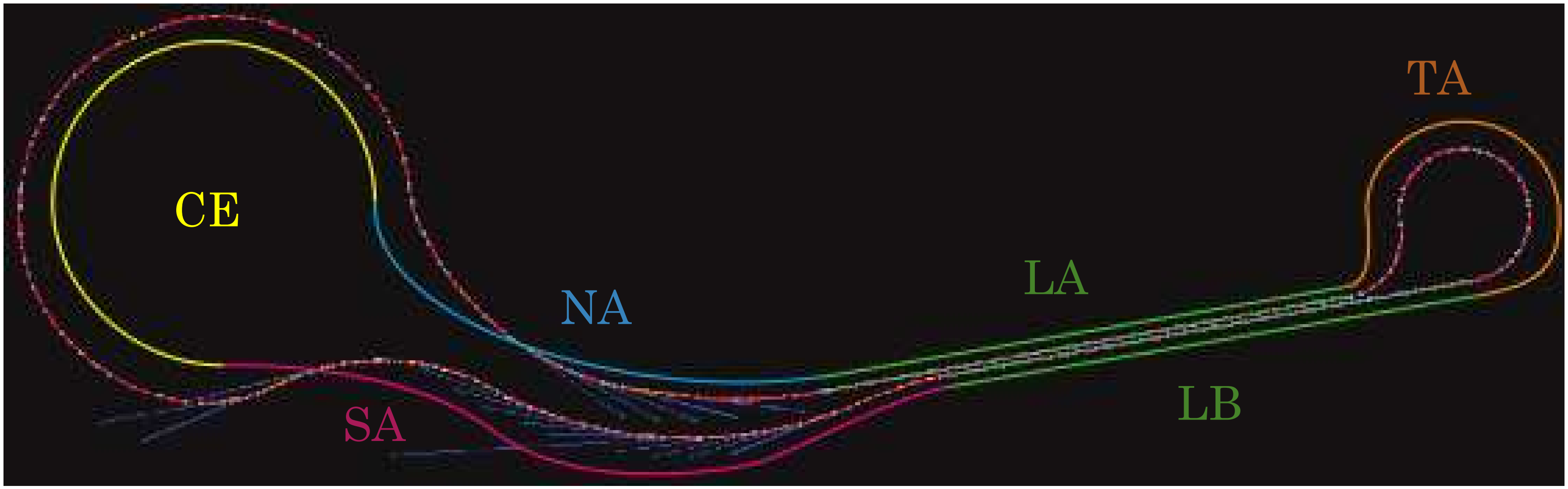
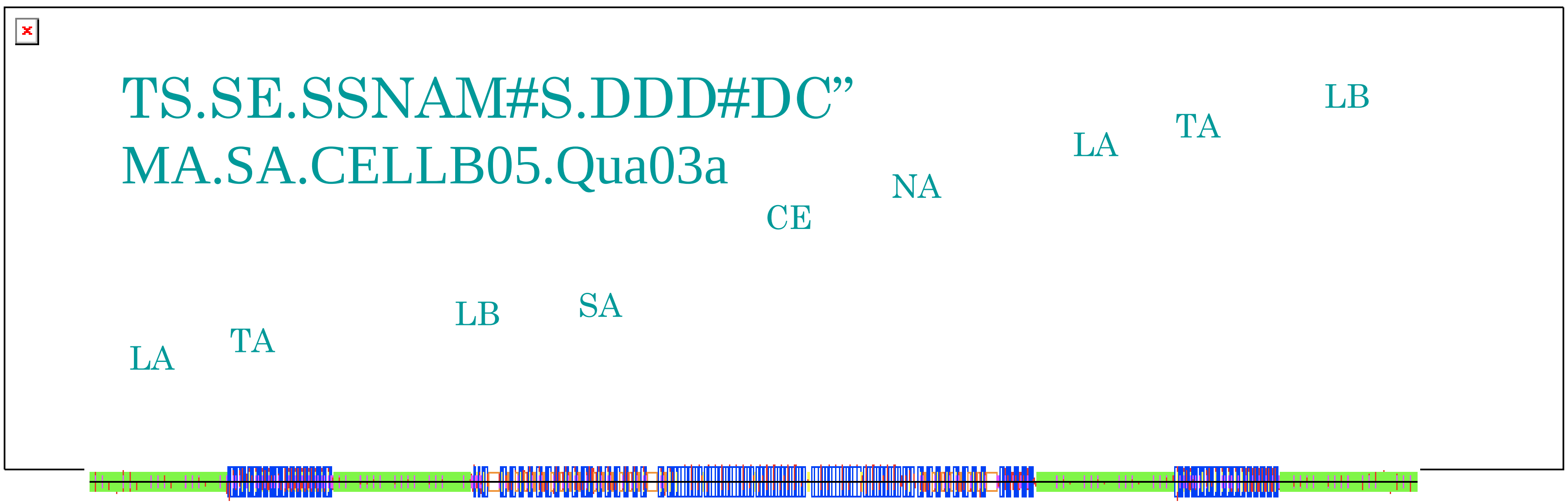
# Incoherent Synchrotron Radiation

