Quadrupole Errors

$$\vec{z}' = \underline{L}(s)\,\vec{z} + \Delta\vec{f}(\vec{z},s)$$

$$\vec{z}(s) = \vec{z}_H(s) + \int_0^s \underline{M}(s,\hat{s})\Delta\vec{f}(\vec{z},\hat{s})\,d\hat{s} \approx \vec{z}_H(s) + \int_0^s \underline{M}(s,\hat{s})\Delta\vec{f}(\vec{z}_H,\hat{s})\,d\hat{s}$$

$$x'' = -(\kappa^2 + k)x - \Delta k(s)\,x \quad \Rightarrow \quad \begin{pmatrix} x' \\ a' \end{pmatrix} = \begin{pmatrix} a \\ -(\kappa^2 + k)x \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ \Delta k(s) & 0 \end{pmatrix}\begin{pmatrix} x \\ a \end{pmatrix}$$

$$\vec{z}(s) = \left\{ \underline{M}(s) - \int_0^s \underline{M}(s,\hat{s}) \begin{pmatrix} 0 & 0 \\ \Delta k(\hat{s}) & 0 \end{pmatrix} \underline{M}(\hat{s},0)\,d\hat{s} \right\} \vec{z}_0$$

One quadrupole error:

$$\underline{M}(s,\hat{s}) + \Delta\underline{M}(s,\hat{s}) = \underline{M}(s,\hat{s}) - \underline{M}(s,\hat{s}) \begin{pmatrix} 0 & 0 \\ \Delta kl(\hat{s}) & 0 \end{pmatrix}$$

Quadrupole Error and Phase advance

$$\Delta \underline{M}(s,\hat{s}) = -\underline{M}(s,\hat{s})\begin{pmatrix} 0 & 0 \\ \Delta kl(\hat{s}) & 0 \end{pmatrix}$$

$$\underline{M}(s)=\begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}[\cos\tilde{\psi}+\alpha_0\sin\tilde{\psi}] & \sqrt{\beta_0\beta}\sin\tilde{\psi} \\ \sqrt{\frac{1}{\beta_0\beta}}[(\alpha_0-\alpha)\cos\tilde{\psi}-(1+\alpha_0\alpha)\sin\tilde{\psi}] & \sqrt{\frac{\beta_0}{\beta}}[\cos\tilde{\psi}-\alpha\sin\tilde{\psi}] \end{pmatrix}$$

$$\Delta \underline{M}(s,\hat{s}) = -\Delta kl(\hat{s})\begin{pmatrix} \sqrt{\hat{\beta}\beta}\sin\tilde{\psi} & 0 \\ \sqrt{\frac{\hat{\beta}}{\beta}}[\cos\tilde{\psi}-\alpha\sin\tilde{\psi}] & 0 \end{pmatrix} \quad , \qquad \tilde{\psi}=\psi-\hat{\psi}$$

$$= \begin{pmatrix} \frac{\frac{1}{2}\Delta\beta[\cos\tilde{\psi}+\hat{\alpha}\sin\tilde{\psi}]+\Delta\psi\beta[\hat{\alpha}\cos\tilde{\psi}-\sin\tilde{\psi}]}{\sqrt{\hat{\beta}\beta}} & \sqrt{\hat{\beta}}\left(\frac{\frac{\Delta\beta}{2}\sin\tilde{\psi}+\Delta\psi\beta\cos\tilde{\psi}}{\sqrt{\beta}}\right) \\ \ldots & \ldots \end{pmatrix}$$

$$\Delta\psi = -\frac{\Delta\beta}{2\beta}\tan\tilde{\psi}$$

$$\tfrac{1}{2}\Delta\beta\cos\tilde{\psi}+\tfrac{1}{2}\Delta\beta\frac{\sin^2\tilde{\psi}}{\cos\tilde{\psi}} = \tfrac{1}{2}\Delta\beta\frac{1}{\cos\tilde{\psi}} = -\Delta kl(\hat{s})\beta\hat{\beta}\sin\tilde{\psi}$$

Quadrupole Error correction

$$\Delta\psi = \Delta kl(\hat{s})\hat{\beta}\sin^2(\psi - \hat{\psi})$$

→ More focusing always increases the tune

$$\frac{\Delta\beta}{\beta} = -\Delta kl(\hat{s})\hat{\beta}\sin(2[\psi - \hat{\psi}])$$

→ Beta beat oscillates twice as fast as orbit.

