathrm{m})}}} \vec{H}^{(\mathrm{m})}(\vec{r}) \qquad \vec{\mathrm{e}}^{(\mathrm{m})}(\vec{r}) = \sqrt{\frac{\varepsilon_0}{2U^{(\mathrm{m})}}} \vec{E}^{(\mathrm{m})}(\vec{r})$$

we get the relations

$$\vec{\nabla} \times \vec{h}^{(m)} = \frac{\omega^{(m)}}{c} \vec{e}^{(m)} \qquad \vec{\nabla} \times \vec{e}^{(m)} = \frac{\omega^{(m)}}{c} \vec{h}^{(m)}$$

A Circuit Model: Step 1
Eigenmodes: Example: Pillbox-Cavity



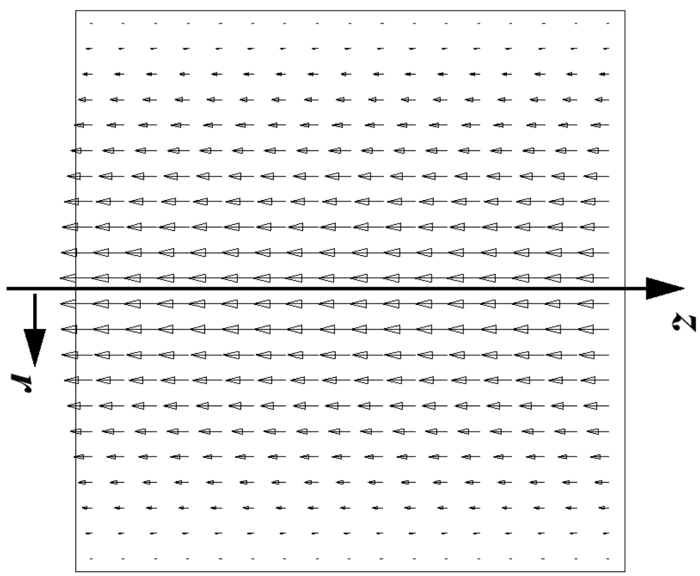
lowest frequency mode: **TM010** ← z

transverse magnetic → TM

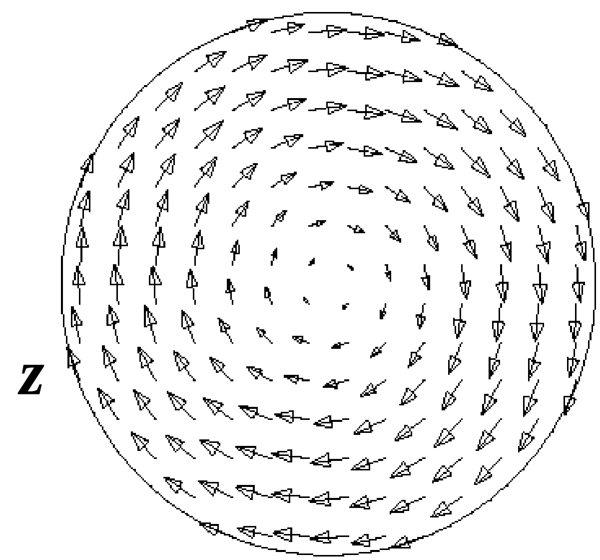
0 ⇒ monopole mode

r → 1

electric field



magnetic field

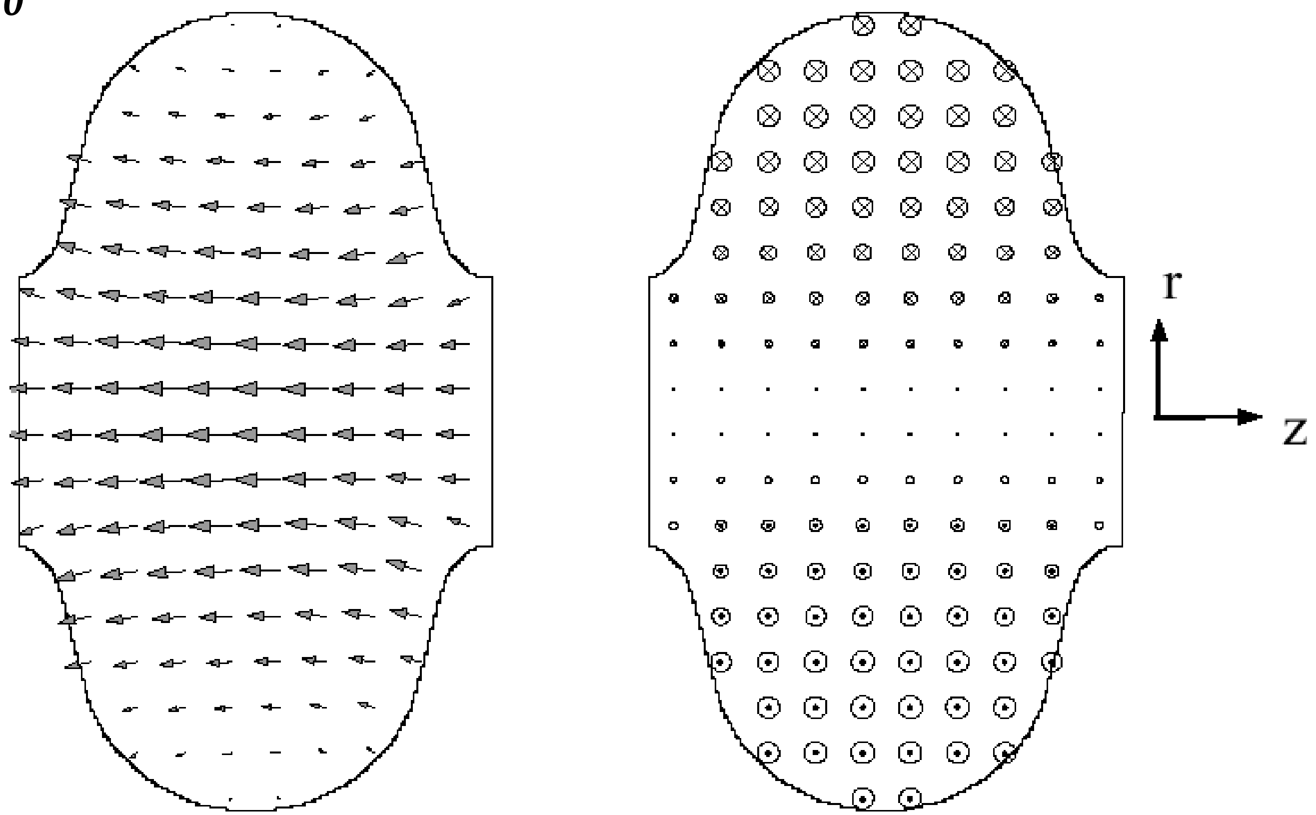


$$\vec{E}(r,t) = E_0 J_0\left(\frac{\omega_0}{c} r\right)\cos(\omega_0 t)\vec{e}_z \quad \Rightarrow \textbf{acceleration}$$

Eigenmodes: Example: Single Cell TESLA Cavity

Accelerating mode:

TM_{010}: *electric field* *magnetic field*



$$f_0 = 1.3 \text{ GHz für TESLA}$$

