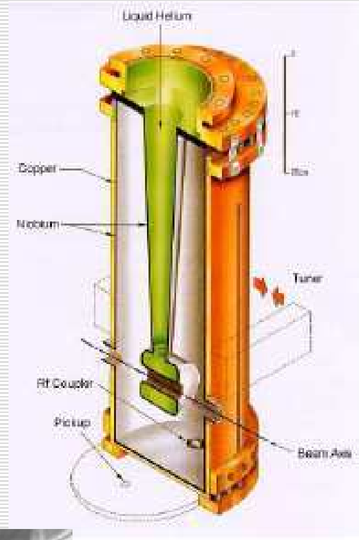
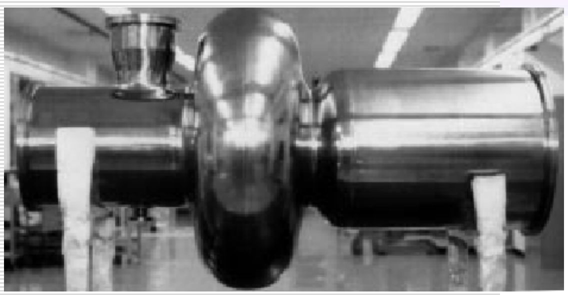
S. Belomestnykh

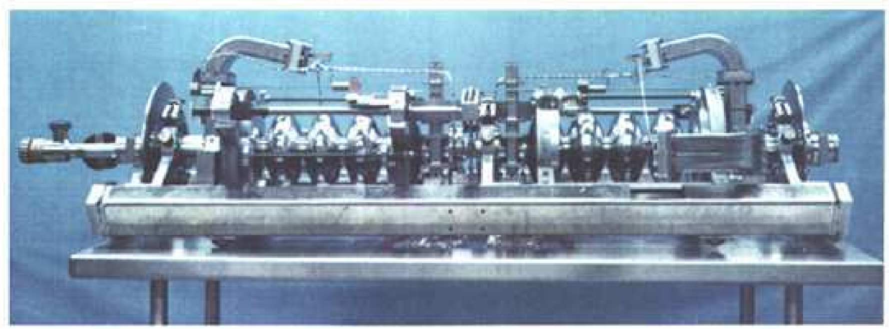
Superconducting RF Technology 1:

- Basics of RF Superconductivity

Superconducting cavity shapes

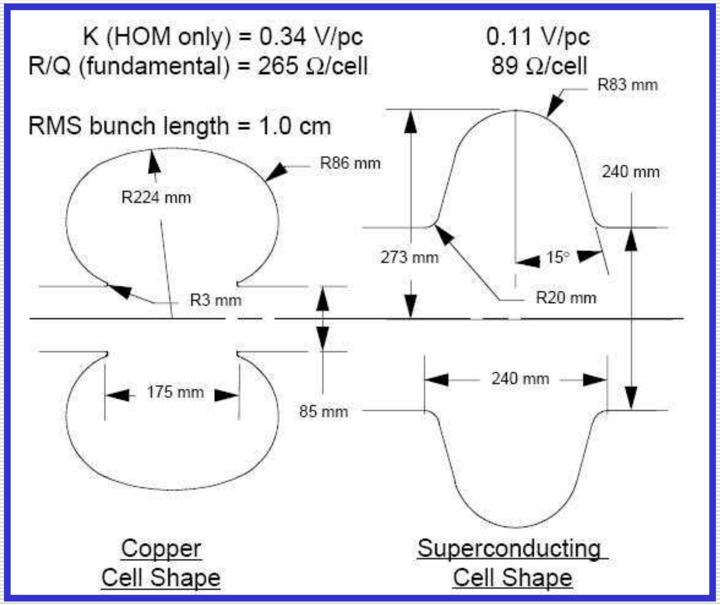






Benefits of RF superconductivity

- Superconducting (SC) cavities excel in applications requiring continuous wave (CW) or long-pulse accelerating fields above a few MV/m.
- For normal conducting (NC) cavities (usually made of copper) power dissipation in cavity walls is a huge constrain in these cases → cavity design is driven by this fact, optimized for lowest possible wall dissipation → small beam aperture.
- The surface resistance of SC cavities is many orders of magnitude less than that of copper → SC accelerating system is more economical: less wall plug power, fewer cavities required, etc.
- Additional benefit: the cavity design decouples from the dynamic losses (wall losses associated with RF fields) → free to adapt design to a specific application.
- The presence of accelerating structures has a disruptive effect on the beam and may cause various instabilities, dilute beam emittance and produce other undesirable effects. Fewer SC cavities → less disruption. SC cavities can trade off some of wall losses to a larger beam pipe → reduce disruption more.



Example:
RF options for a 3rd generation light source

Suppose we are designing a third generation X-ray light source based on an electron storage ring. RF system is required to provide the total accelerating voltage of 2.4 MV and deliver 300 kW of RF power to beam. Let us compare two state-of-the-art 500 MHz RF systems: normal conducting PEP-II cavity and superconducting CESR-B cavity, operating at 4.5 K.

Parameter	PEP-II cavity	CESR-B cavity
Frequency	500 MHz	500 MHz
Number of cavities	3	1
R/Q	230 Ohm	89 Ohm
Q_0	3×10^4	1×10^9
Acc. voltage per cavity	0.8 MV	2.4 MV
Cavity wall power dissipation	278 kW	65 W
Total RF power	578 kW	300 kW
RF efficiency	0.5	0.5
Refrigeration efficiency	-	5×10^{-3}
Total AC power	1156 kW	613 kW

Figures of merit

OR what you have learned at previous lectures:

- Transit time factor, $T = 2/\pi$ for pillbox cavity
- Accelerating voltage V_{acc}
- Accelerating field $E_{acc} = V_{acc}/d$, d is the cavity length, typically $\lambda/2$
- Stored energy U
- Dissipation in the cavity walls P_c
- Quality factor Q_0, determined by material properties and cavity geometry, ~10^4 for NC cavities, ~10^{10} for SC cavities
- Surface resistance R_s – material properties
- Geometry factor G – cavity geometry, 257 Ohm for pillbox cavity
- Shunt impedance R_a
- R/Q – cavity geometry, 196 Ohm/cell for pillbox cavity
- Peak surface fields E_{pk}/E_{acc} and H_{pk}/E_{acc}, 1.6 and 30.5 Oe/(MV/m) for pillbox

$$V_{cav} = \mathrm{Re}\left[\int_0^d E_z(\rho=0,z)e^{i\omega_0 z/c}dz\right] = d\cdot E_0 \frac{\sin\frac{\omega_0 d}{c}}{\frac{\omega_0 d}{c}} = d\cdot E_0 T$$

$$U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2 dv$$

$$P_c = \frac{1}{2}R_s \int_S |\mathbf{H}|^2 ds$$

$$Q_0 = \frac{\omega_0 U}{P_c}$$

$$G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{\int_S |\mathbf{H}|^2 ds}.$$

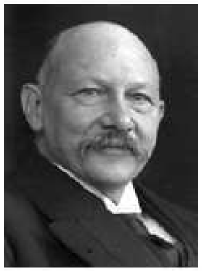
$$R_a = \frac{V_c^2}{P_c}$$

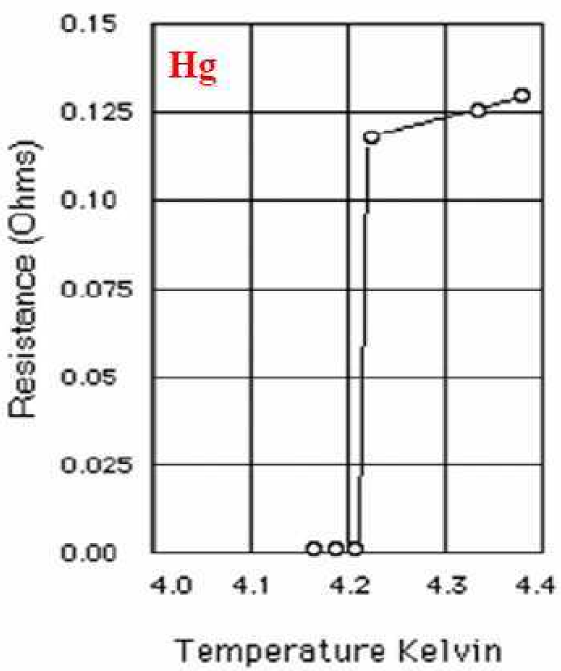
$$\frac{R_a}{Q_0} = \frac{V_c^2}{\omega_0 U},$$