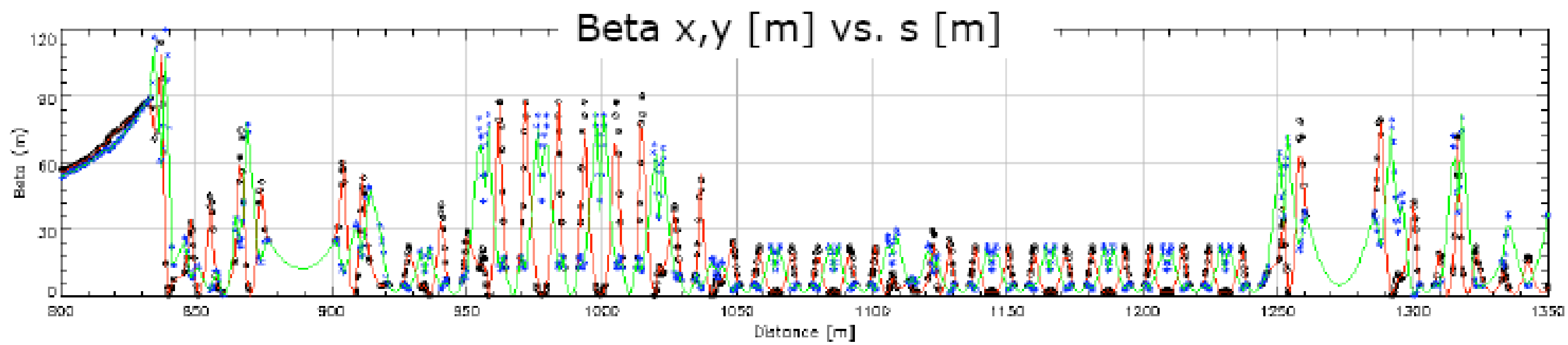
CHESS & LEPP

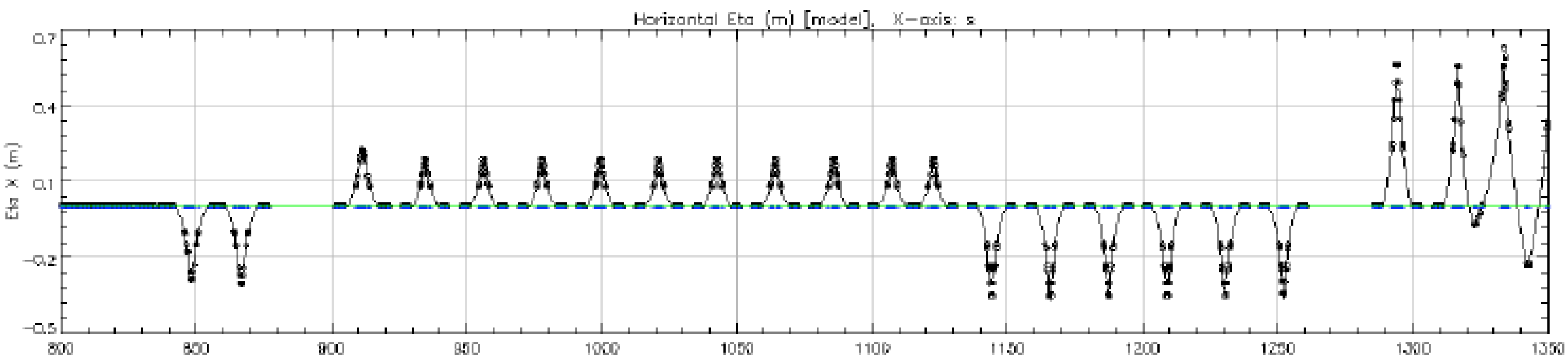


# ERL Optics – South Arc Achromats





# ERL Optics – South Arc Insertion Sections

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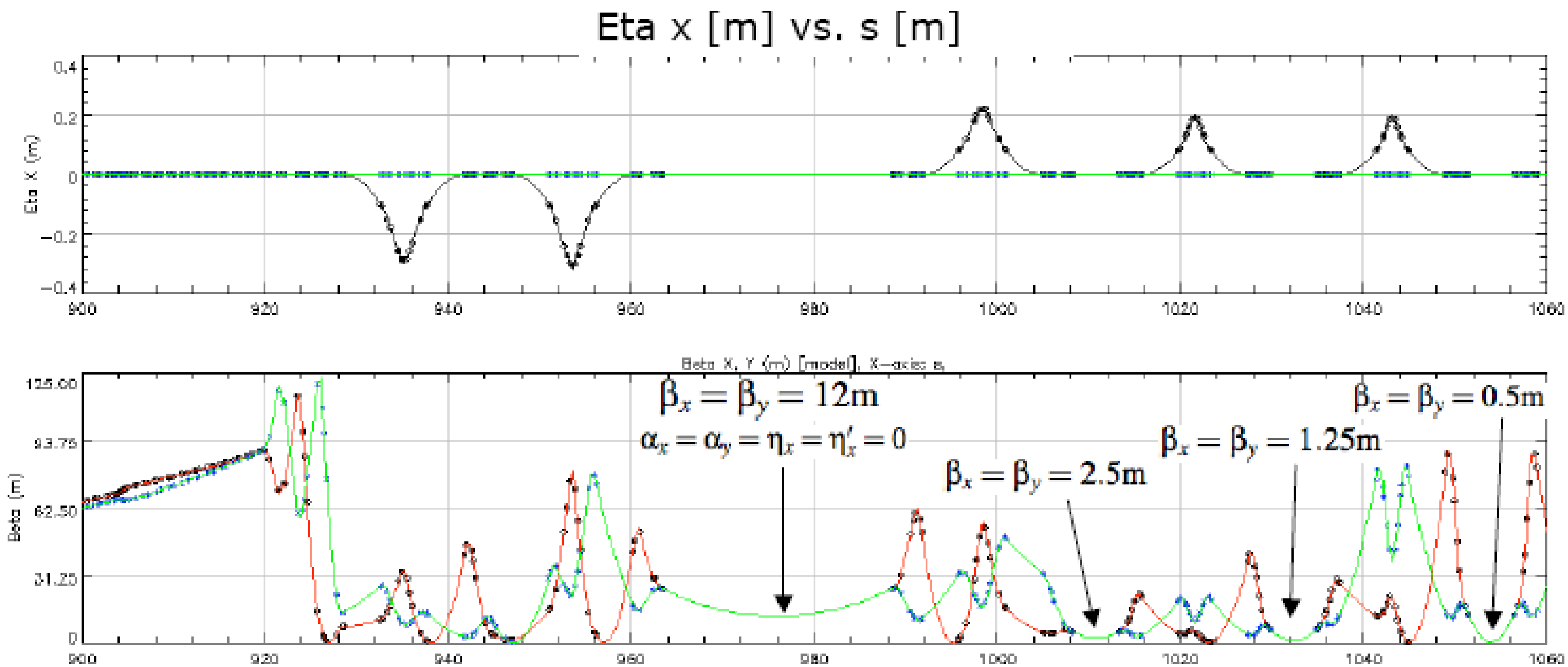
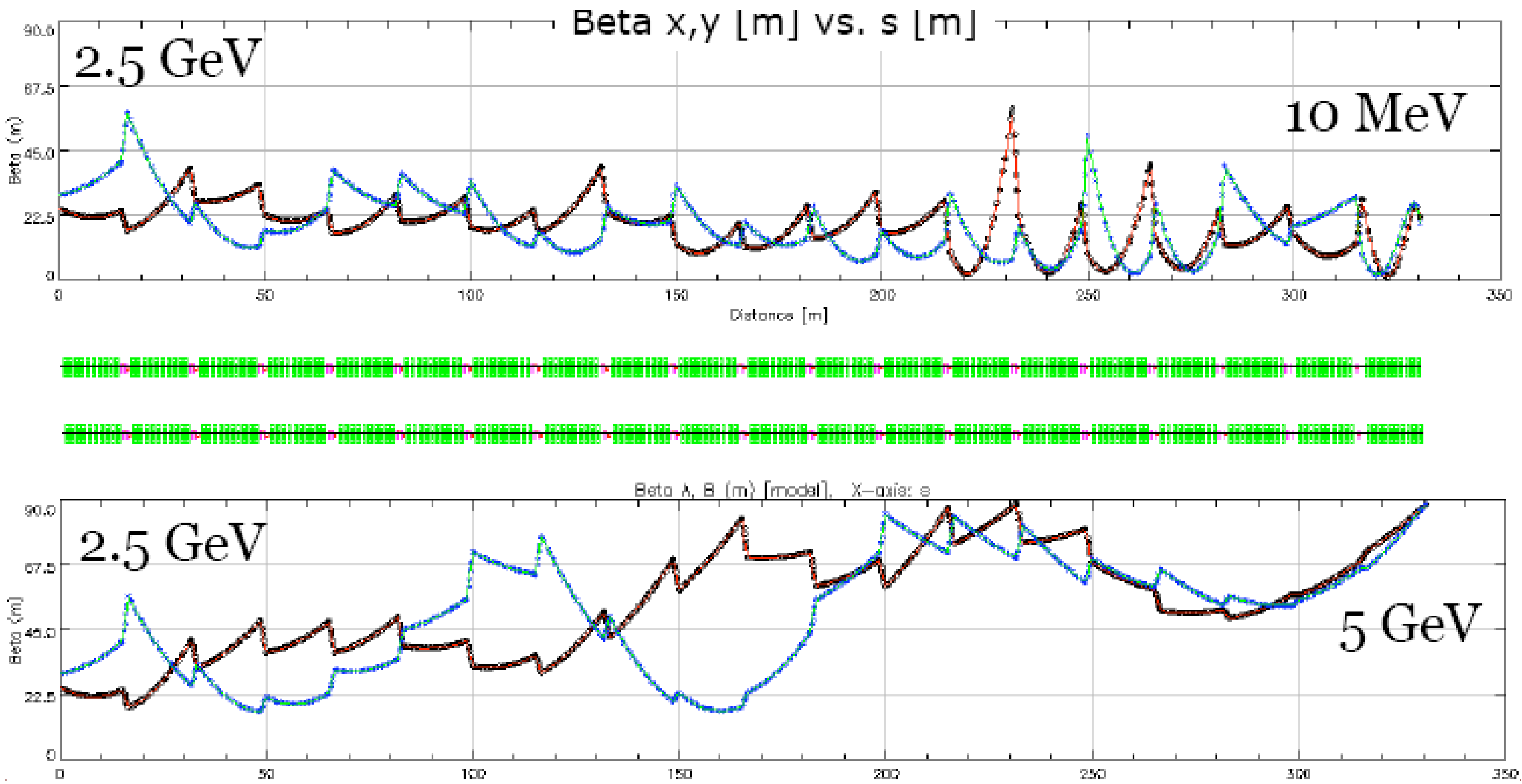