Each *time dependent field* in a cavity can be written as a *sum of the cavity eigenfunctions* with *time dependent amplitudes*:

$$\vec{E}(\vec{r},t) = \sum_m \hat{E}^{(m)}(t)\vec{e}^{(m)}(\vec{r})$$

$\hat{E}^{(m)}(t)$: ***time dependent amplitude***

$\vec{e}^{(m)}(\vec{r})$: ***eigenfunction***

$$\vec{H}(\vec{r},t) = \sum_m \hat{H}^{(m)}(t)\vec{h}^{(m)}(\vec{r})$$

$\hat{H}^{(m)}(t)$: ***time dependent amplitude***

$\vec{h}^{(m)}(\vec{r})$: ***eigenfunction***

Insert expansion of fields into Maxwell's equations in vacuum:

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

insert eigenmode-expansion of fields… $\Downarrow$ …and the equations from page 37

$$\sum_m \hat{H}^{(m)}(t) \frac{\omega^{(m)}}{c} \vec{e}^{(m)}(\vec{r}) = \sum_m \varepsilon_0 \frac{d\hat{E}^{(m)}(t)}{dt} \vec{e}^{(m)}(\vec{r})$$

use orthogonality of eigenmodes: $\Downarrow$ multiply by $e^{(m)}$ and integrate over cavity volume

$$\frac{\omega^{(m)}}{c} \hat{H}^{(m)}(t) - \varepsilon_0 \frac{d}{dt} \hat{E}^{(m)}(t) = 0 \quad , m = 1,2...$$