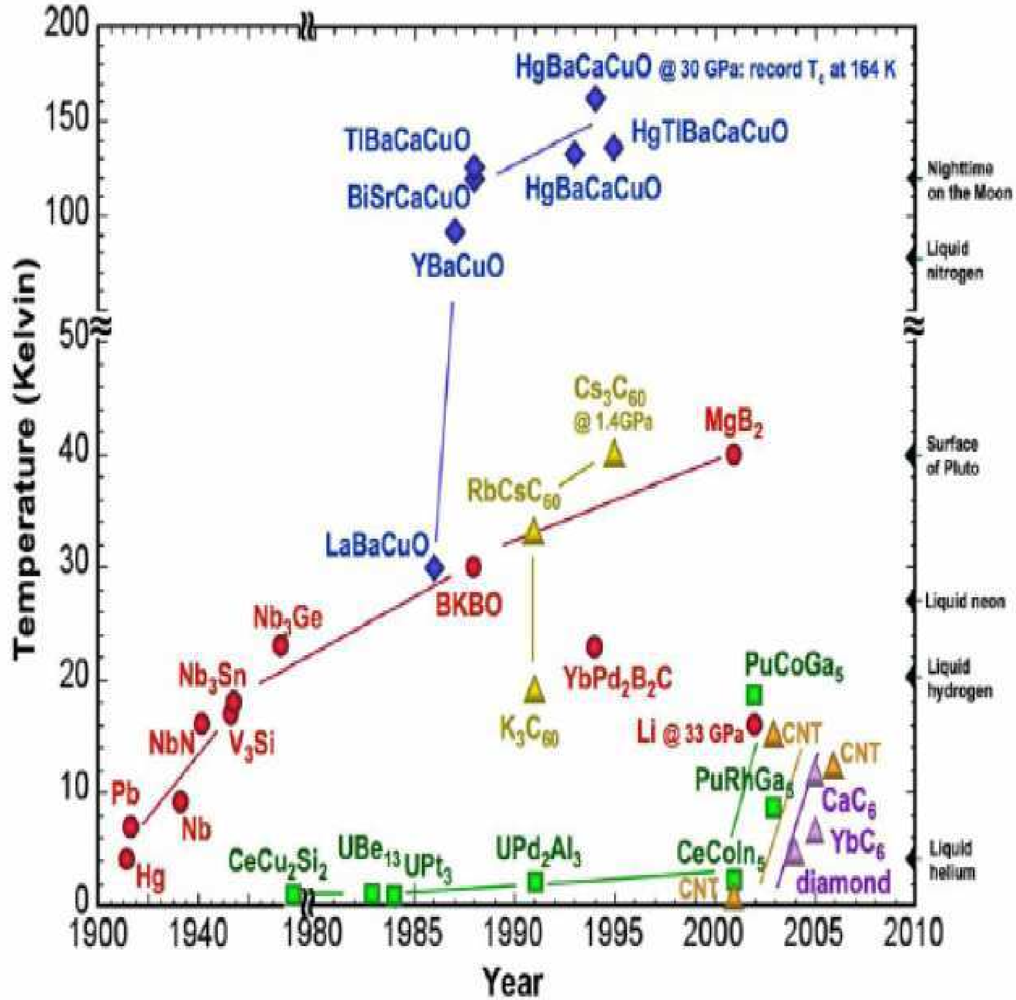
$$P_c = \frac{V_c^2}{R_a} = \frac{V_c^2}{Q_0\cdot(R_a/Q_0)} = \frac{V_c^2}{(R_s\cdot Q_0)(R_a/Q_0)/R_s} = \frac{V_c^2\cdot R_s}{G\cdot(R_a/Q_0)}$$

Discovery of superconductivity

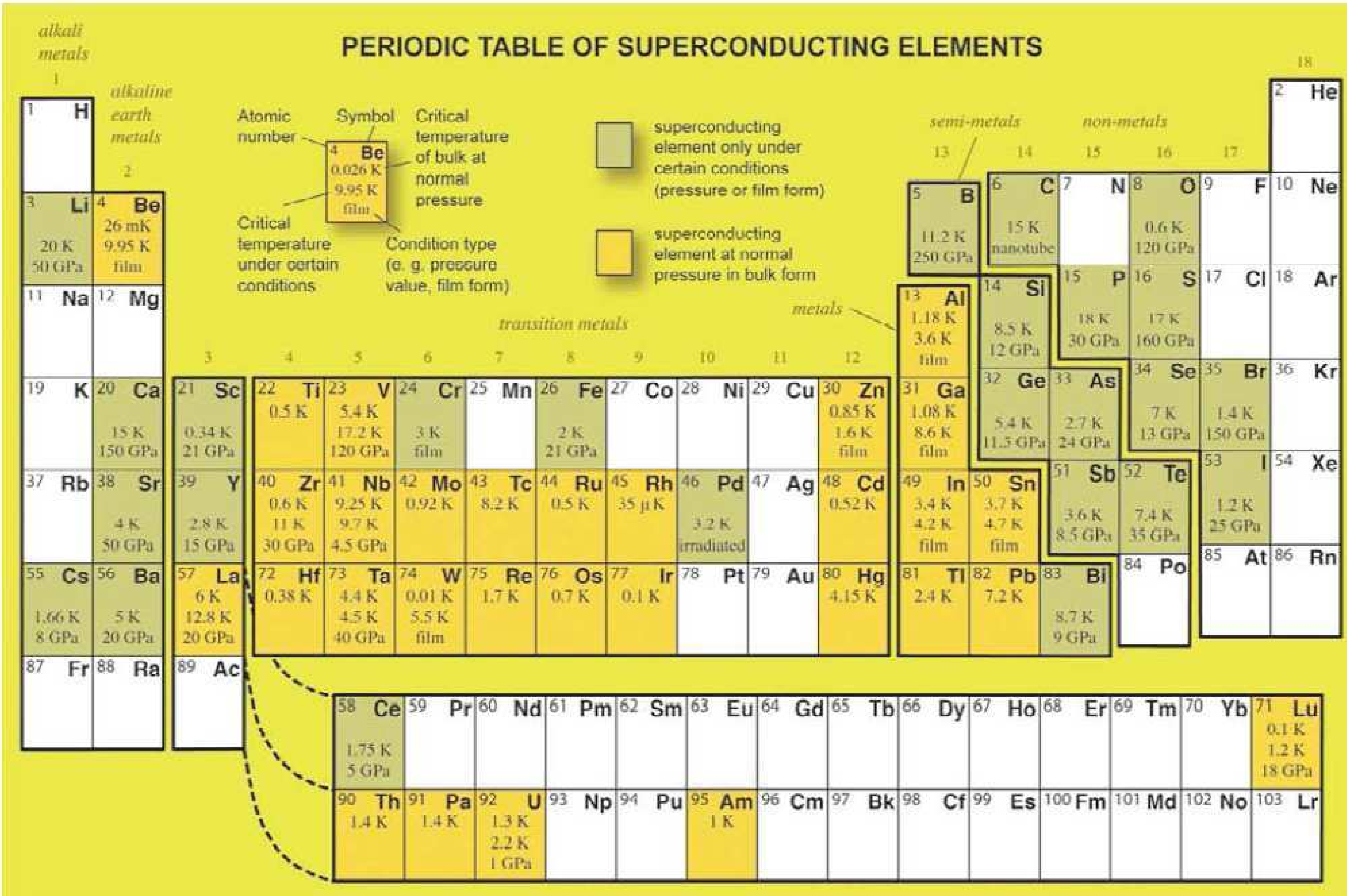
Discovered in 1911 by Heike Kamerlingh Onnes and Giles Holst after Onnes was able to liquify helium in 1908. Nobel prize in 1913







Superconducting elements



Superconducting state

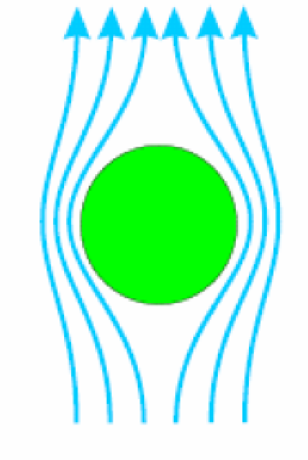
The superconducting state is characterized by the critical temperature T_c and field H_c

$$H_c(T) = H_c(0) \cdot \left(1 - (T/T_c)^2\right)$$

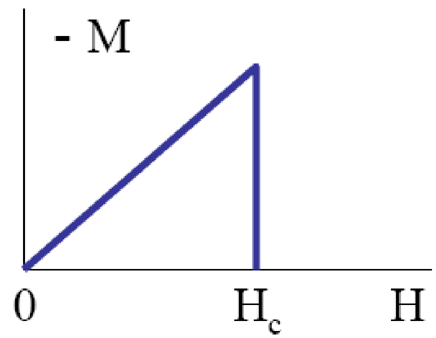
The external field is expelled from a superconductor if $H_{ext} < H_c$ for Type I superconductors.