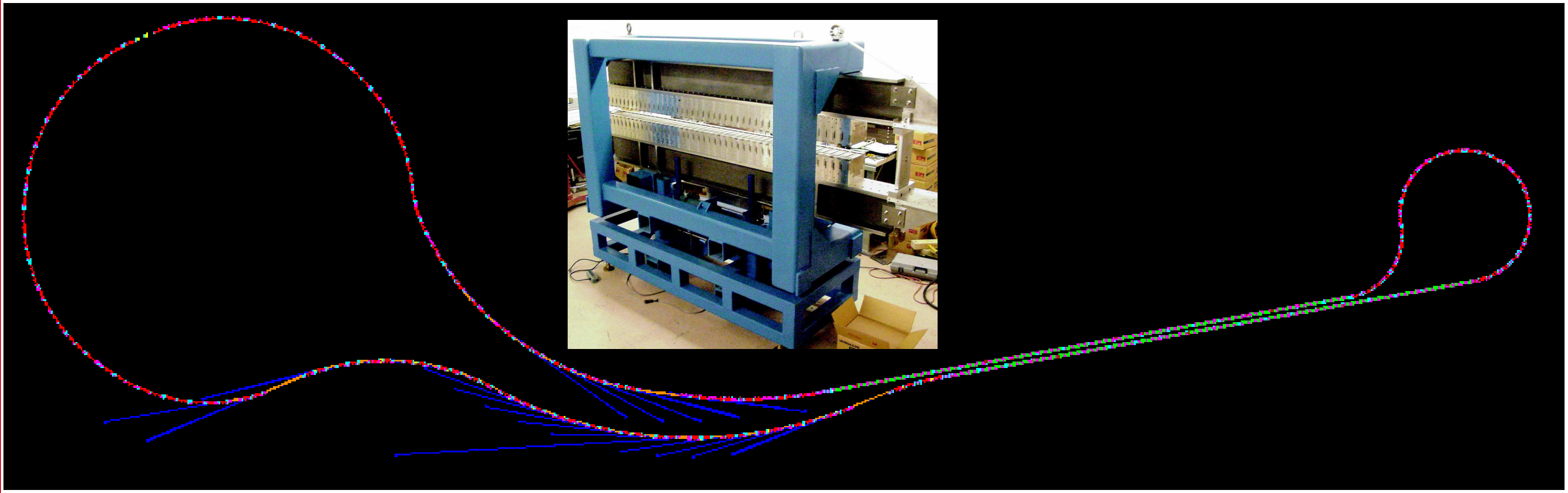
$$\Delta\psi = \sum_j \Delta kl_j \beta_j \tfrac{1}{2}[1 - \cos(2[\psi - \psi_j])]$$

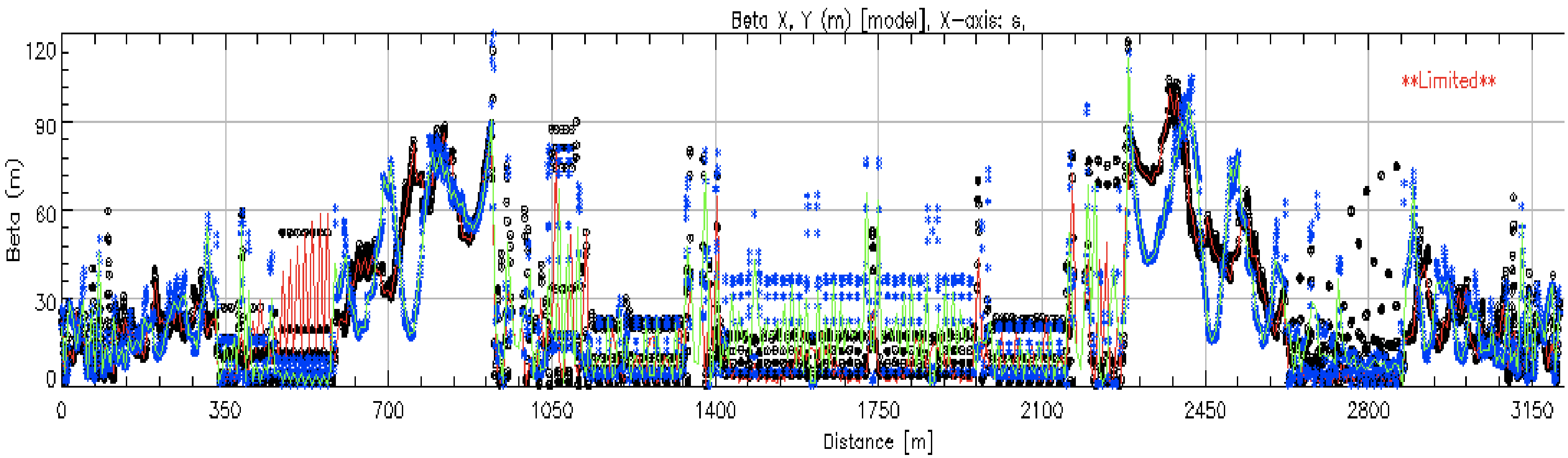
$$\frac{\Delta\beta}{\beta} = -\sum_j \Delta kl_j(\hat{s})\beta_j \sin(2[\psi - \psi_j])$$