Insert expansion of fields into Maxwell's equations:

$$\vec{\nabla}\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

insert eigenmode-expansion of fields… $\Downarrow$ …and the equations from page 37

$$\sum_m \hat{E}^{(m)}(t)\frac{\omega^{(m)}}{c}\vec{h}^{(m)}(\vec{r}) = -\sum_m \mu_0 \frac{d\hat{H}^{(m)}(t)}{dt}\vec{h}^{(m)}(\vec{r})$$

use orthogonality of eigenmodes: $\Downarrow$ multiply by $h^{(m)}$ and integrate over cavity volume

$$\frac{\omega^{(m)}}{c}\hat{E}^{(m)}(t) + \mu_0 \frac{d}{dt}\hat{H}^{(m)}(t) = 0 \quad , m = 1,2...$$

Differential Equation for the Eigenmode Amplitudes

$$\frac{\omega^{(m)}}{c}\hat{H}^{(m)}(t)-\varepsilon_0\frac{d}{dt}\hat{E}^{(m)}(t)=0 \quad ,\mathrm{m}=1,2...$$

$$\frac{\omega^{(m)}}{c}\hat{E}^{(m)}(t)+\mu_0\frac{d}{dt}\hat{H}^{(m)}(t)=0 \quad ,\mathrm{m}=1,2...$$

$$\Downarrow$$

$$\frac{d^2}{dt^2}\hat{E}^{(m)}(t)+\left(\omega^{(m)}\right)^2\hat{E}^{(m)}(t)=0 \quad ,\mathrm{m}=1,2...$$

amplitude of eigenmode #m

⇒ Differential equation of an oscillator (we will add losses and a generator later)!

TM010 modes in couples cells:

N coupled cells ⇒ N coupled oscillators

⇒ N coupled differential equations:

$$\frac{d^2}{dt^2}\hat{E}_n + \omega_n^2 \hat{E}_n + \omega_n^2 \frac{K}{2}\left(\hat{E}_n - \hat{E}_{n-1}\right) + \omega_n^2 \frac{K}{2}\left(\hat{E}_n - \hat{E}_{n+1}\right) = 0 \quad , n = 1,2...,N$$

TM_{010} eigenfrequency of cell #n

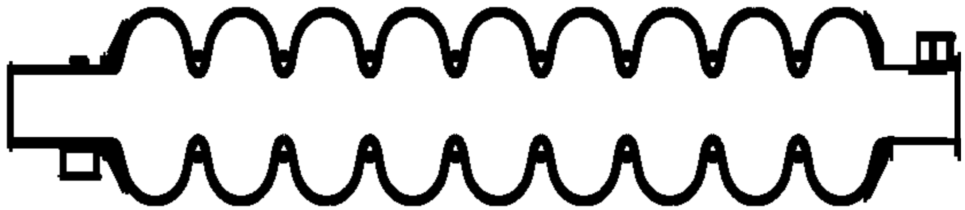
cell-to-cell coupling factor

TM_{010} field amplitude in cell #n

N-cell structure

⇒ N coupled differential equations

Circuit Model: Step 5
Equivalent Circuit Model (TM010 Modes only)