For Type II superconductors the external field will partially penetrate for $H_{ext} > H_{c1}$ and will completely penetrate at H_{c2}

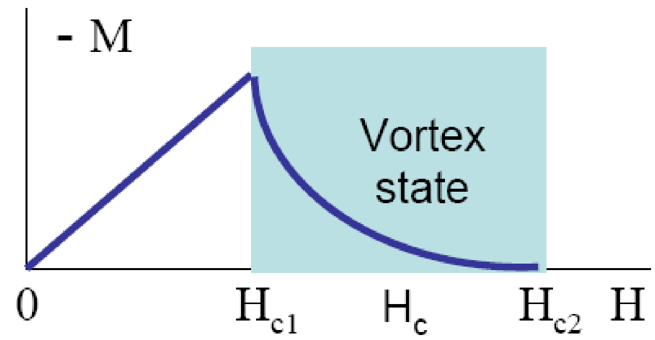
Type I and Type II superconductors



Superconductor in Meissner state = ideal diamagnetic



Complete Meissner effect in type-I superconductors



High-field partial Meissner effect in type-II superconductors

- Type-I: Meissner state $B = H + M = 0$ for $H < H_c$; normal state at $H > H_c$
- Type-II: Meissner state $B = H + M = 0$ for $H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$

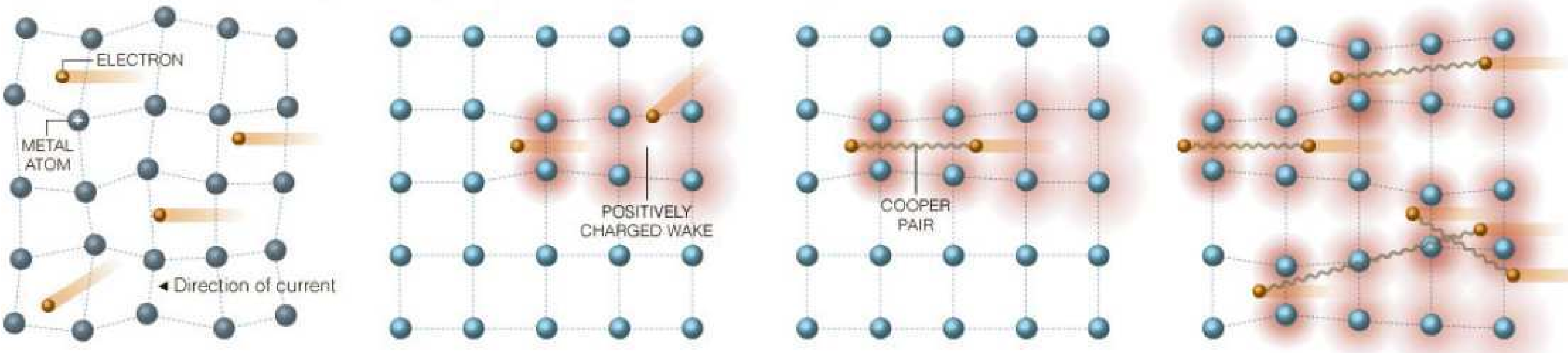
50 years of BCS theory

Bardeen-Cooper-Schrieffer (BCS) theory (1957). Nobel prize in 1972

January 7, 2008

Low-Temperature Superconductivity

December was the 50th anniversary of the theory of superconductivity, the flow of electricity without resistance that can occur in some metals and ceramics.



ELECTRICAL RESISTANCE
Electrons carrying an electrical current through a metal wire typically encounter resistance, which is caused by collisions and scattering as the particles move through the vibrating lattice of metal atoms.

CRITICAL TEMPERATURE
As the metal is cooled to low temperatures, the lattice vibration slows. A moving electron attracts nearby metal atoms, which create a positively charged wake behind the electron. This wake can attract another nearby electron.

COOPER PAIRS
The two electrons form a weak bond, called a Cooper pair, which encounters less resistance than two electrons moving separately. When more Cooper pairs form, they behave in the same way.

SUPERCONDUCTIVITY
If a pair is scattered by an impurity, it will quickly get back in step with other pairs. This allows the electrons to flow undisturbed through the lattice of metal atoms. With no resistance, the current may persist for years.

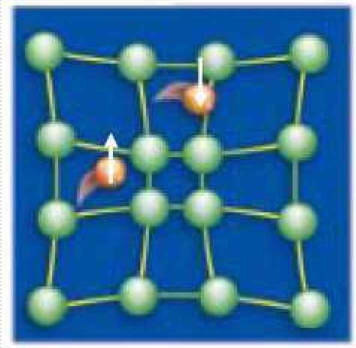
Sources: Oak Ridge National Laboratory; Philip W. Phillips

JONATHAN CORUM/THE NEW YORK TIMES

BCS theory

- Attraction between electrons with antiparallel momenta **k** and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs
- Bose condensation of overlapping Cooper pairs in a coherent superconducting state.
- Scattering on electrons does not cause the electric resistance because it would break the Cooper pair

The strong overlap of many Cooper pairs results in the macroscopic phase coherence



What is the phase coherence?

Incoherent (normal) crowd: each electron for itself

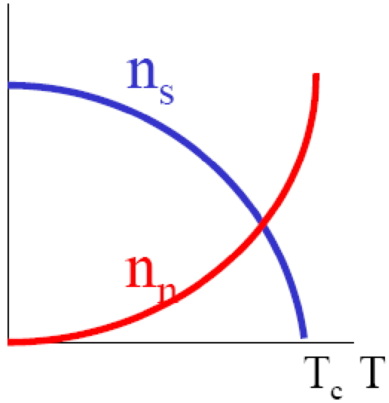
Phase-coherent (superconducting) condensate of electrons

London equations (1935)

- Two-fluid model: coexisting SC and N "liquids" with the densities $n_s(T) + n_n(T) = n$.

- Electric field E accelerates only the SC component, the N component is short circuited.

- Second Newton law for the SC component: $m d\mathbf{v}_s/dt = e\mathbf{E}$ yields the first London equation:



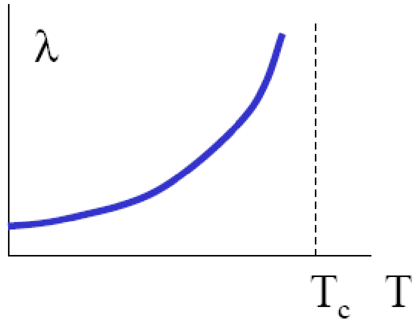
$$dJ_s/dt = (e^2 n_s/m)\mathbf{E} \quad \Longleftrightarrow \quad \mathbf{J} = \sigma \mathbf{E}$$

(ballistic electron flow in SC) (viscous electron flow in metals)

- Using the Maxwell equations, $\nabla\times \mathbf{E} = -\mu_0 \partial_t \mathbf{H}$ and $\nabla\times \mathbf{H} = \mathbf{J}_s$ we obtain the second London equation:

$$\lambda^2 \nabla \mathbf{H} - \mathbf{H} = 0$$

- London penetration depth: $$\lambda = \left(\frac{m}{e^2 n_s(T) \mu_0} \right)^{1/2}$$