When beta functions and betatron phases have been measured at many places, quadrupoles can be changed with these formulas to correct the Twiss errors.

Twiss Parameters in Energy Recovery Linacs

CHESS & LEPP

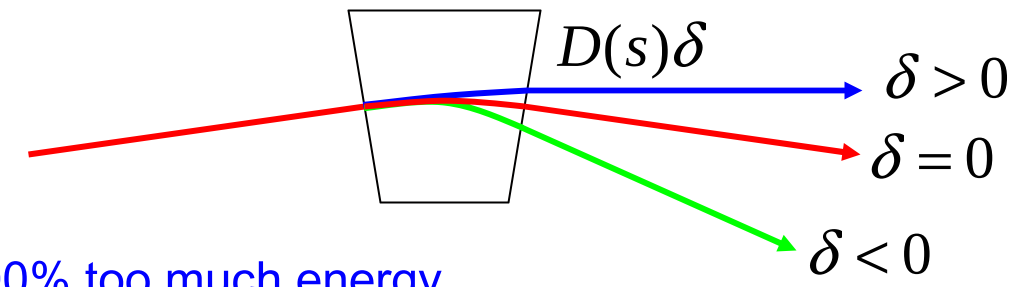




Longitudinal Beam Dynamics

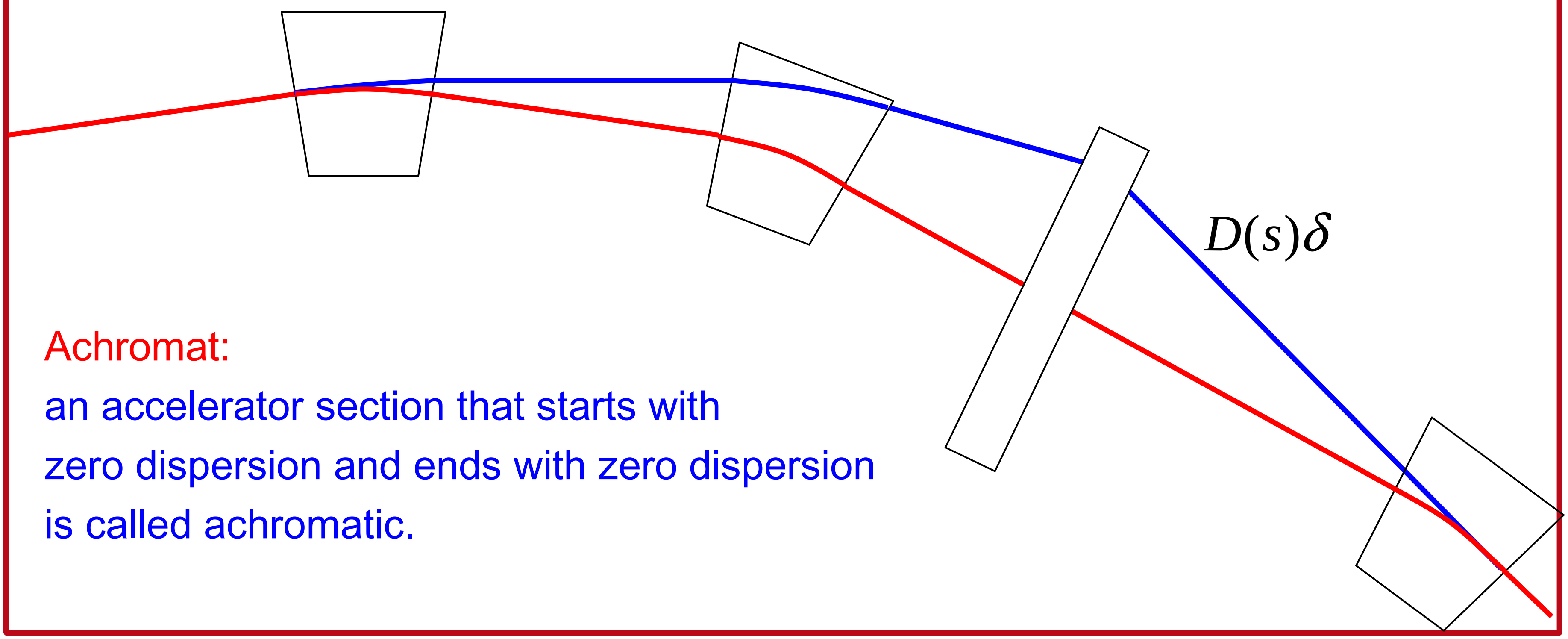
$$x' = a$$

$$a' = -(\kappa^2 + k)x + \kappa\delta$$



Dispersion:

path (for linearized motion) for 100% too much energy.



Achromat:

an accelerator section that starts with zero dispersion and ends with zero dispersion is called achromatic.