# ERL Optics – The Linacs





# The Dipole Equation with Time of Flight



$$x'' = -x\,\kappa^2 + \delta\kappa$$

$$y'' = 0$$

$$\tau' = -x\,\kappa$$

Homogeneous solution:

$$x_H'' = -x_H\,\kappa^2 \quad \Rightarrow \quad x_H = A\cos(\kappa s) + B\sin(\kappa s)$$ (natural ring focusing)

Variation of constants:

$$x = A(s)\cos(\kappa s) + B(s)\sin(\kappa s)$$

$$x' = -A\kappa\sin(\kappa s) + B\kappa\cos(\kappa s) + \underbrace{A'\cos(\kappa s) + B'\sin(\kappa s)}_{\equiv 0}$$

$$x'' = -\kappa^2 x \underbrace{- A'\kappa\sin(\kappa s) + B'\kappa\cos(\kappa s)}_{=\delta\kappa} = -\kappa^2 x + \delta\kappa$$

$$\begin{pmatrix} \cos(\kappa s) & \sin(\kappa s) \\ -\sin(\kappa s) & \cos(\kappa s) \end{pmatrix} \begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} 0 \\ \delta \end{pmatrix}$$

# The Dipole Matrix with Time of Flight



$$\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} \cos(\kappa s) & -\sin(\kappa s) \\ \sin(\kappa s) & \cos(\kappa s) \end{pmatrix} \begin{pmatrix} 0 \\ \delta \end{pmatrix}$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \delta\,\kappa^{-1} \begin{pmatrix} \cos(\kappa s) \\ \sin(\kappa s) \end{pmatrix} + \begin{pmatrix} A_H \\ B_H \end{pmatrix} \quad \text{with} \quad x = A\cos(\kappa s) + B\sin(\kappa s)$$

$$\tau' = -x\kappa$$

$$\underline{M} = \begin{pmatrix} \begin{matrix} \cos(\kappa s) & \frac{1}{\kappa}\sin(\kappa s) \\ -\kappa\sin(\kappa s) & \cos(\kappa s) \end{matrix} & \underline{0} & \begin{matrix} 0 & \kappa^{-1}[1-\cos(\kappa s)] \\ 0 & \sin(\kappa s) \end{matrix} \\ \underline{0} & \begin{matrix} 1 & s \\ 0 & 1 \end{matrix} & \underline{0} \\ \begin{matrix} -\sin(\kappa s) & \kappa^{-1}[\cos(\kappa s)-1] \\ 0 & 0 \end{matrix} & \underline{0} & \begin{matrix} 1 & \kappa^{-1}[\sin(\kappa s)-s\kappa] \\ 0 & 1 \end{matrix} \end{pmatrix}$$

# Time of Flight from Symplecticity



$$\underline{M} = \begin{pmatrix} \underline{M}_4 & \vec{0} & \vec{D} \\ \vec{T}^T & 1 & M_{56} \\ \vec{0}^T & 0 & 1 \end{pmatrix}$$ is in SU(6) and therefore $\underline{M}\,\underline{J}\,\underline{M}^T = \underline{J}$

$$\begin{pmatrix} \underline{M}_4 \underline{J}_4 & -\vec{D} & \vec{0} \\ \vec{T}^T \underline{J}_4 & -M_{56} & 1 \\ \vec{0}^T & -1 & 0 \end{pmatrix} \begin{pmatrix} \underline{M}_4^T & \vec{T} & \vec{0} \\ \vec{0}^T & 1 & 0 \\ \vec{D}^T & M_{56} & 1 \end{pmatrix} = \begin{pmatrix} \underline{J}_4 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \underline{M}_4 \underline{J}_4 \underline{M}_4^T & \underline{M}_4 \underline{J}_4 \vec{T} - \vec{D} & \vec{0} \\ \vec{T}^T \underline{J}_4 \underline{M}_4^T + \vec{D}^T & 0 & 1 \\ \vec{0}^T & -1 & 0 \end{pmatrix} = \begin{pmatrix} \underline{J}_4 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\vec{T} = -\underline{J}_4 \underline{M}_4^{-1} \vec{D}$$

It is sufficient to compute the 4D map $\underline{M}_4$, the Dispersion $\vec{D}$ and the time of flight term $M_{56}$

# Time of Flight and Bunch Compression

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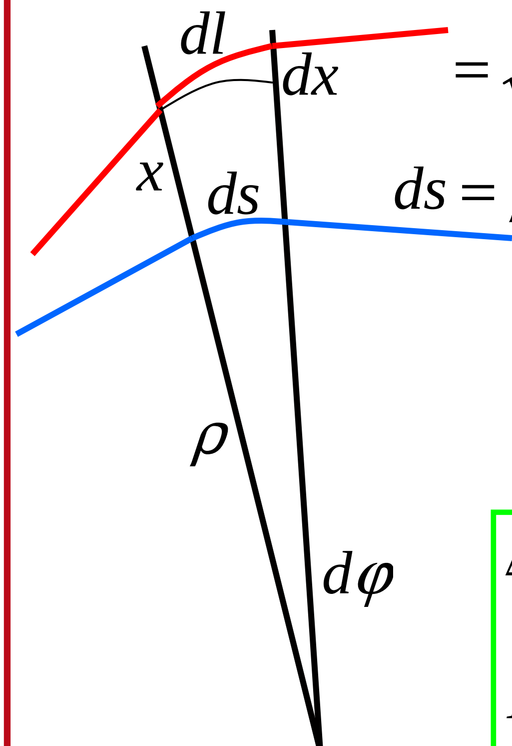
$$dl = \sqrt{dx^2 + dy^2 + [(\rho + x)d\varphi]^2}$$

$$= \sqrt{x'^2 + y'^2 + (1+\tfrac{x}{\rho})^2}\, ds = (1+\tfrac{x}{\rho})ds + O^2$$

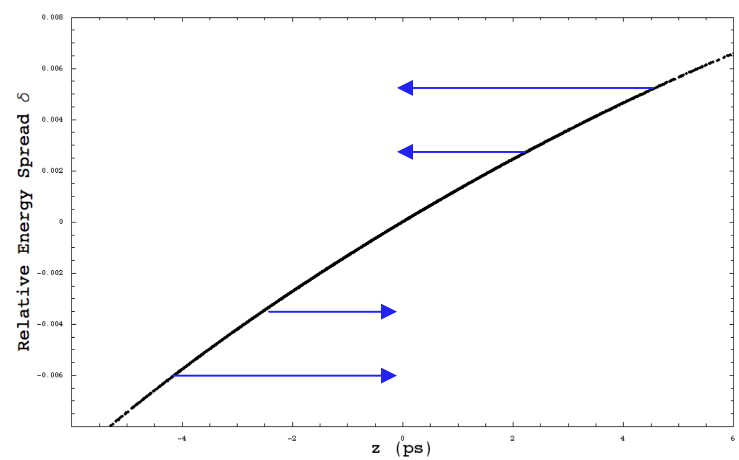



$$ds = \rho\, d\varphi$$

$$\Delta l = \int_0^L \frac{x(s)}{\rho} ds = x_0 \int_0^L \frac{M_{11}(s)}{\rho} ds + x'_0 \int_0^L \frac{M_{12}(s)}{\rho} ds + \delta \int_0^L \frac{D(s)}{\rho} ds$$