GL theory

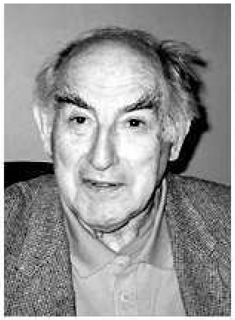
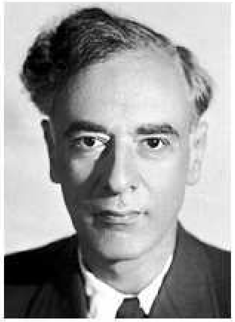
- the linear London equations

$$\frac{\partial \vec{J}_s}{\partial t} = -\frac{\vec{E}}{\lambda^2 \mu_0}, \qquad \lambda^2 \nabla^2 \vec{H} - \vec{H} = 0$$

along with the Maxwell equations describe the electrdynamics of SC at all T if:

- J_s is much smaller than the depairing current density J_d
- the superfluid density n_s is unaffected by current

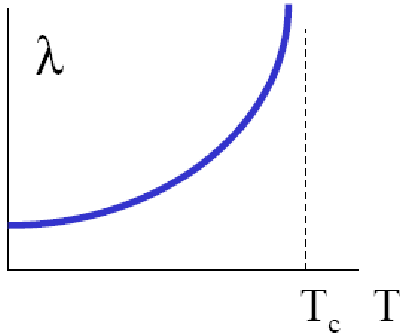
- Generalization of the London equations to nonlinear problems
- Phenomenological Ginzburg-Landau theory (1950, Nobel prize 2003) was developed before the microscopic BCS theory (1957).
- GL theory is one of the most widely used theories

Fundamental lengths and GL parameter

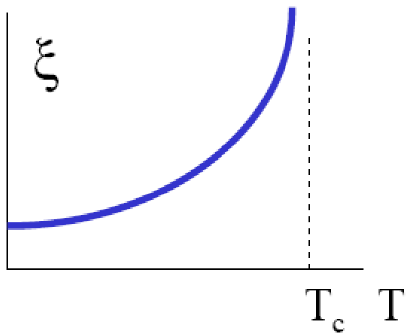
- Magnetic London penetration depth:

$$\lambda(T) = \left(\frac{m\beta}{2e^2\mu_0\alpha_0}\right)^{1/2}\sqrt{\frac{T_c}{T_c - T}}$$



- Coherence length – a new scale of spatial variation of the superfluid density $n_s(r)$ or superconducting gap $\Delta(r)$:

$$\xi(T) = \left(\frac{\hbar^2}{4m\alpha_0}\right)^{1/2}\sqrt{\frac{T_c}{T_c - T}}$$

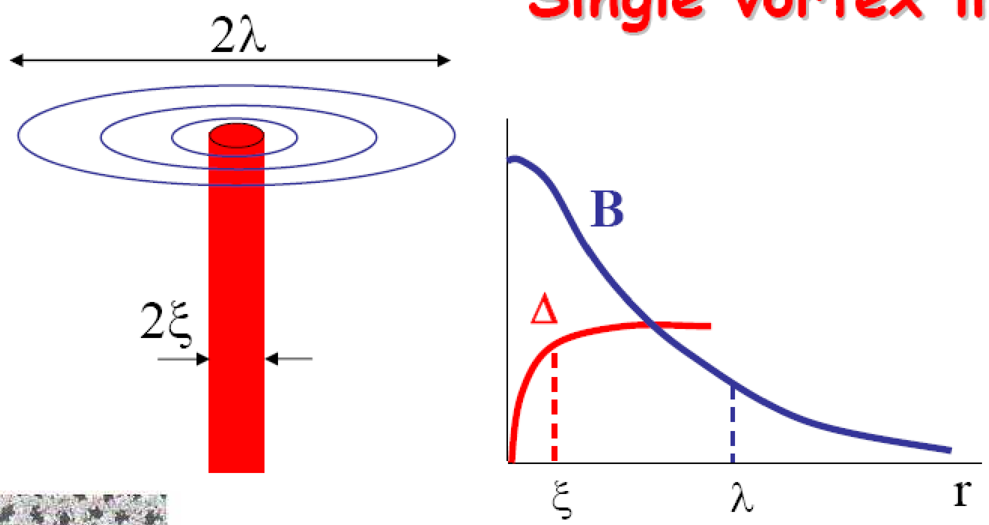


- The GL parameter $\kappa = \lambda/\xi$ is independent of T.
- Critical field $H_c(T)$ in terms of λ and ξ:

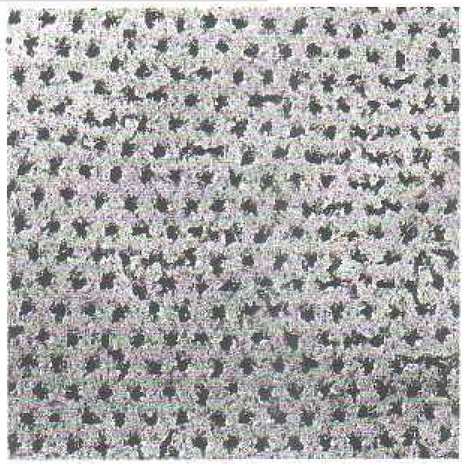
$$B_c(T) = \frac{\phi_0}{2\sqrt{2}\pi\xi(T)\lambda(T)}$$

Vortex state

Single vortex line



- Small core region r < ξ where Δ(r) is suppressed
- Region of circulating supercurrents, r < λ.
- Each vortex carries the flux quantum ϕ_0



Important lengths and fields

- Coherence length ξ and magnetic (London) penetration depth λ

$$B_{c1} = \frac{\phi_0}{2\pi\lambda^2}\left(\ln\frac{\lambda}{\xi} + 0.5\right), \qquad B_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}, \qquad B_{c2} = \frac{\phi_0}{2\pi\xi^2}$$

Type-II superconductors: $\lambda/\xi > 1/\sqrt{2}$: For clean Nb, $\lambda \approx 40$ nm, $\xi \approx 38$ nm

What happens if AC field is applied?

Two fluid model considers both superconducting and normal conducting components:

- At $0 < T < T_c$ not all electrons are bonded into Cooper pairs. The density of *unpaired,* "normal" electrons is given by the Boltzman factor

$$n_{normal} \propto e^{\left(-\frac{\Delta}{k_B T}\right)}$$

  where 2Δ is the energy gap around Fermi level between the ground state and the excited state.

- Cooper pairs move without resistance, and thus dissipate no power. In DC case the lossless Cooper pairs short out the field, hence the normal electrons are not accelerated and the SC is lossless even for $T > 0$ K.
- The Cooper pairs do nonetheless have an inertial mass, and thus they cannot follow an AC electromagnetic fields instantly and do not shield it perfectly. A residual EM field remains and acts on the unpaired electrons as well, therefore causing power dissipation.
- We expect the surface resistance to drop exponentially below T_c.

Losses in normal conductors

- For simplicity, use the nearly-free electron model
- Losses given by Ohm's law $\mathbf{j} = \sigma\mathbf{E} = \dfrac{n_n e^2 \tau}{m}\mathbf{E}, \qquad \tau = \text{scattering time}$
- The electrons have a time τ between scattering events to gain energy $\Delta\mathbf{v} = \dfrac{-e\mathbf{E}\,\tau}{m}$

- In a cavity, the magnetic field drives an oscillating current in the wall
  - → Start with Maxwell's equations

