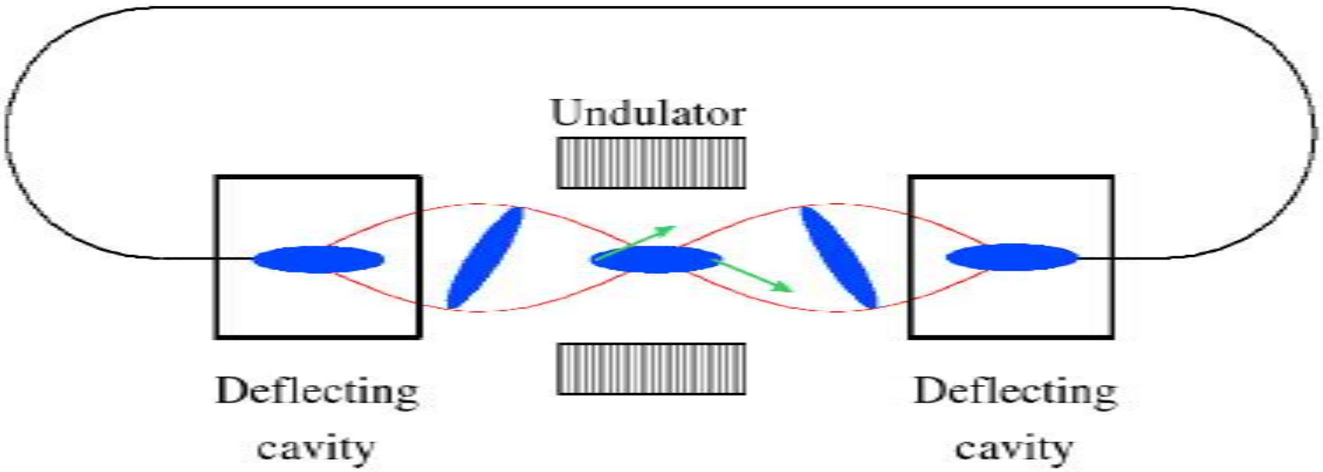
Short Pulses in Rings



1) The time varying fields in a transverse mode cavity kick the front of a bunch up, and the back of the bunch down.
2) A betatron phase advance of π later, the bunch radiates in an undulator
3) The vertical photon angles are correlated with the source point

CHESS & LEPP

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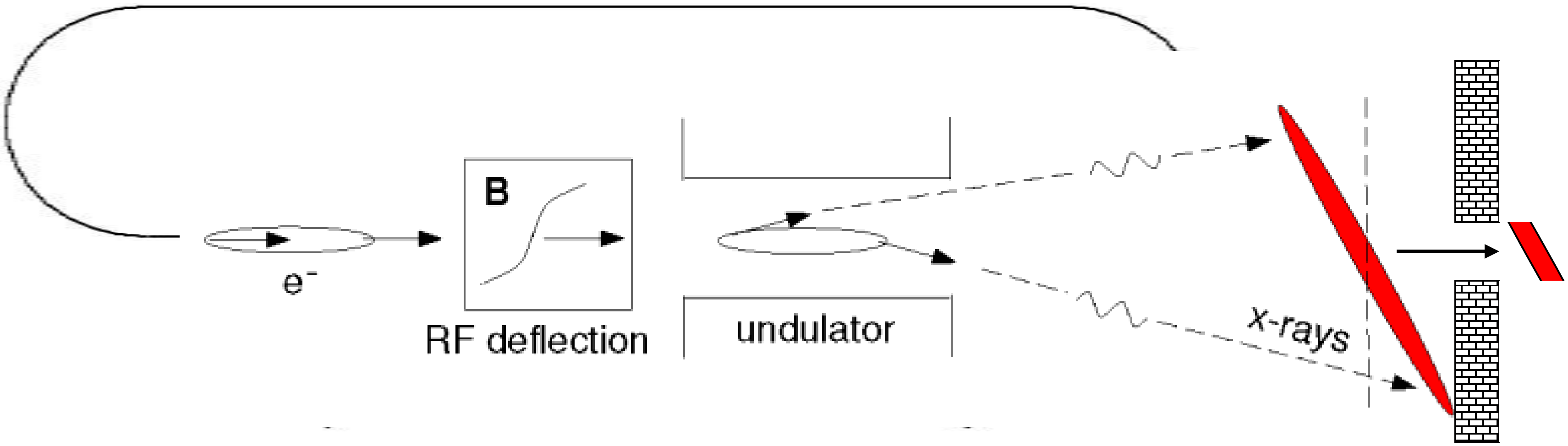