Achromatic Sections in Light Sources

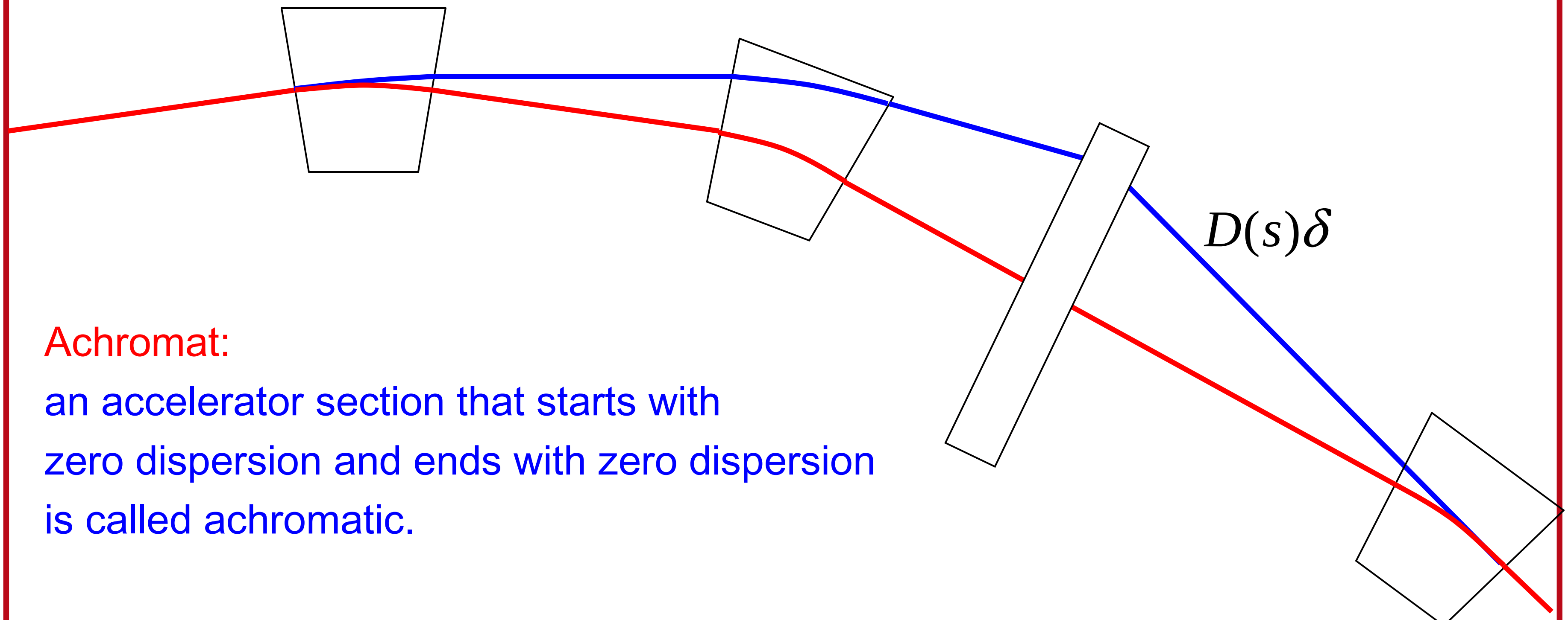
Dispersion free sections for Insertion Devices:

It is good to have zero dispersion at the source of x-ray radiation because:

A) The beam position becomes independent to beam-energy fluctuations.

B) The beam size is not made wider by energy spread in the beam.

C) Dispersion is small in most dipoles, and synchrotron photons therefore do not increase the beam emittance much.



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