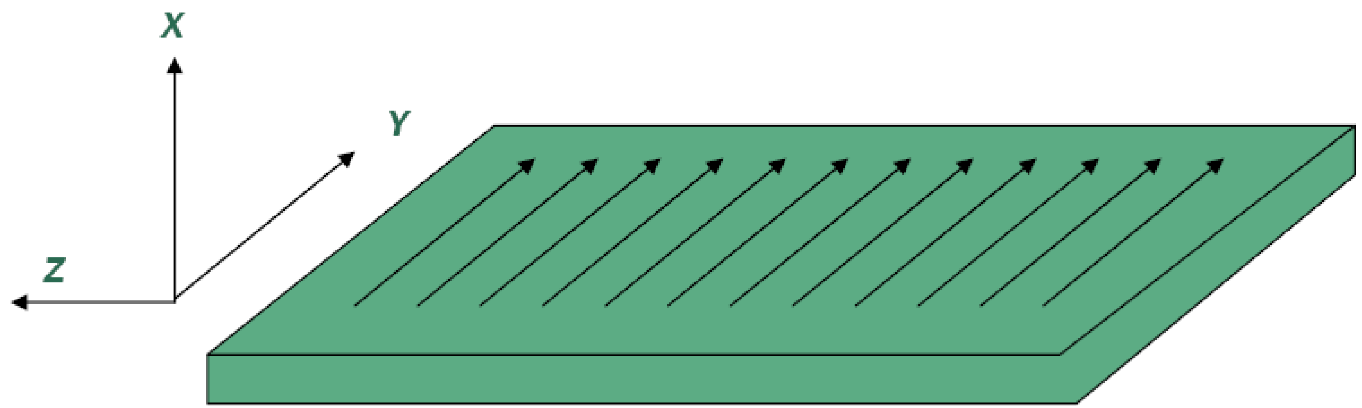
$$\nabla\times\mathbf{B} = \mu\mathbf{j} + \mu\varepsilon\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

- Combine the two and take the exp(iωt) dependence into account

$$-\nabla^2\mathbf{B} = \mu\nabla\times\mathbf{j} - \mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial^2 t} = -i\mu\sigma\omega\mathbf{B} + \mu\varepsilon\omega^2\mathbf{B}$$

- Look at a typical copper RF cavity: $\sigma = 5.8\times10^7\,\dfrac{\text{A}}{\text{Vm}} \qquad \omega\varepsilon_0 = 0.08\,\dfrac{\text{A}}{\text{Vm}}$ at 1.5 GHz

Field near conductor



Consider now a uniform magnetic field (y-direction) at the surface of a conductor.

Solving $\nabla^2 \mathbf{B} - i\mu\sigma\omega\mathbf{B} = 0$

yields $H_y = H_0 e^{-x/\delta} e^{-ix/\delta}$

where the field decays into the conductor with over a skin depth of $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

Similarly, from Maxwell find that $E_z = -\frac{(1+i)}{\sigma\delta} H_y$

So that a small, tangential component of ***E*** also exists which decays into the conductor

NC surface impedance

$$H_y = H_0 e^{-x/\delta} e^{-ix/\delta} \qquad E_z = -\frac{(1+i)}{\sigma\delta} H_y$$

- The losses per area are simply $$P'_{\text{diss}} = \frac{1}{2}\int_{x=0}^{\infty} J_z^* E_z \, dx = \frac{1}{2}\int_{x=0}^{\infty} \sigma |E_z|^2 dx$$

$$= \frac{1}{2\sigma\delta} H_0^2 \quad = \frac{1}{2} R_s H_0^2 \qquad R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}$$

- Note: Surface resistance is just the real part of the surface impedance:

$$Z_s = \frac{E_z}{H_y} = \frac{1+i}{\sigma\delta} = (1+i)\sqrt{\frac{\pi f \mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \exp(i\pi/4)$$

- Plug in some numbers:
- Copper: f = 1.5 GHz, σ = 5.8 x 10^7 A/Vm, μ_0 = 1.26 x 10^{-6} Vs/Am

→ δ = 1.7 μm, R_s = 10 mΩ
→ Q_0 = G/Rs = 25700

Surface impedance of superconductors

- Calculate surface impedance of a superconductor

  → Must take into account the „superconducting" electrons (n_s) in the 2-fluid model
- For these there is no scattering
- Thus: $\mathbf{j}_s = n_s e \mathbf{v}$

$$m\frac{\partial \mathbf{v}}{\partial t} = -e\mathbf{E} \quad \Rightarrow \frac{\partial \mathbf{j}_s}{\partial t} = \frac{n_s e^2}{m}\mathbf{E}$$

First London Equation

- In an RF field with exp(iωt) dependance $\Rightarrow \mathbf{j}_s = -i\frac{n_s e^2}{m\omega}\mathbf{E}$

Acts as the AC conductivity of the superconducting fluid. „Collision time" is the RF period.

or $\mathbf{j}_s = \frac{-i}{\omega\mu_0\lambda_L^2}\mathbf{E}$ where $\lambda_L = \frac{m}{\mu_0 n_s e^2}$ is the London penetration depth

- Total current: Just add the currents due to both „fluids": $\mathbf{j} = \mathbf{j}_n + \mathbf{j}_s = (\sigma_n - i\sigma_s)\mathbf{E}$

Surface impedance of superconductors

- Thus, the treatment with a superconductor is the same as before, only that we have to change: $\sigma \to (\sigma_n - i\sigma_s)$
- Impedance $Z_s = \sqrt{\frac{\omega\mu}{\sigma}} \exp\left(i\pi/4\right) \to \sqrt{\frac{\omega\mu}{\sigma_n - i\sigma_s}} \exp\left(i\pi/4\right)$
- Penetration depth $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \to \frac{1}{\sqrt{\pi f \mu (\sigma_n - i\sigma_s)}}$
- Where $H_y = H_0 \exp\left(-\frac{(1+i)}{\delta} x\right)$, $\sigma_n = \frac{n_n e^2 \tau}{m}$ and $\sigma_s = \frac{n_s e^2}{m\omega}$
- Note that 1/ω is of order 100 ps whereas for normal conducting electrons *τ* is of order few 10 fs. Also, $n_s >> n_n$ for $T << T_c$. Hence $\sigma_n << \sigma_s$
- As a result one finds that: $\delta \approx (1+i)\lambda_L \left(1 + i\frac{\sigma_n}{2\sigma_s}\right)$ $H_y = H_0 e^{-x/\lambda_L} e^{-ix\sigma_n/2\sigma_s\lambda_L}$
- Again, the field decays rapidly but now over the London penetration depth

Surface impedance of superconductors

Surface resistance of the superconductor

- For the impedance we get: $Z_s \approx \sqrt{\frac{\omega\mu}{\sigma_s}}\left(\sigma_n / 2\sigma_s + i\right)$ $\quad X_s = \omega\mu_0\lambda_L$ $\quad R_s = \frac{1}{2}\sigma_n\omega^2\mu_0^2\lambda_L^3$
- Lets look at some numbers:

  For niobium λ_L = 36 nm, for Copper the penetration depth was 1.7 µm (@ 1.5 GHz)

  → The field penetrates over a much shorter distance than for a normal conductor

- At 1.5 GHz: X_s = 0.43 mΩ, whereas R_s is < 1µΩ

  → The superconductor is mostly reactive in line with our previous explanation of losses in a superconductor

- The surface resistance is proportional to the conductivity of the normal fluid!

  → If the normal-state resistivity is low, the superconductor is more lossy!

- Explanation: For „residual“ field not shielded by the Cooper pairs more „normal current“ flows → more dissipation $P_{diss} \propto \sigma_n E^2$

BCS surface resistance

Calculation of surface resistance must take into account numerous parameters. Mattis and Bardeen developed theory based on BCS, which predicts

$$R_{BCS} = A\frac{\omega^2}{T}e^{-\left(\frac{\Delta}{k_B T_c}\right)\frac{T_c}{T}},$$

where A is the material constant.

While at low frequencies (≤ 500 MHz) it may be efficient to operate at 4.2 K (liquid helium at atmospheric pressure), higher frequency structures favor lower operating temperatures (typically superfluid LHe at 2 K, below lambda point).

Approximate expression for Nb:

$$R_{BCS} \approx 2\times 10^{-4}\,\Omega\left(\frac{f}{1500MHz}\right)^2\frac{1}{T}e^{\left(\frac{-17.67}{T}\right)},$$

Why Nb?