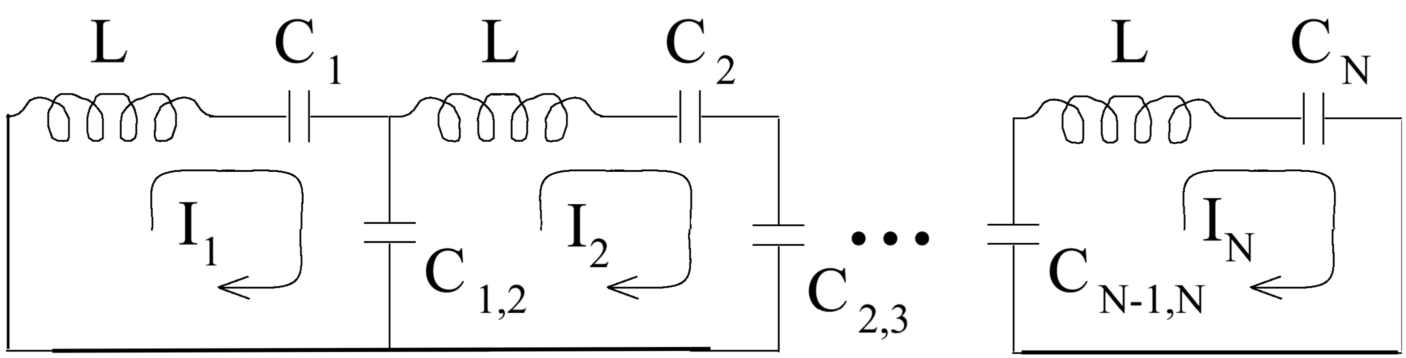


$$\frac{d^2}{dt^2}\hat{E}_n + \omega_n{}^2 \hat{E}_n + \omega_n{}^2 \frac{K}{2}\left(\hat{E}_n - \hat{E}_{n-1}\right) + \omega_n{}^2 \frac{K}{2}\left(\hat{E}_n - \hat{E}_{n+1}\right) = 0 \quad , n = 1,2...,N$$

substitute:

$$\omega_n{}^2 = \frac{1}{LC_n}$$

$$K_{n,n+1} = 2\frac{C_n}{C_{n,n+1}} = 2k_{n,n+1}$$



Circuit Model: Step 6
Eigenmode Equation for the TM010 Modes

$$\frac{d^2}{dt^2}\hat{E}_n + \omega_0{}^2\hat{E}_n + \omega_0{}^2\frac{K}{2}\left(\hat{E}_n - \hat{E}_{n-1}\right) + \omega_0{}^2\frac{K}{2}\left(\hat{E}_n - \hat{E}_{n+1}\right) = 0 \quad , \mathrm{n} = 1,2...,\mathrm{N}$$

steady state ansatz

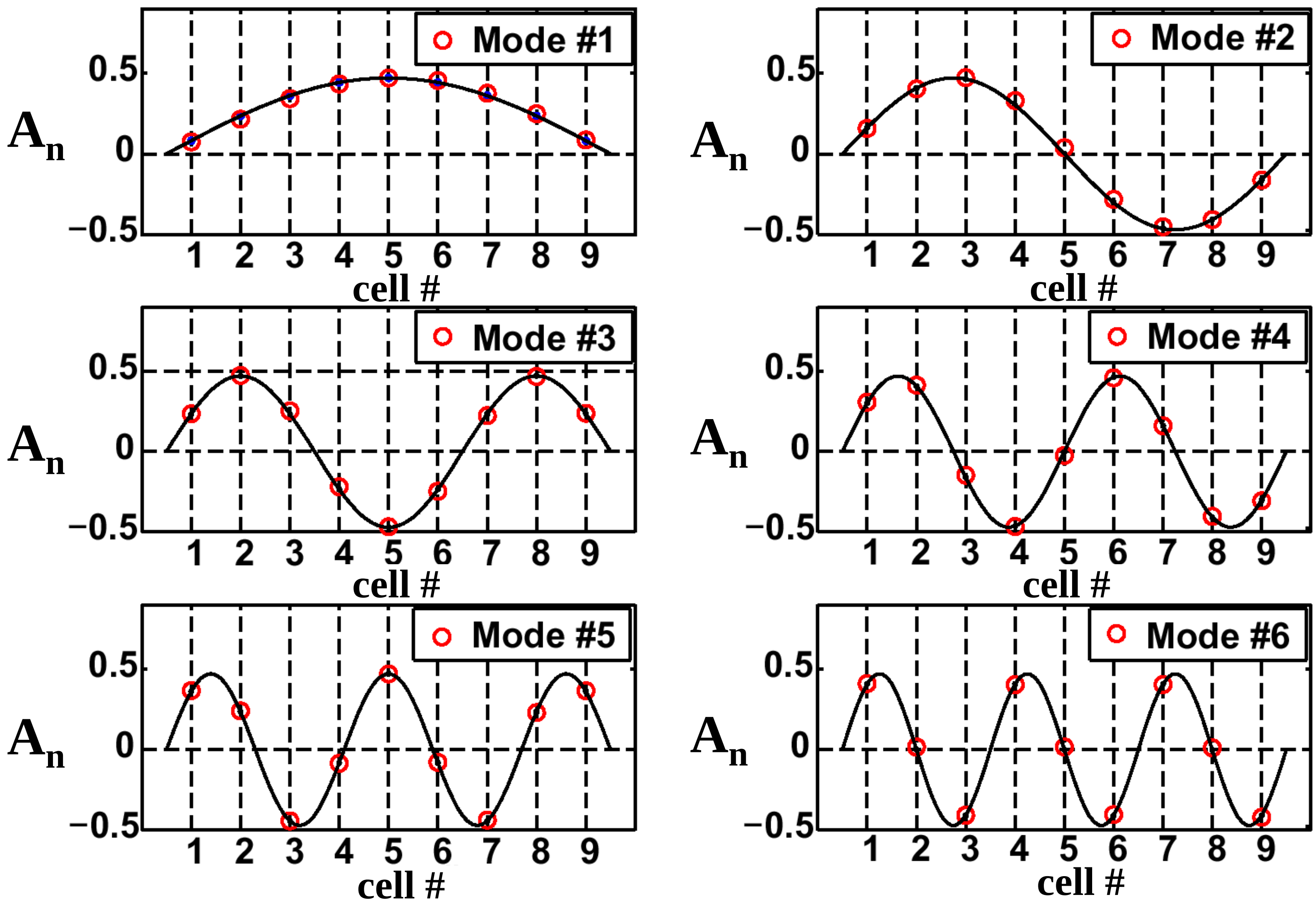
$$\begin{bmatrix} 1+k_{1,2} & -k_{1,2} & & & \\ -k_{1,2} & 1+k_{1,2}+k_{2,3} & -k_{2,3} & & \\ & & \ddots & & \\ & & k_{N-2,N-1} & 1+k_{1,2}+k_{2,3} & k_{N-1,N} \\ & & & k_{N-1,N} & 1+k_{N-1,N} \end{bmatrix} \begin{bmatrix} A_1 \\ \vdots \\ A_N \end{bmatrix}^{(j)} = \left(\frac{\omega^{(j)}}{\omega_0}\right)^2 \begin{bmatrix} A_1 \\ \vdots \\ A_N \end{bmatrix}^{(j)}$$