$$\Delta l = R_{56}\delta$$

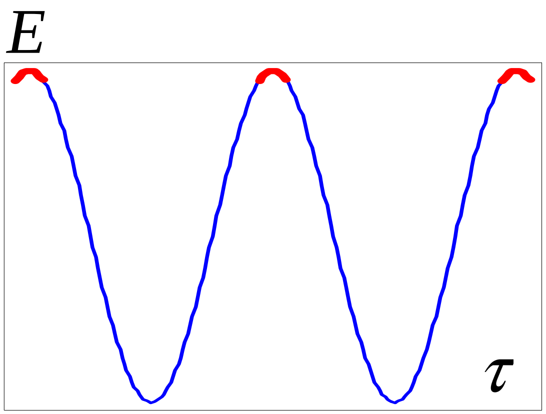
$$R_{56} = \int_0^L \frac{D(s)}{\rho} ds$$




Energy - position correlation leads to bunch compression

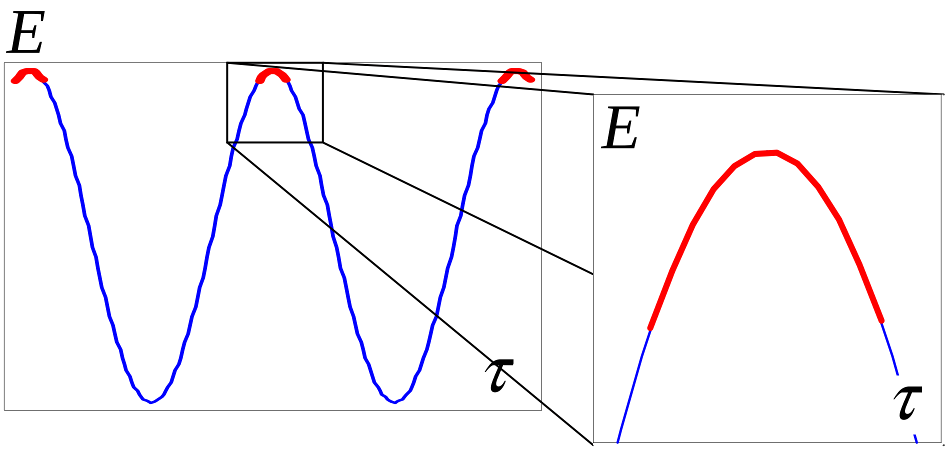
# Bunch Compression





# Bunch Compression

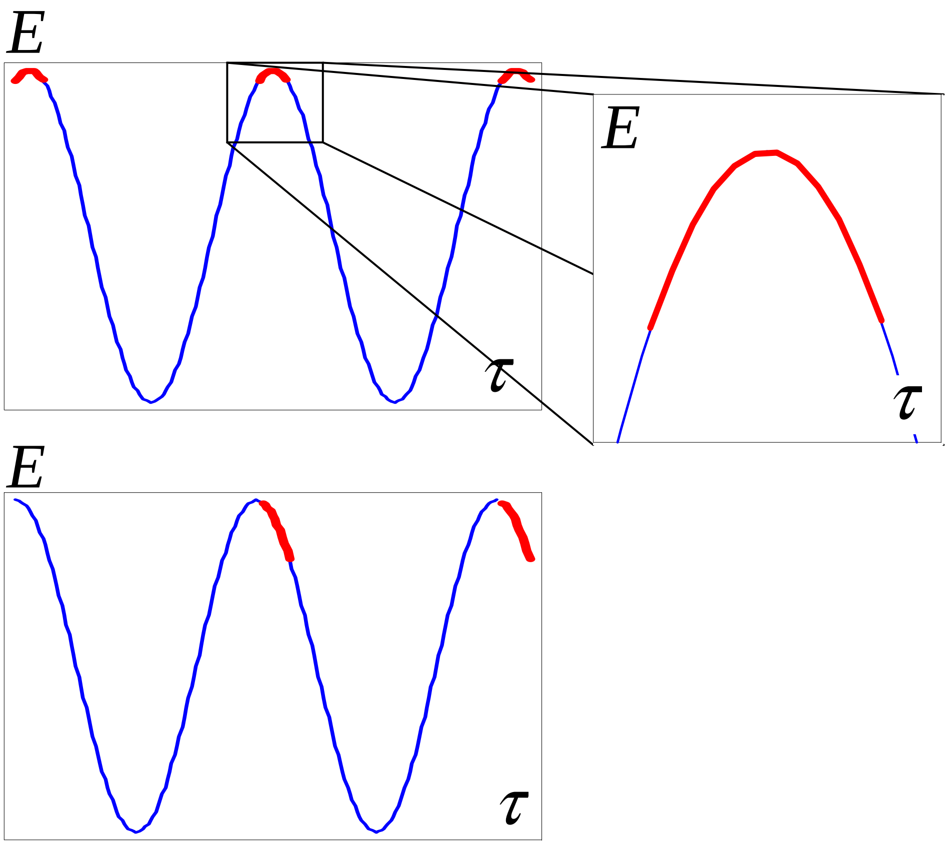





- On crest acceleration leads to long Bunches with small energy spread.