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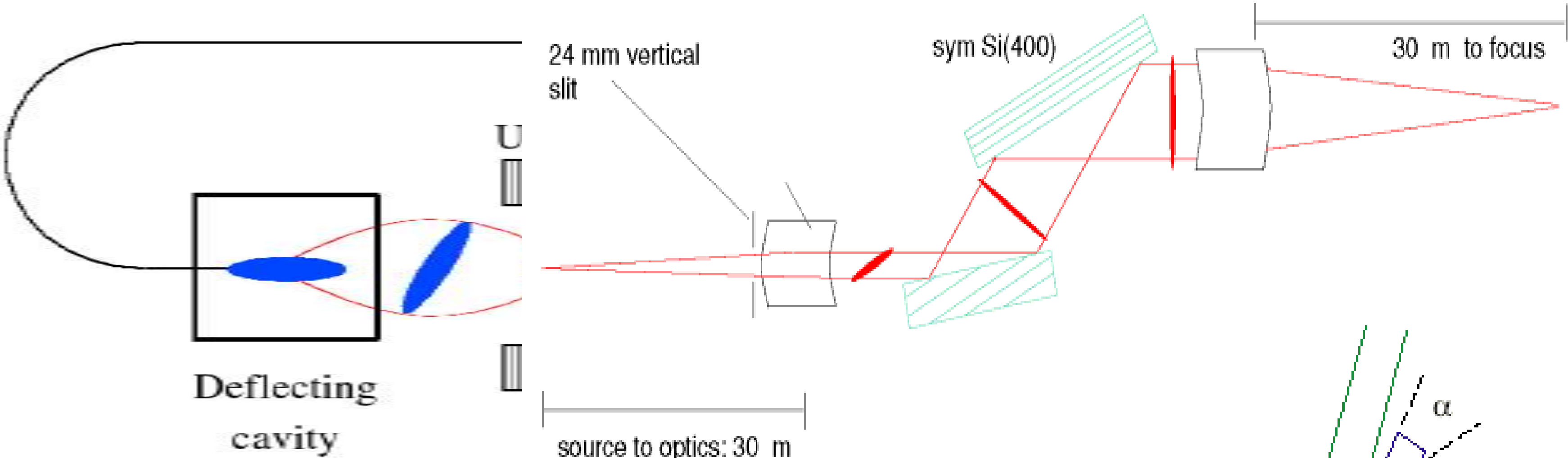
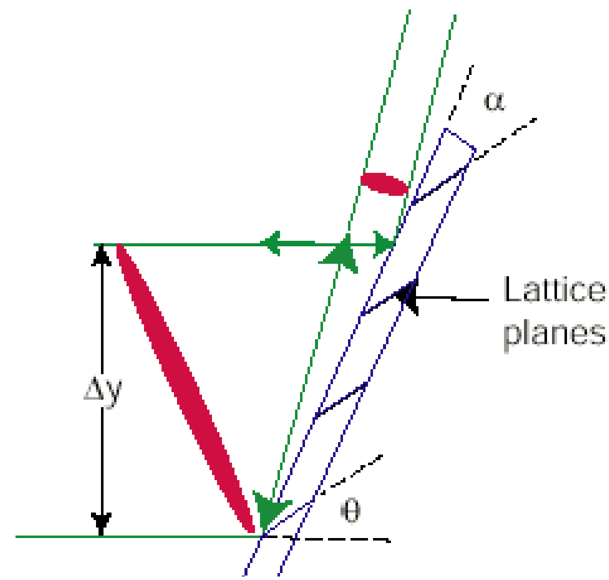
2) A betatron phase advance of π later, the bunch radiates in an undulator
3) The vertical photon angles are correlated with the source point
4) A slit, selecting only a short range of vertical angles, selects photons from a small range of source points along the bunch.
5) A second crab cavity, a betatron phase advance of 2p after the first, kicks the tail up and the front down, compensating the vertical oscilations.
6) The bunch is typically about 100ps long, selecting 1ps reduces the intensity to approximately 1%.

Short Pulses in Rings





X-ray compression in asymmetric-cut crystals

1) Instead of a slit, one can use an x-ray bunch compressor. It produces a time of flight that depends on the vertical angle to eliminate the correlation between vertical angle and source point location.
1) Realistically: transmits up to 5% of beam due to collimation and losses.

Optics 1: Where is the vertical Dipole?