eigenvector #j

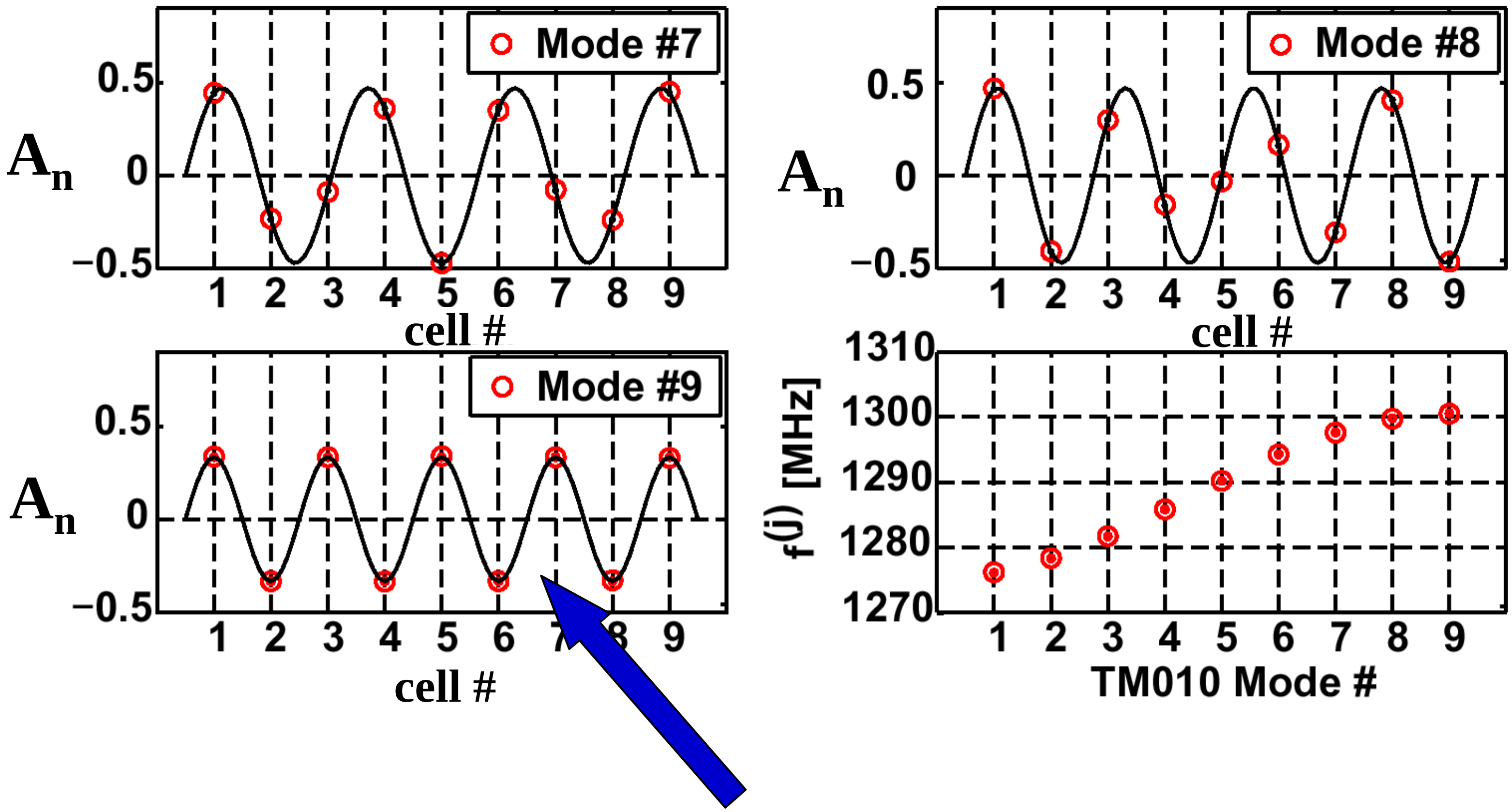
eigenfrequency of mode #j

Eigenvector: TM_{010} cell amplitudes of a