# Nonlinear Bunch Compression

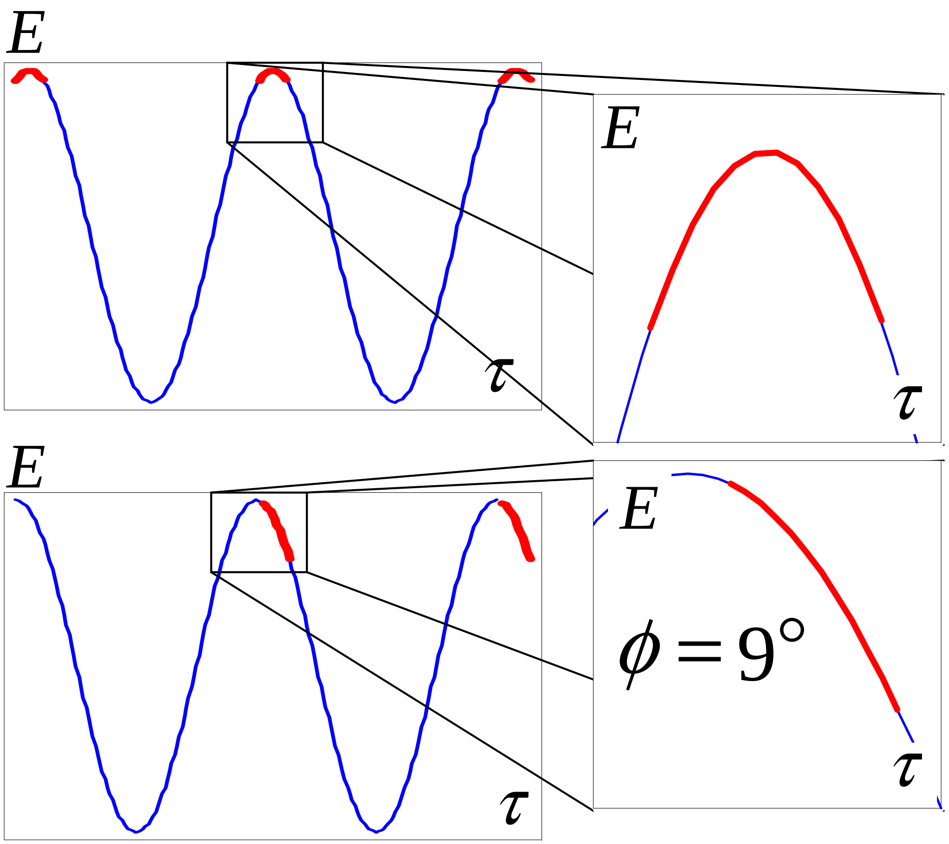





- **On crest acceleration leads to long Bunches with small energy spread.**

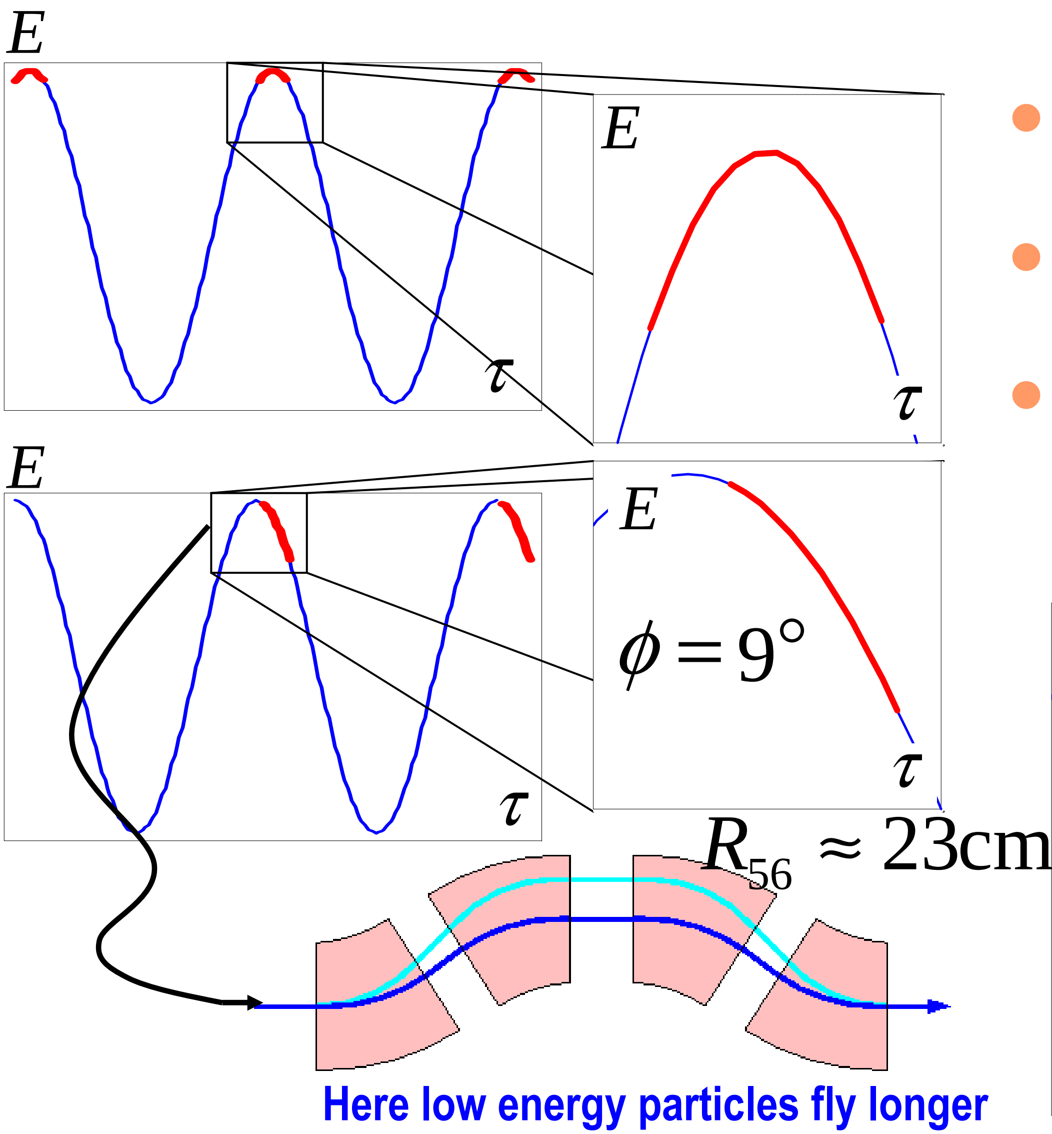
# Nonlinear Bunch Compression



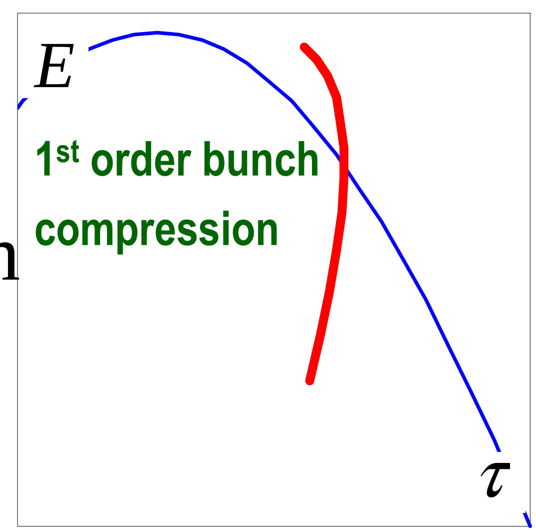



- On crest acceleration leads to long Bunches with small energy spread.
- Off crest acceleration leads to short Bunches with more energy spread.