	Type	T_c	H_{c1}	H_c	H_{c2}	Fabrication
	-	K	mOe	mOe	mOe	-
Nb	II	9.25	1700	2060	4000	bulk, film
Pb	I	7.20	-	803	-	electroplating
Nb_3Sn*	II	18.1	380	5200	250000	film
MgB_2	II	39.0	300	4290		film
Hg	I	4.15	-	411/339	-	-
Ta	I	4.47	-	829	-	-
In	I	3.41	-	281.5	-	-

**) Other compounds with the same β-tungsten or A15 structure are under investigation as well.*

- High critical temperature (cavities with High-T_c sputter coatings on copper have shown much inferior performance in comparison to niobium cavities) → lower RF losses → smaller heat load on refrigeration system.
- High RF critical field, which of the order of H_c. Strong flux pinning associated with high H_{c2} is undesirable as it is coupled with losses due to hysteresis. Hence a 'soft' superconductor must be used.
- Good formability is desirable for ease of cavity fabrication. Alternative is a thin superconducting film on a copper substrate.
- Pure niobium is the best candidate, although its critical temperature Tc is only 9.25 K, and the thermodynamic critical field about 200 mT. Nb_3Sn with a critical temperature of 18.1 K looks more favorable at first sight, however the gradients achieved in Nb_3Sn coated niobium cavities were always below 15 MV/m, probably due to grain boundary effects in the Nb3Sn layer. For this reason niobium is the preferred superconducting material.

Residual surface resistance

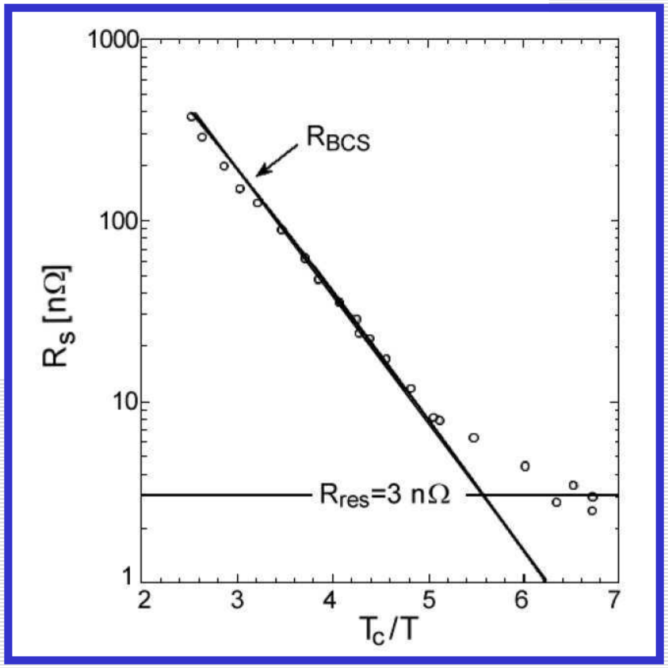
At low temperatures the measured surface resistance is larger than predicted by theory:

$$R_s = R_{BCS}(T) + R_{res}$$

where R_{res} is the temperature independent residual resistance. It can be as low as 1 nOhm.

Causes for this are:

- **magnetic flux frozen in at cooldown**
- **surface contaminations**
- **hydrogen precipitates**

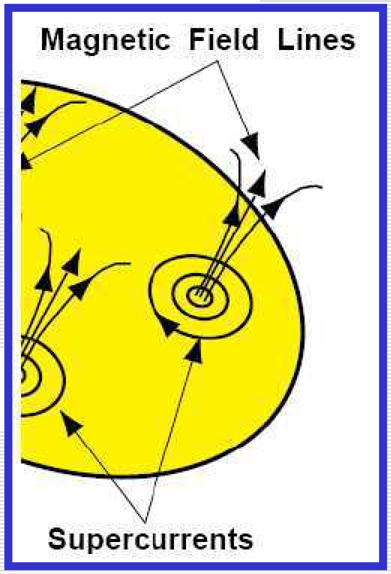


Trapped magnetic flux

Ideally, if the external magnetic fiels is less than H_{c1}, the DC flux will be expelled due to Meissner effect. In reality, there are lattice defects and other inhomogeneities, where the flux lines may be "pinned" and trapped within material.

The resulting contribution to the residual resistance

$$R_{mag} = \frac{H_{ext}}{2H_{c2}} R_n$$

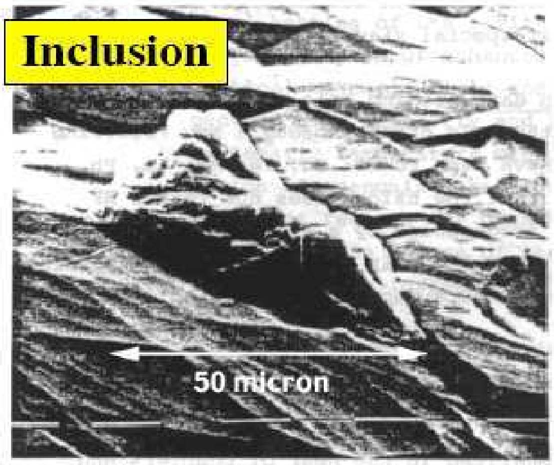


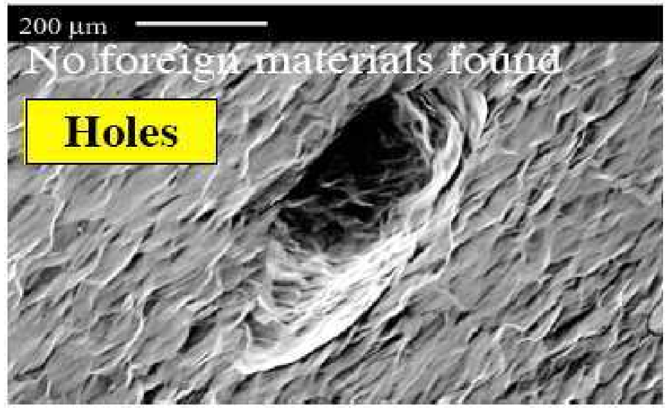
For high purity (RRR=300) Nb one gets

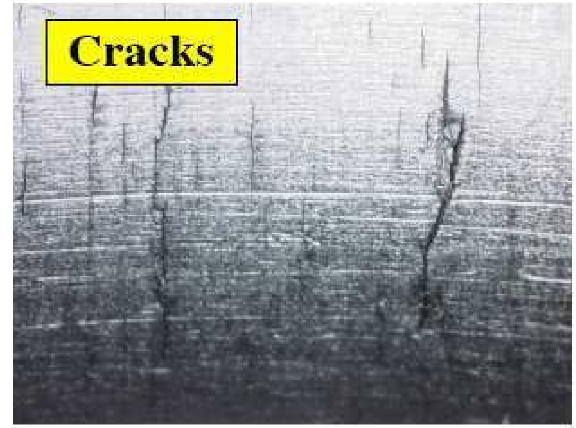
$$R_{mag} = 0.3(n\Omega) H_{ext}(mOe)\sqrt{f(GHz)}$$

Earth's field is 0.5 G → residual resistance of 150 nOhm at 1 GHz → $Q_0 < 2\times10^9$ → need magnetic shielding around the cavity to reach quality factor in the 10^{10} range. Usually the goal is to have residual magnetic field of less than 10 mG.