TM_{010} eigenmode of a multicell cavity.

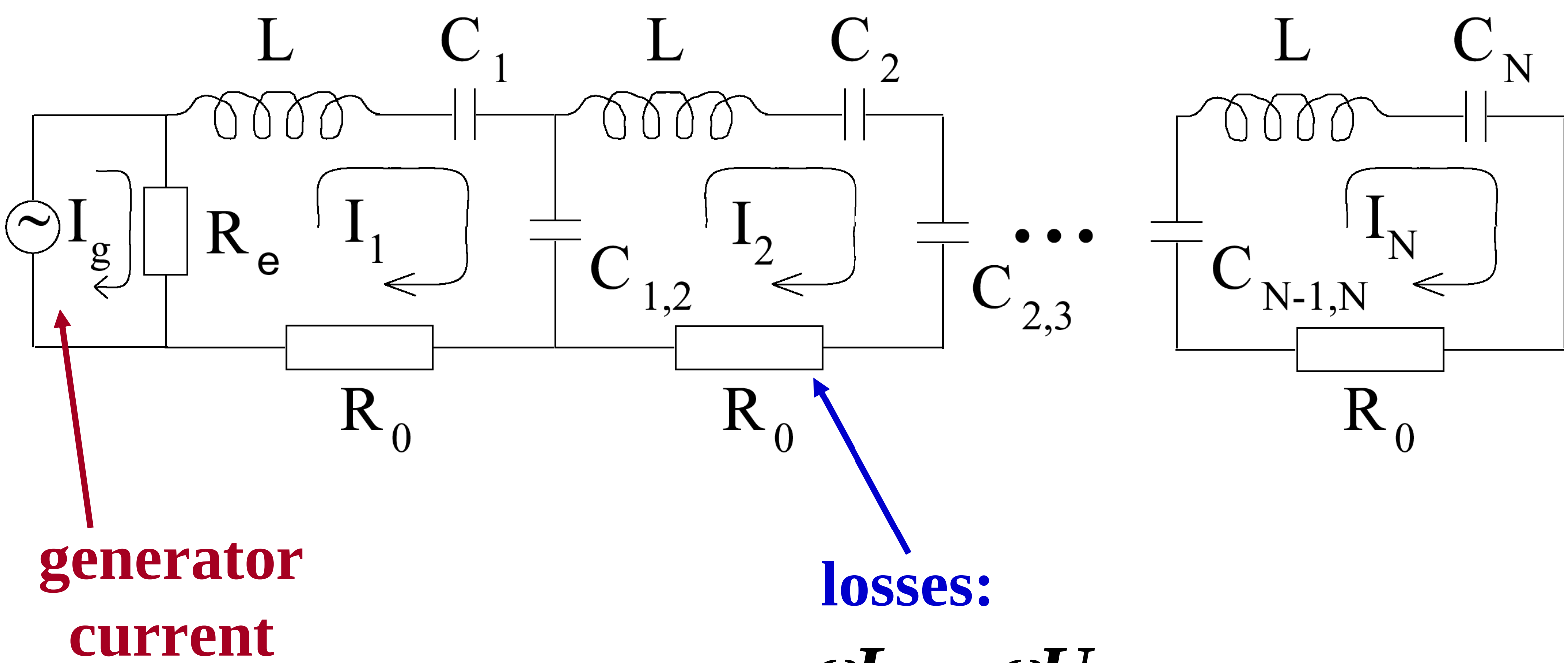
TM010 Eigenmodes Example: 9-Cell Cavity (1)