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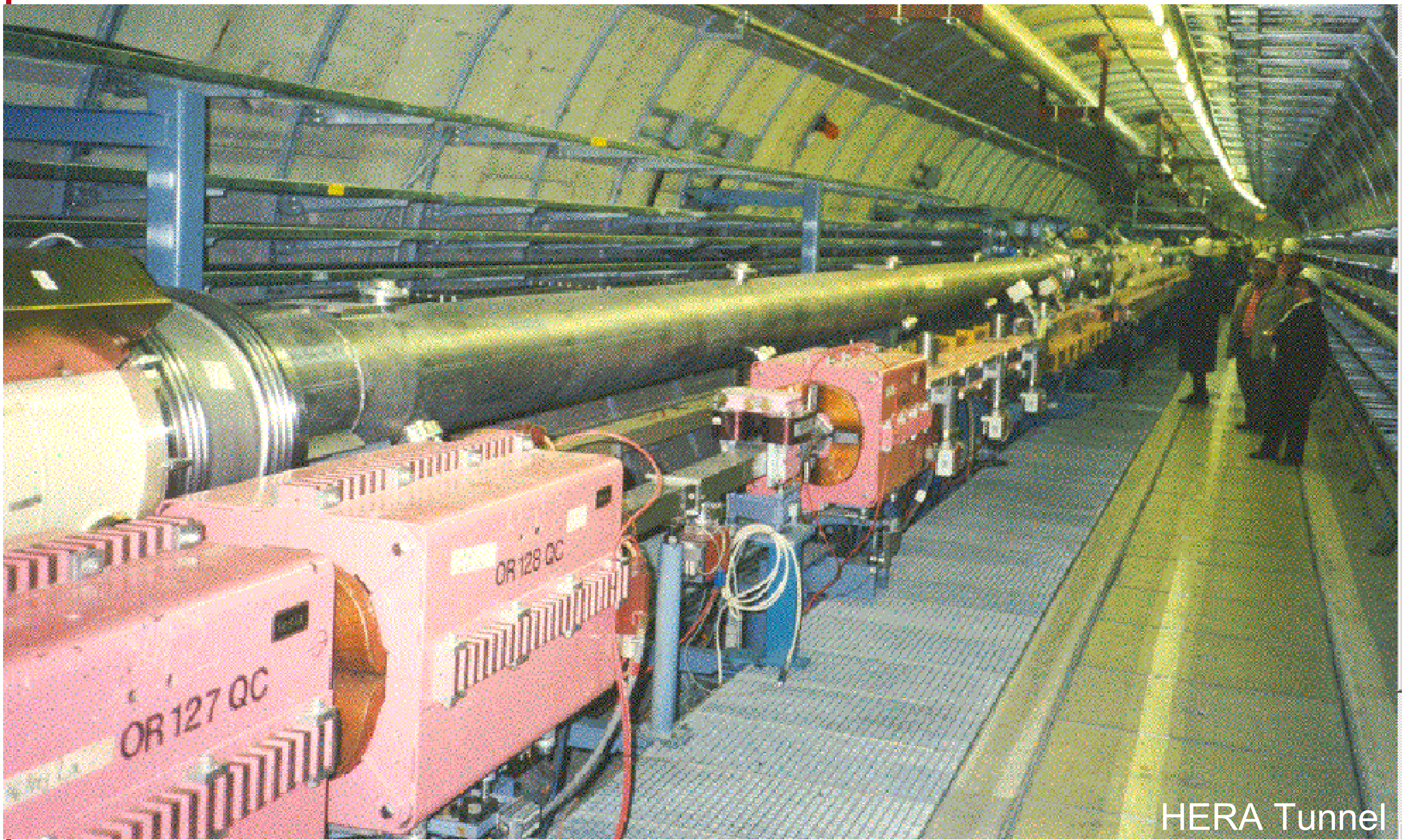
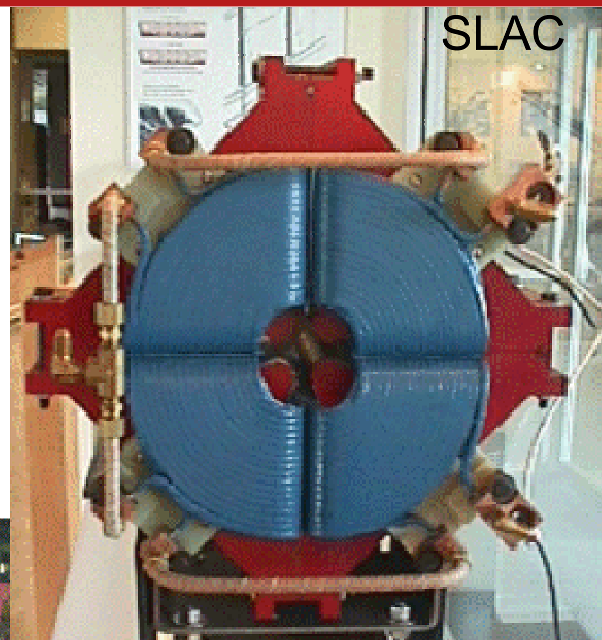