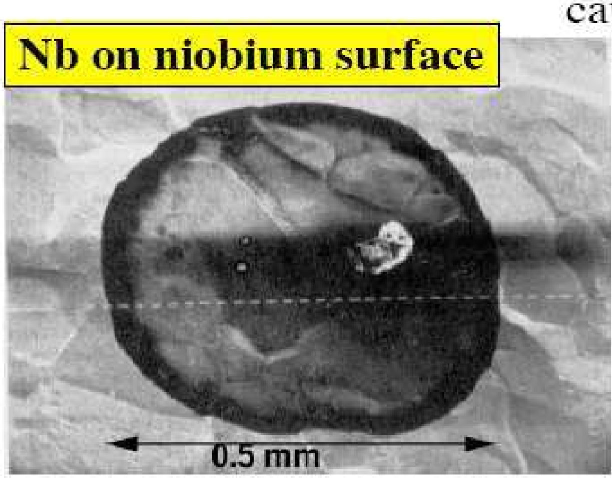
Examples of surface defects

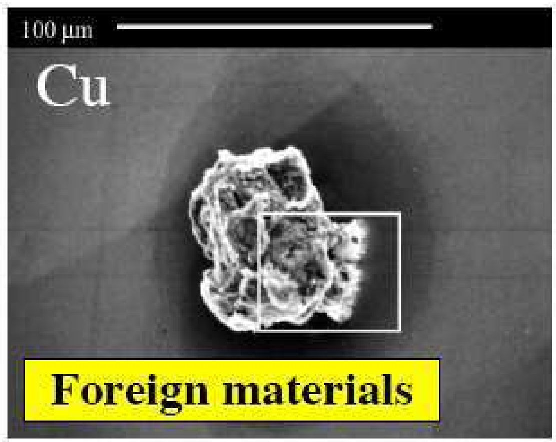


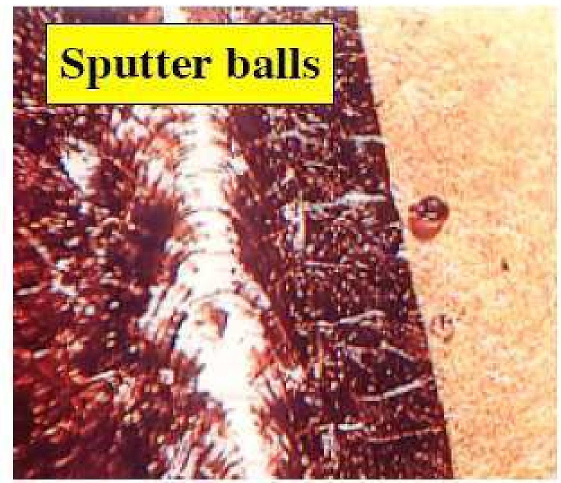




Surface defects, holes can also cause TB

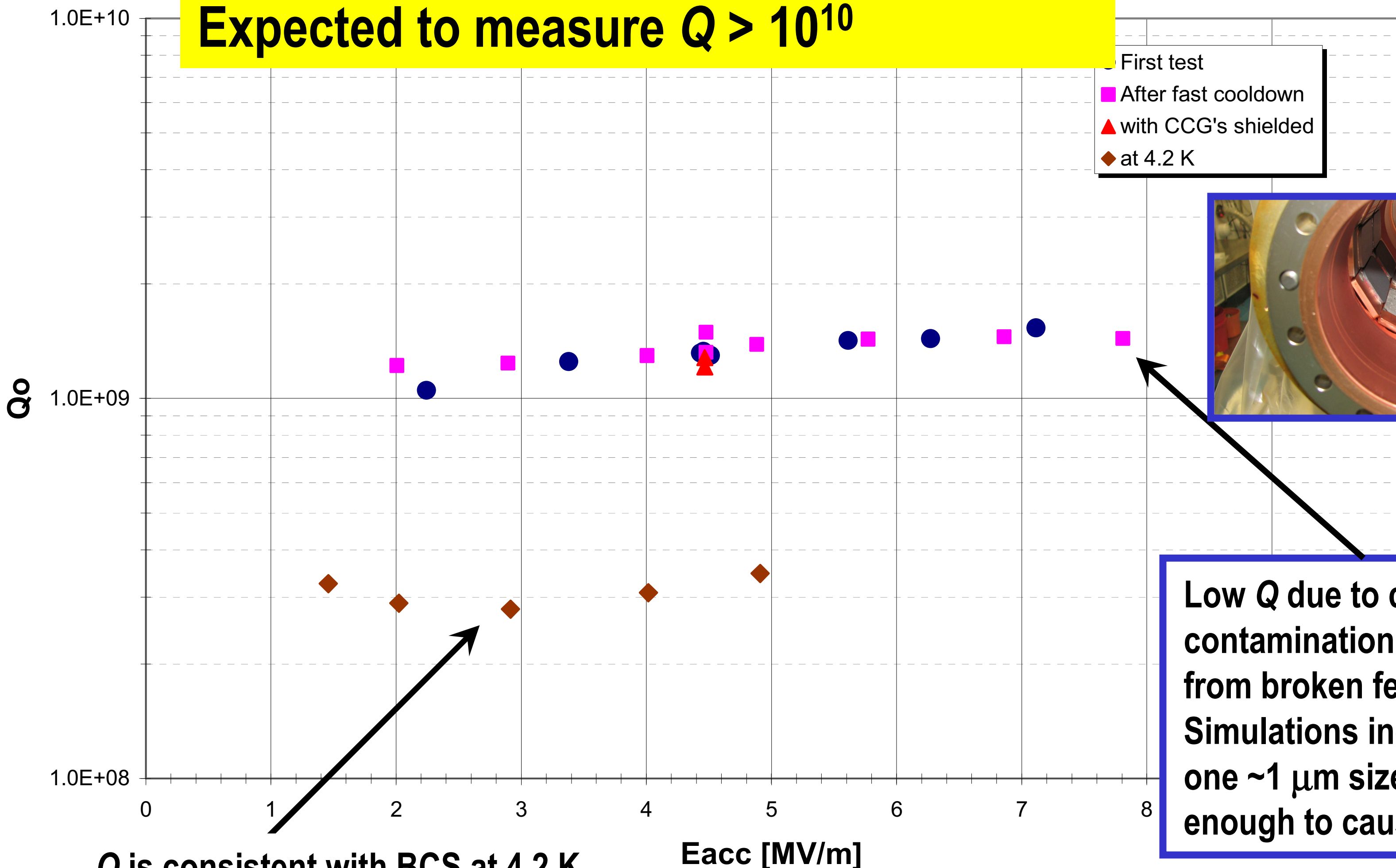


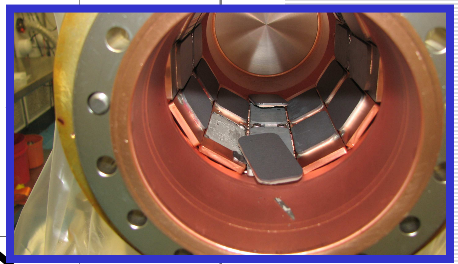




Surface contamination

1.3 GHz two-cell ERL injector cavity test
Expected to measure $Q > 10^{10}$



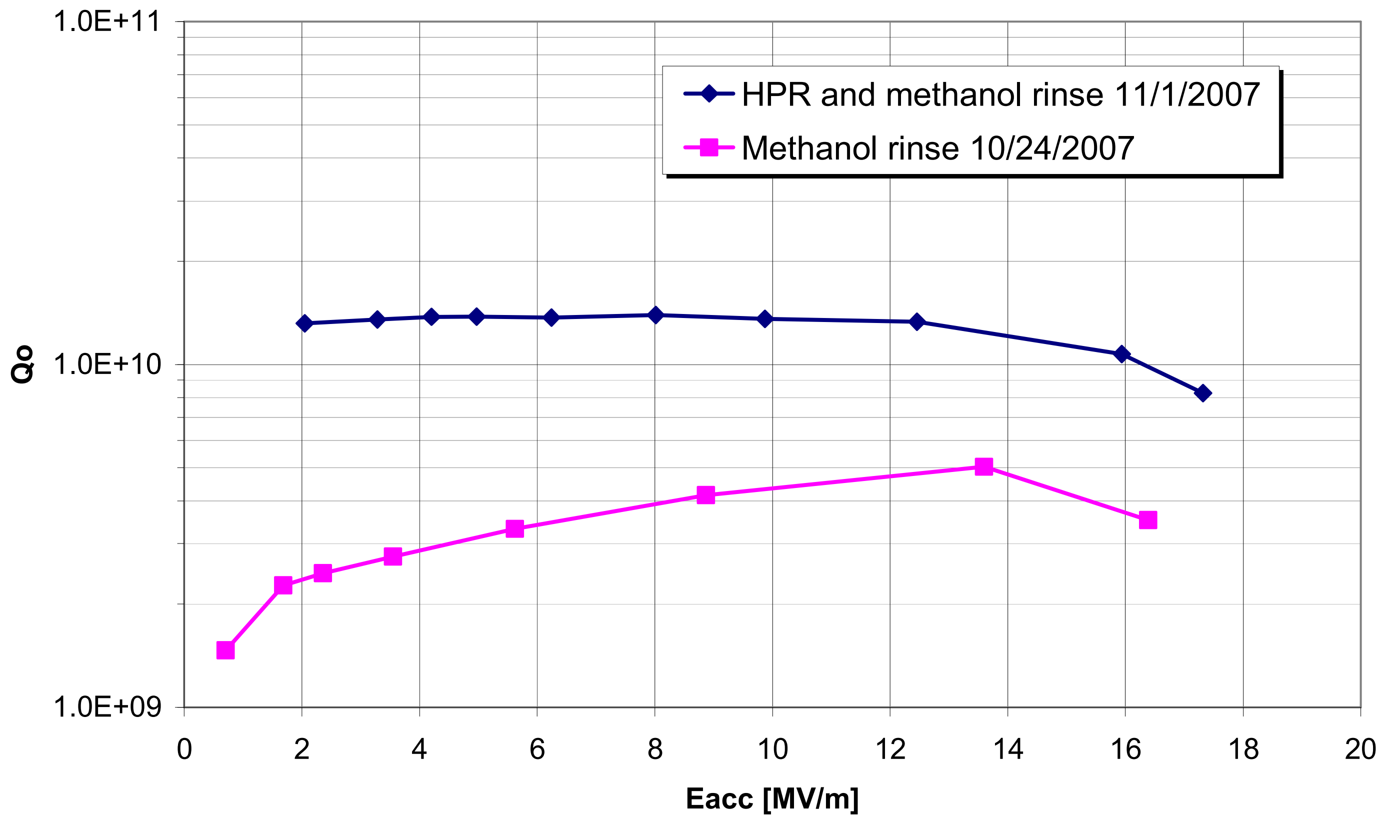


Low Q due to cavity contamination by dust from broken ferrite tiles. Simulations indicate that one ~1 μm size particle is enough to cause this!

Q is consistent with BCS at 4.2 K

Cavity re-test

- **Re-tested the cavity after removing ferrite and rinsing it with methanol: *Q* was still low.**
- **HPR'd the cavity and tested again: Q is back to normal.**

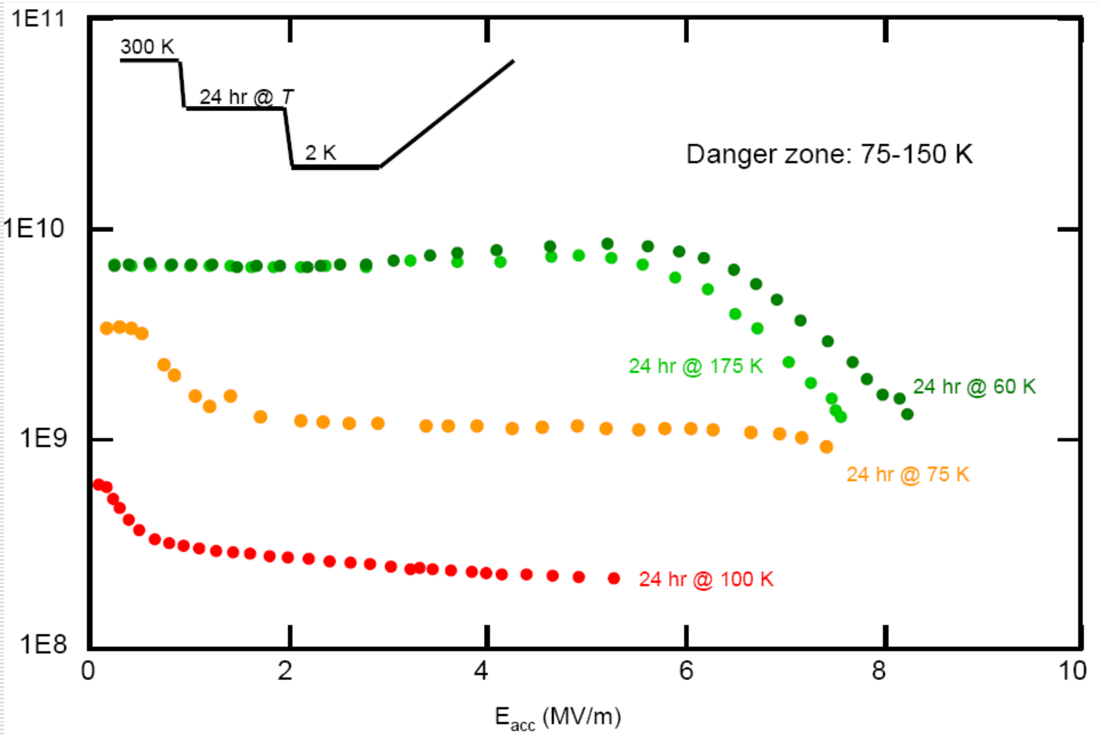


Q disease

The hydrogen dissolved in bulk niobium can under certain conditions during cooldown precipitate as a lossy hydride at the niobium surface. This is known as the "*Q*-diseas." At temperatures above 150 K too high concentration of hydrogen is required to form the hydride phase. However, in the temperature range from 60 to 150 K the required hydrogen concentrations drops to as low as 2 wt ppm while its diffusion rate remains significant. This is the danger zone.

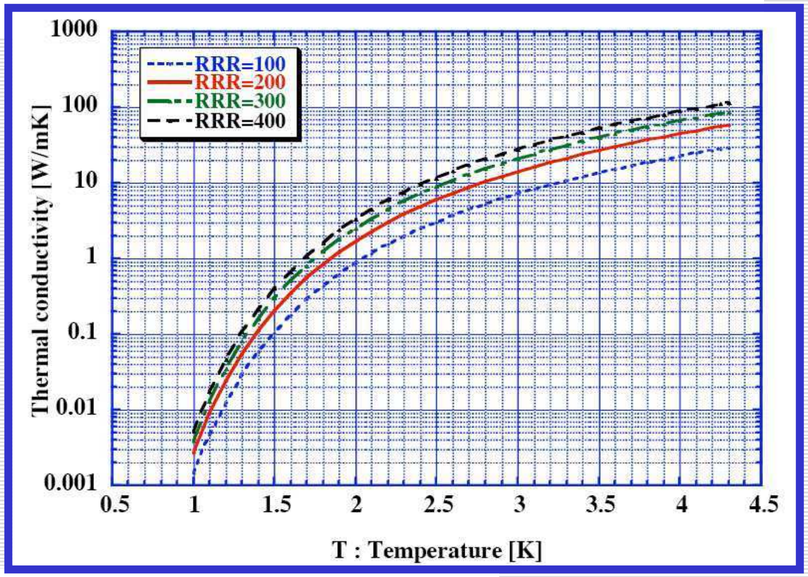
Mitigation:

- rapid cooldown through the danger temperature zone
- degassing hydrogen by heating the Nb cavity in vacuum of better than 10^{-6} Torr at 600°C for 10 h or at 800°C for 1 – 2 h.
- keep the acid temperature below 15C during chemical etching



RRR