# Nonlinear Bunch Compression

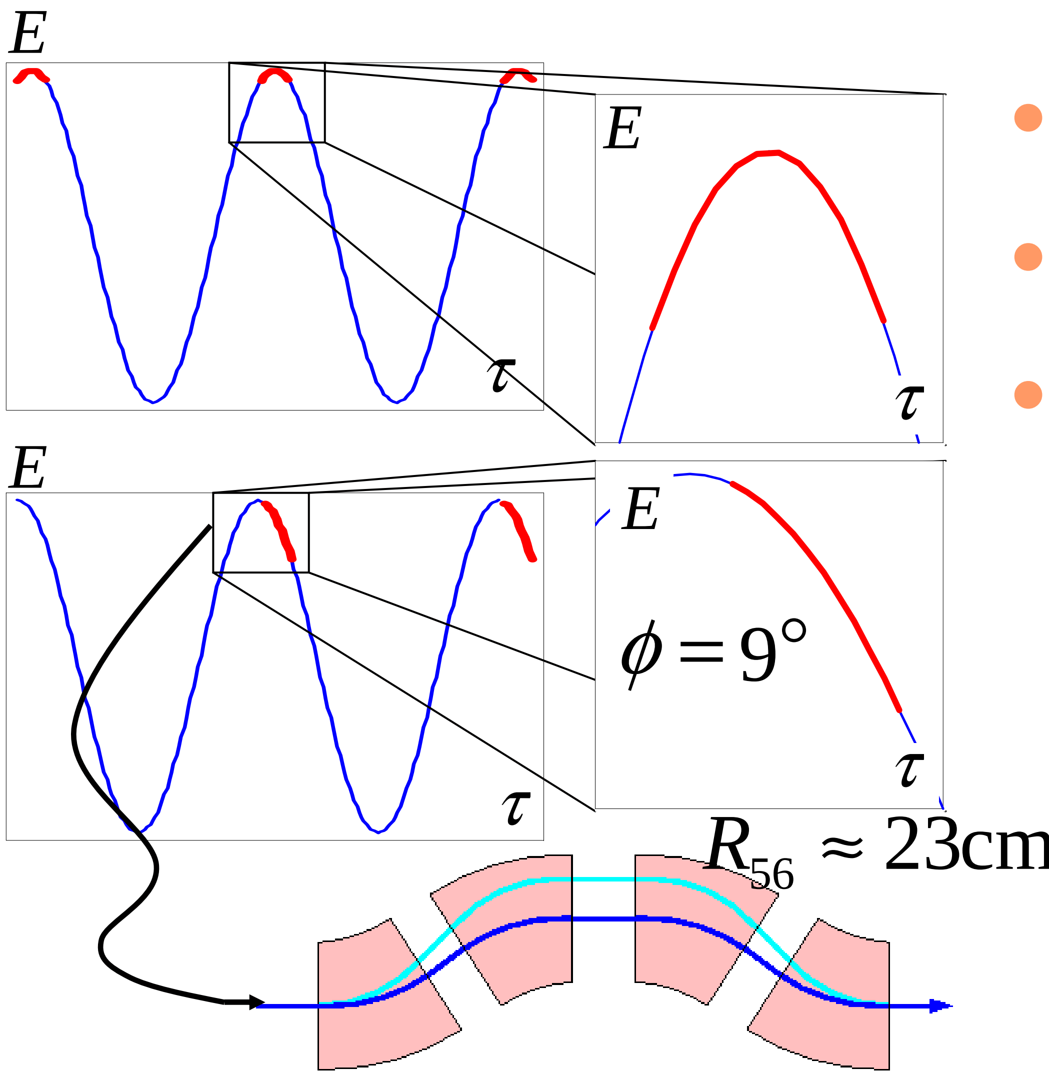



- On crest acceleration leads to long Bunches with small energy spread.
- Off crest acceleration leads to short Bunches with more energy spread.
- The bunch length can be made even shorter by nonlinear optics



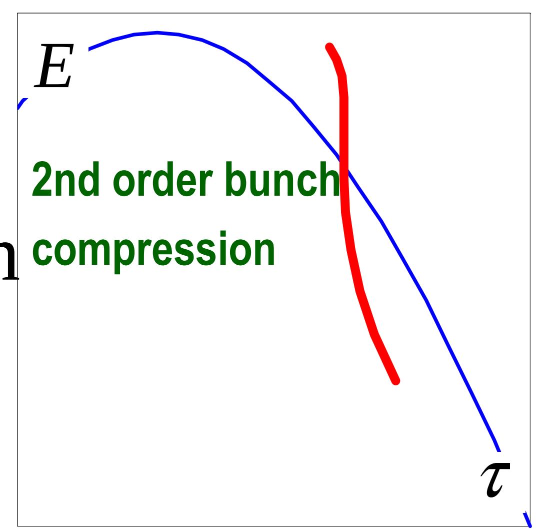
# Nonlinear Bunch Compression





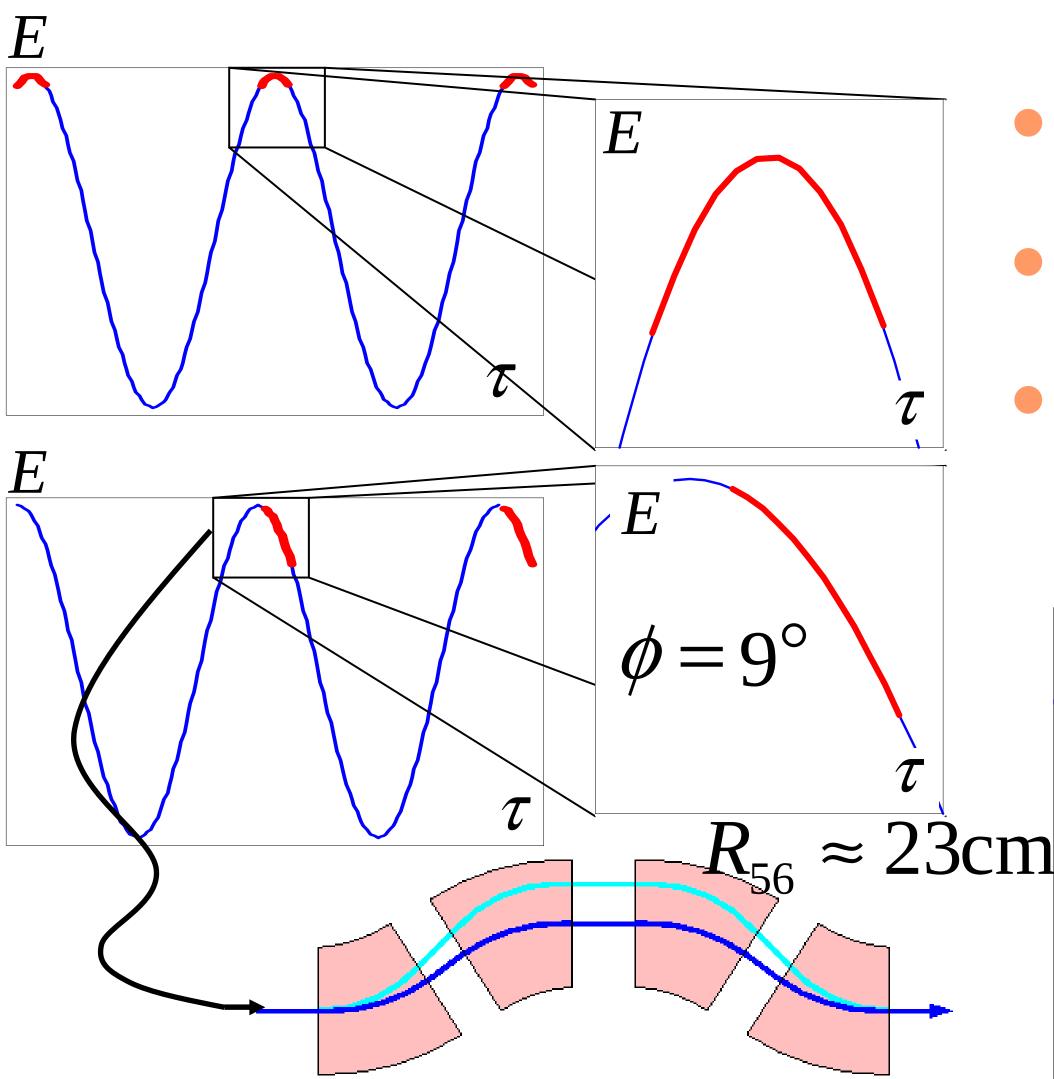

Here low energy particles fly longer

- On crest acceleration leads to long Bunches with small energy spread.
- Off crest acceleration leads to short Bunches with more energy spread.
- The bunch length can be made even shorter by nonlinear optics