HERA Tunnel

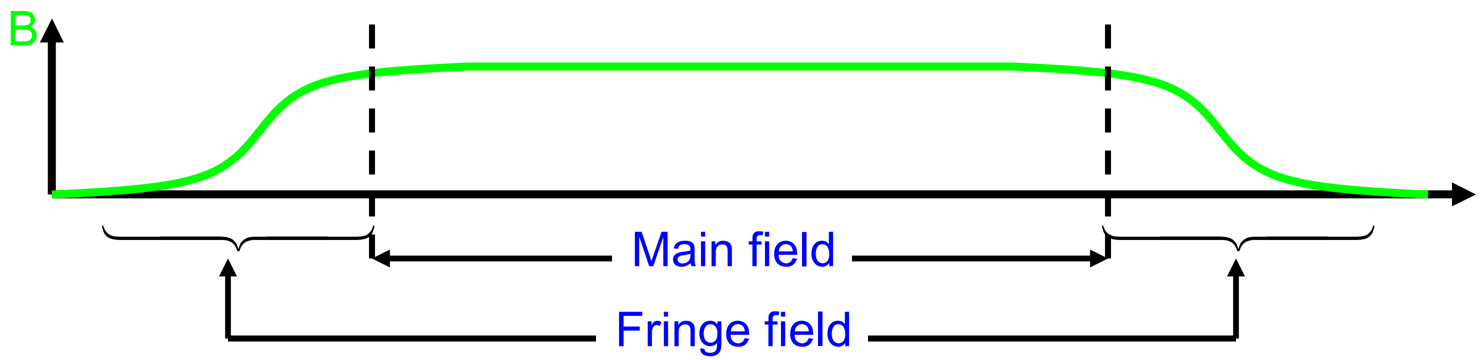
Optics 2: Real Quadrupoles



The coils show that this is an upright quadrupole not a rotated or skew quadrupole.

Optics 3: Real Sextupoles

Fringe Fields and Main Fields



Only the fringe field region has terms with $\partial_z^2 \psi \neq 0$

Main fields in accelerator physics: $\partial_z^2 \psi = 0$

Complex Potentials

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$$w = x + iy \quad , \quad \overline{w} = x - iy$$

$$\partial_x = \partial_w + \partial_{\overline{w}} \ , \quad \partial_y = i\partial_w - i\partial_{\overline{w}} = i(\partial_w - \partial_{\overline{w}})$$

$$\vec{\nabla}^2 = \partial_x^2 + \partial_y^2 + \partial_z^2 = (\partial_w + \partial_{\overline{w}})^2 - (\partial_w - \partial_{\overline{w}})^2 + \partial_z^2 = 4\partial_w\partial_{\overline{w}} + \partial_z^2$$

$$\psi = \mathrm{Im}\{\sum_{\nu,\lambda=0}^{\infty} a_{\nu\lambda}(z)\cdot(w\overline{w})^{\lambda}\,\overline{w}^{\nu}\} \approx \mathrm{Im}\{\sum_{\nu,\lambda=0}^{\infty} a_{\nu\lambda}\cdot(w\overline{w})^{\lambda}\,\overline{w}^{\nu}\}$$

$$\vec{\nabla}^2\psi = \mathrm{Im}\{\sum_{\nu=0,\lambda=1}^{\infty} 4a_{\nu\lambda}(\lambda+\nu)\lambda(w\overline{w})^{\lambda-1}\,\overline{w}^{\nu}\} = 0$$

Iteration equation: $a_{\nu\lambda} = 0 \text{ for } \lambda \geq 1 \quad , \quad a_{\nu 0} = \Psi_\nu$

The functions Ψ_ν determine the complete field inside a magnet.