- Residual Resistivity Ratio (RRR) is a measure of material purity and is defined as the ratio of the resistivity at 273 K (or at 300 K) to that at 4.2 K in normal state.
- High purity materials have better thermal conductivity, hence better handling of RF losses.
- The ideal RRR of niobium due to phonon scattering is 35,000. Typical "reactor grade" Nb has RRR ≈ 30. Nb sheets used in cavity fabrication have RRR ≥ 200.



$$\lambda(4.2K) \approx 0.25 \cdot RRR \quad [W/(m \cdot K)]$$

$$RRR = \left(\sum_i f_i / r_i \right)^{-1}$$

where the f_i denote the fractional contents of impurity i (measured in weight ppm) and the r_i the corresponding resistivity coefficients which are listed in the following table.

Table II Weight factor r_i of some impurities (see equation (4))

Impurity atom i	N	O	C	H	Ta
r_i in 10^4 wt. ppm	0.44	0.58	0.47	0.36	111

What have we learned so far?

- Superconducting RF systems are more efficient than normal conducting systems.
- SC cavities are less disruptive to the beam than NC cavities.
- They operate at cryogenic temperatures, typically immersed in a bath of liquid helium.
- SC cavities are appropriate for use in CW and long-pulse accelerators.
- Nb is a material of choice in either bulk form or as a film on a copper substrate. Other materials are being investigated.
- Material quality (impurities, mechanical damage) plays important role.
- Performance of SC cavities is dependant on the quality of a thin surface layer, hence it is very important to properly process cavities and carefully follow all preparation steps.
- Cavities have to be shielded from external magnetic field.

✪ We will discuss further only cavities made of bulk Nb for β = 1 accelerators (elliptical shape cavities).