# Nonlinear Bunch Compression



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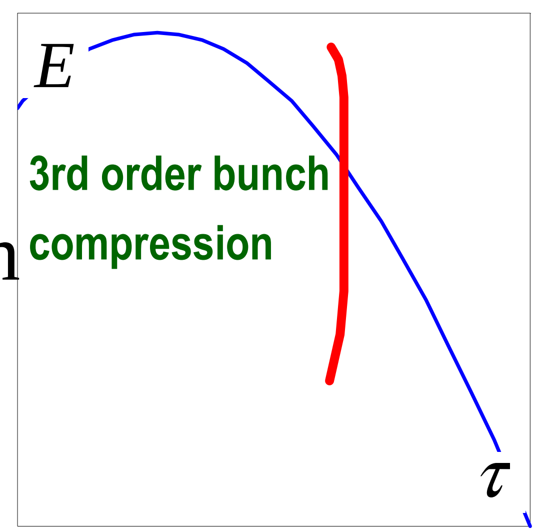
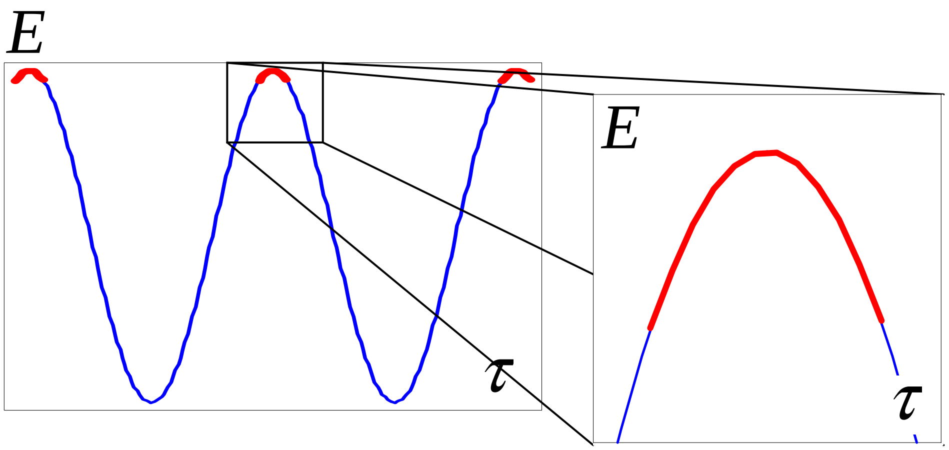
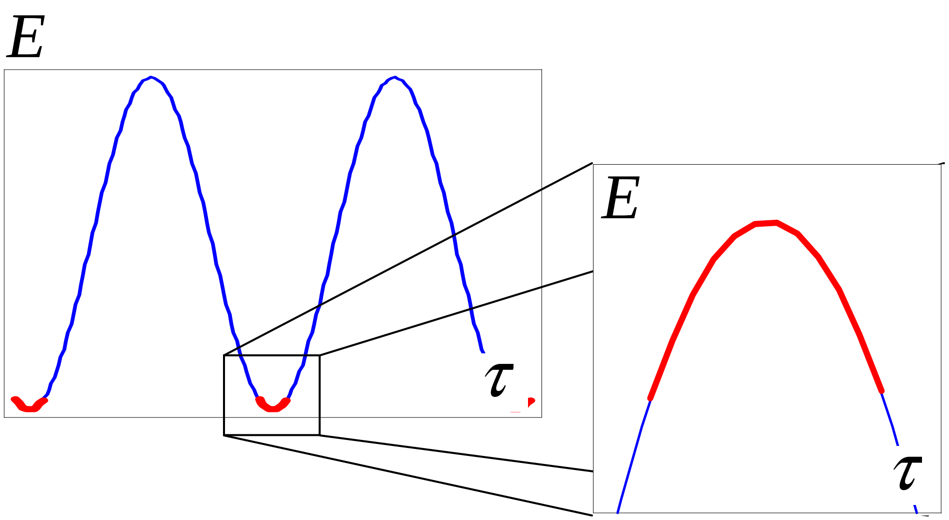


# Nonlinear Bunch Compression





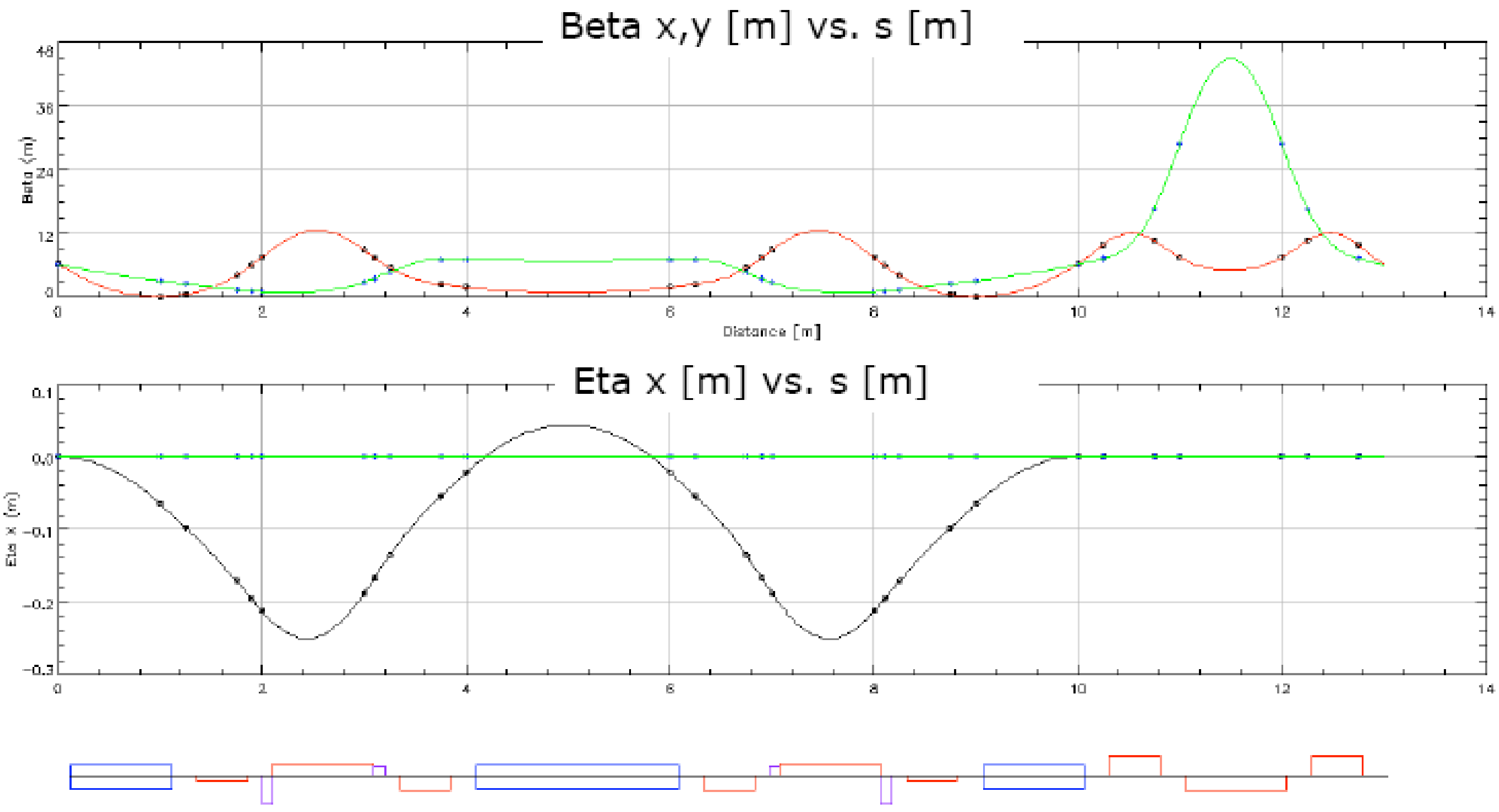

- On crest acceleration leads to long Bunches with small energy spread.




- Bunches must arrive for deceleration with the same shape in longitudinal phase space that they had after acceleration.
- The energy dependent time of flight must be zero. The beam transport is then called isochronous.