TM010 Eigenmodes
Example: 9-Cell Cavity (2)

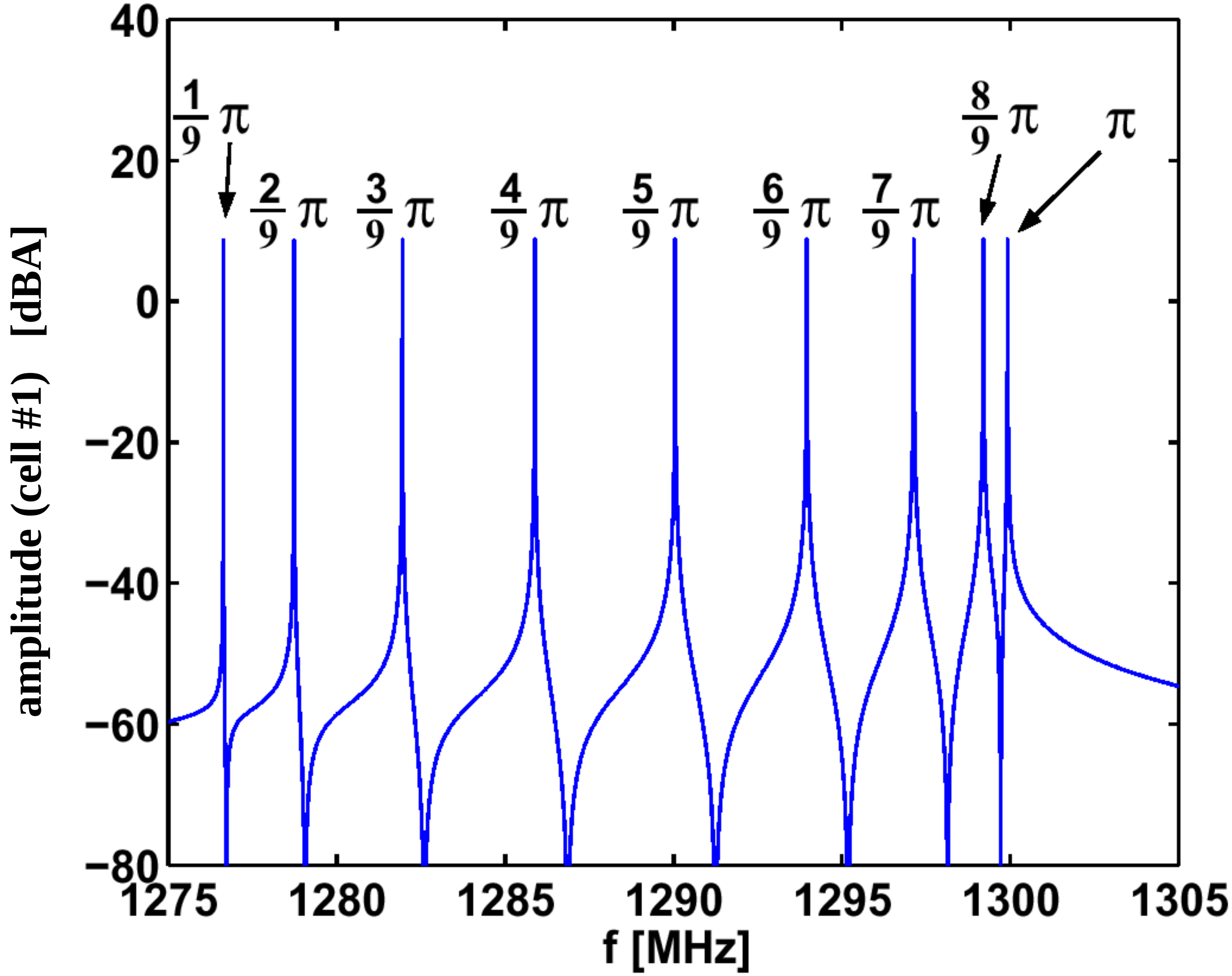


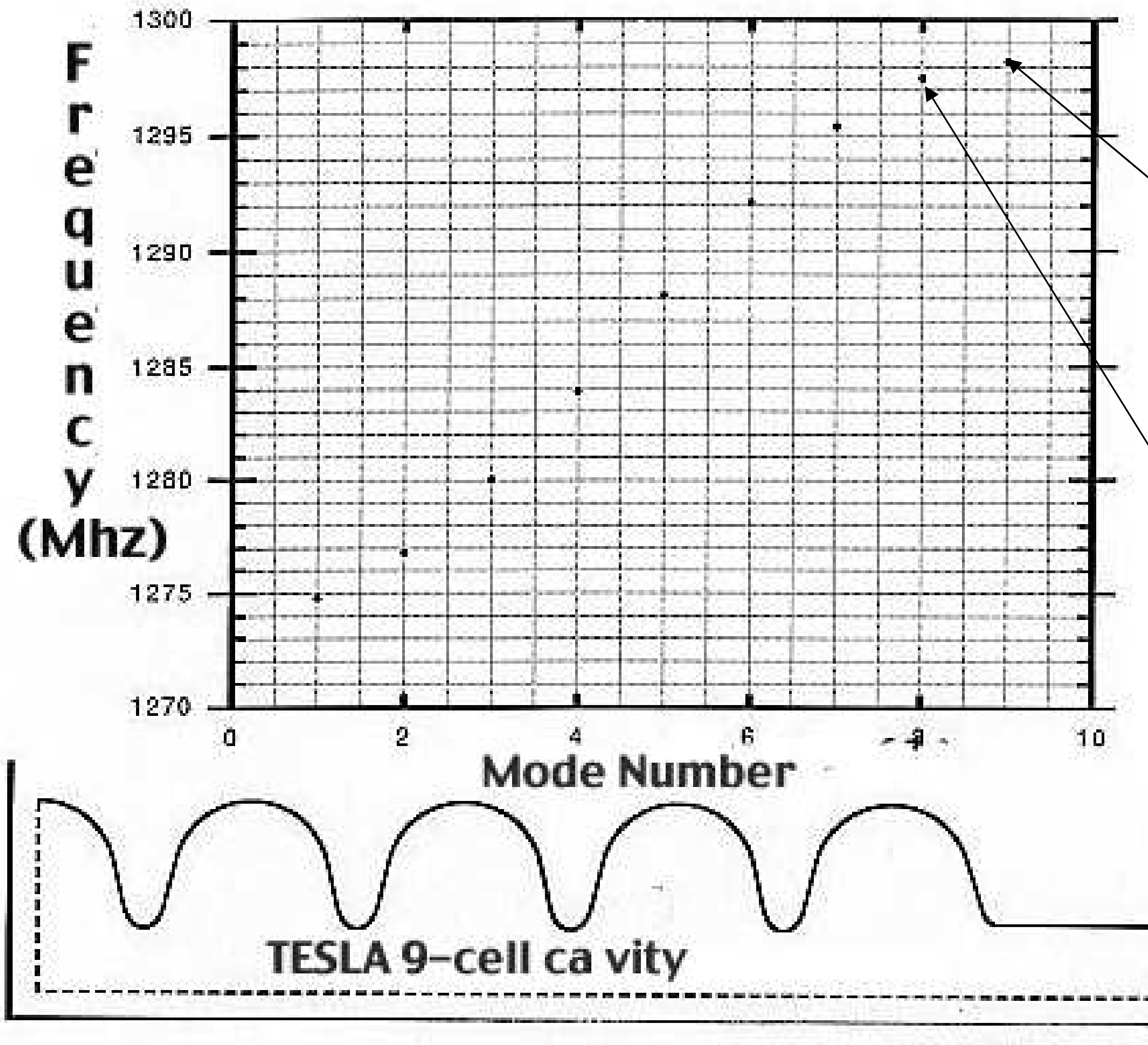
accelerating mode: π cell-to-cell phase advance



$$Q_n = \frac{\omega L}{R_0} = \frac{\omega U_n}{P_{dis,n}}$$

Simulation Example: TM010 Eigenmode-Spectrum of a 9-Cell Cavity





The working point. If it is too close to the neighbor point this neighbor mode can also be excited. To avoid this, more cell-to-cell coupling is needed: broader aperture.

Mode Beating during Cavity Filling

- Modeling of the transient state (mode beating)

  Example: 7-cells, k_{cc}=1.85%, Q_L=3.4 10^6