Fringe Fields and Main Fields

Only the fringe field region has terms with $\partial_z^2 \psi \neq 0$

Main fields in accelerator physics: $\partial_z^2 \psi = 0$

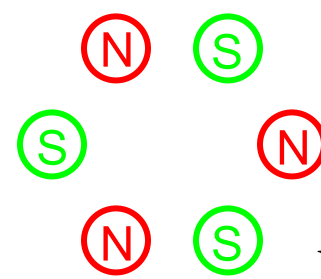
$$\psi \quad = \mathrm{Im}\{\sum_{\nu=1}^{\infty} \Psi_\nu \bar{w}^\nu\}$$

Nice way to derive multipole fields

$$\psi(r,\varphi) = \sum_{\nu=1}^{\infty} r^\nu |\Psi_\nu| \mathrm{Im}\{e^{-i\nu(\varphi-\vartheta_\nu)}\}$$

Relation between radial power and azimuthal symmetry !

The index ν describes C_ν Symmetry around the z-axis $\vec{e}_z$ due to a sign change after $\Delta\varphi = \frac{\pi}{\nu}$



$\nu = 3$

Multipoles in Accelerators ν=1: Dipoles

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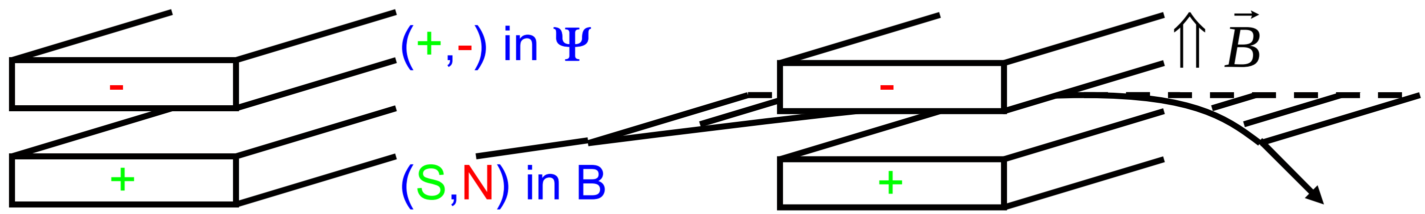
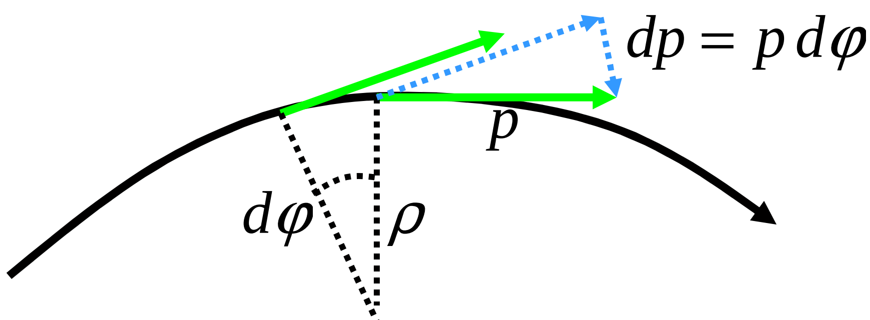
$$\psi = \Psi_1 \, \mathrm{Im}\{x - iy\} = -\Psi_1 \cdot y \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_1 \, \vec{e}_y$$

Equipotential
$y = \mathrm{const.}$

C_1 Symmetry



Dipole magnets are used for steering the beams direction



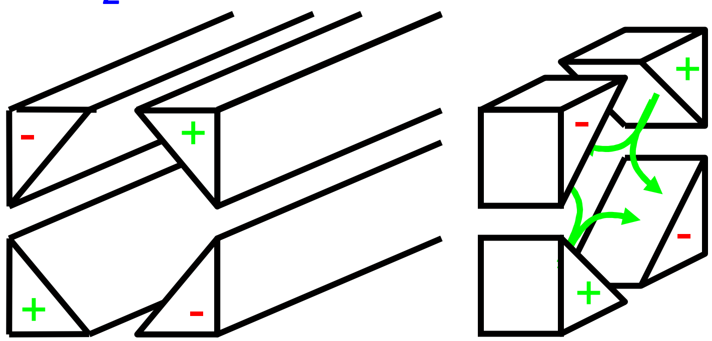
$$\frac{d\vec{p}}{dt} = q\vec{v} \times \vec{B} \quad \Rightarrow \quad \frac{dp}{dt} = qvB_\perp \quad \Rightarrow \quad \rho = \frac{dl}{d\varphi} = \frac{vdt}{dp/p} = \frac{p}{qB_\perp}$$