# ERL Optics – Isochronous Turn Around





# ERL Optics - isochronous CESR

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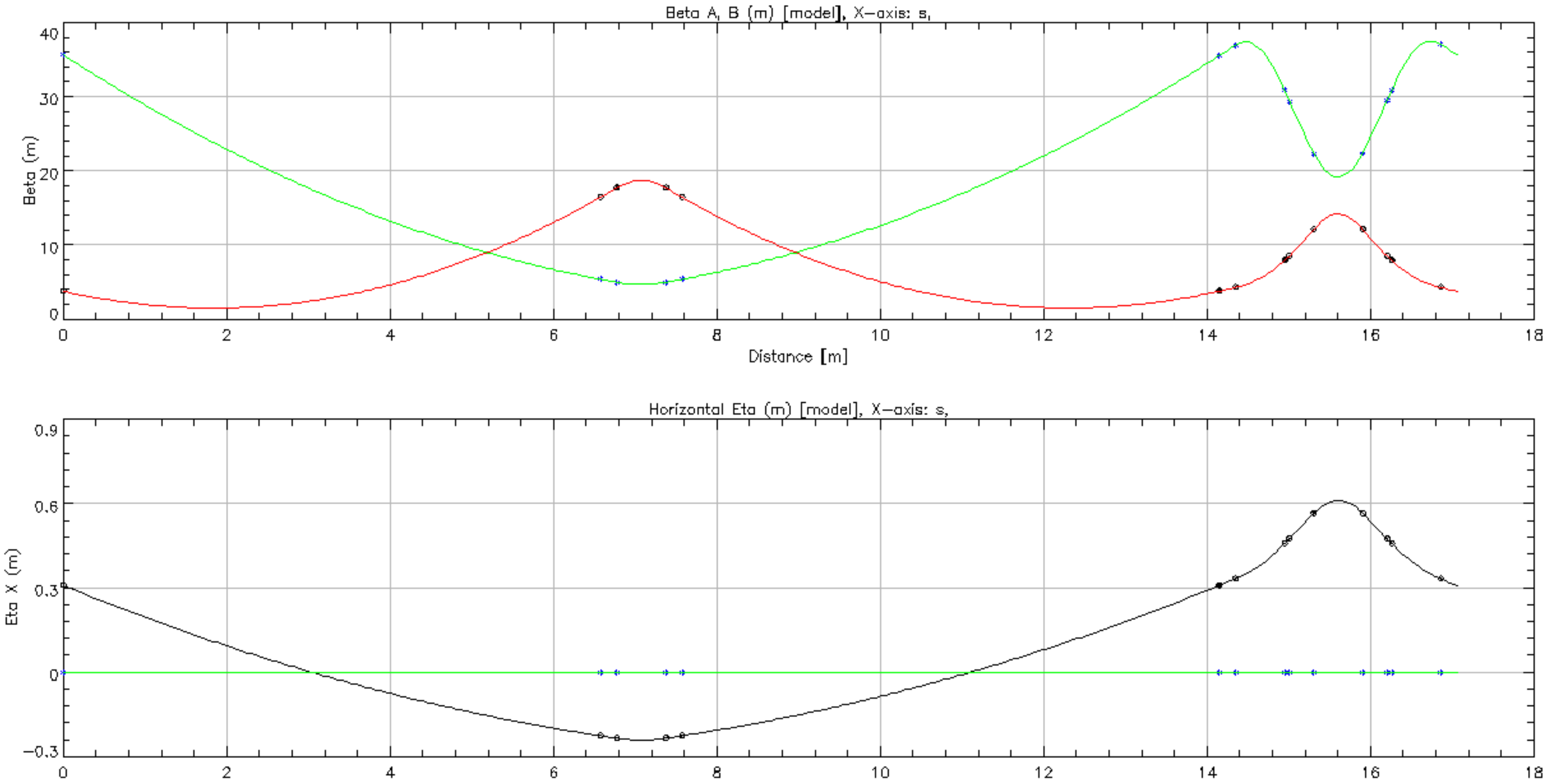
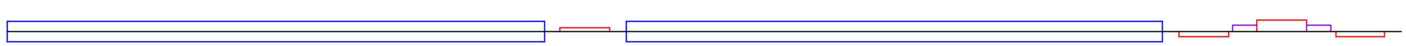
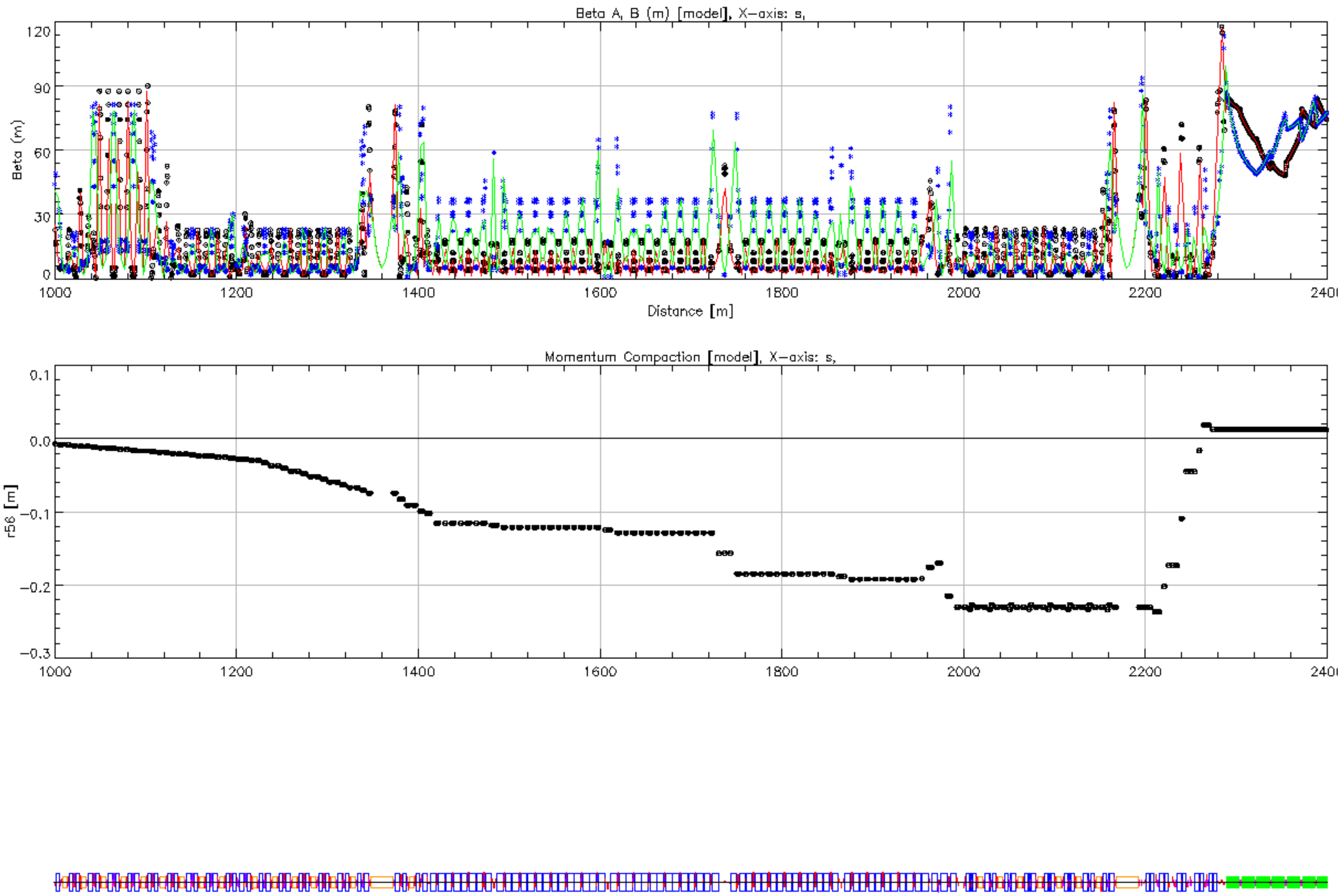


courtesy Christopher Mayes

# ERL Optics – Energy Dependent Time of Flight

CHESS & LEPP



# ERL Optics – Bunch compression in CESR