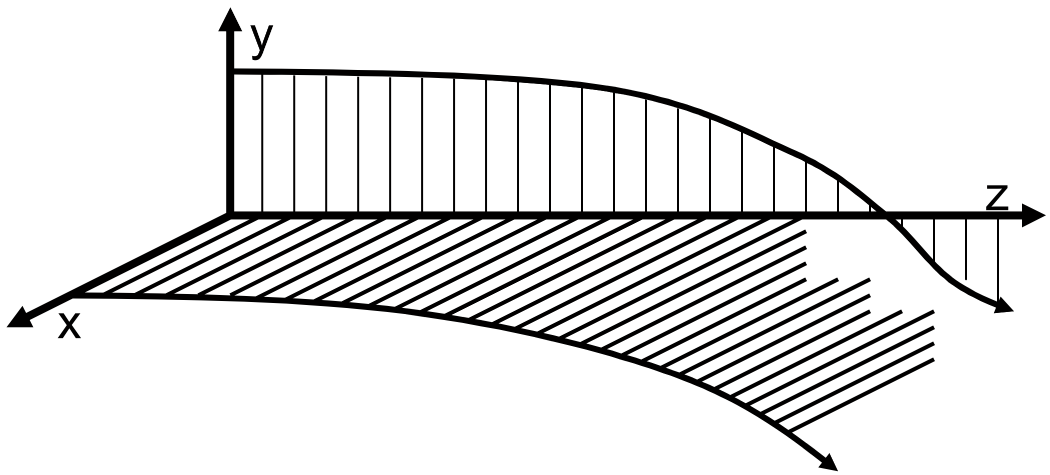
Bending radius: $\rho = \dfrac{p}{qB}$

Multipoles in Accelerators ν=2: Quadrupoles

$$\psi = \Psi_2 \operatorname{Im}\{(x-iy)^2\} = -\Psi_2 \cdot 2xy \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_2\, 2\begin{pmatrix} y \\ x \end{pmatrix}$$

C_2 Symmetry



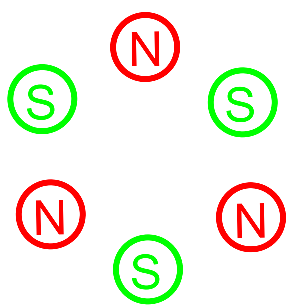


In a quadrupole particles are focused in one plane and defocused in the other plane. Other modes of strong focusing are not possible.

Nonlinear Optics - Sextupoles

$$\psi = \Psi_3\, \mathrm{Im}\{(x - iy)^3\} = \Psi_3 \cdot (y^3 - 3x^2 y) \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_3\, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix}$$

C_3 Symmetry



i) Sextupole fields hardly influence the particles close to the center, where one can linearize in x and y.

ii) In linear approximation a by Δx shifted sextupole has a quadrupole field.

iii) When Δx depends on the energy, one can build an energy dependent quadrupole.

$$\vec{B} = -\vec{\nabla}\psi = \Psi_3\, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix}$$

$$x \mapsto \Delta x + x$$

$$\vec{B} \approx \Psi_3\, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix} + 6\Psi_3 \Delta x \begin{pmatrix} y \\ x \end{pmatrix} + O(\Delta x^2)$$

$$k_2 = 3!\,\Psi_3 \Rightarrow k_1 = k_2 \Delta x$$

Second-Order Dispersion

$$x''+(k_1+\kappa^2)x=0$$

First order in x, x'

$$x''+(k_1+\kappa^2)x=f_1 \qquad f_1(\delta)=\kappa\delta$$

First order in x, x', δ

$$D''\delta+K\cdot D\delta=f_1(\delta) \quad \Rightarrow \quad D''+K\cdot D=\kappa$$

$$D=\int_0^s\hat{\kappa}\sqrt{\beta\hat{\beta}}\sin(\psi-\hat{\psi})d\hat{s}$$

$$x''+(k_1+\kappa^2)x=f_1+f_2$$

Second order in x, x', δ

$$f_2=-\kappa(\delta^2-\tfrac{1}{2}x'^2-2\kappa x\delta+\kappa^2x^2)+k_1x(\delta-2\kappa x)-\tfrac{1}{2}k_2x^2=f_2(x,x',\delta)$$

The energy dependent disperion: $D_2''+K\cdot D_2=f_2(D,D',1)$

$$\Delta D=-\int_0^s[f_2]\sqrt{\beta\hat{\beta}}\sin(\psi-\hat{\psi})d\hat{s}$$