First-Order Dispersion

$$x''+(k_1+\kappa^2)x=0$$

First order in x, x'

$$x''+K\cdot x=f_1 \qquad f_1(\delta)=\kappa\delta$$

First order in x, x', δ

$$\boxed{x''+K\cdot x=\kappa\delta+f_2(x,x',\delta)}$$

Second order in x, x', δ

$$x_\delta(\delta)=D\delta+D_2\delta^2+O^3(\delta)$$

Trajectory of a particle that starts as designed, but has relative energy deviation δ.

$$\boxed{(D''+K\cdot D)\delta+(D_2''+K\cdot D_2)\delta^2=\kappa\delta+f_2(D,D',1)\delta^2}$$

First-order dispersion:

$$\boxed{D''+K\cdot D=\kappa \quad \Rightarrow \quad D=\int_0^s \hat{\kappa}\sqrt{\beta\hat{\beta}}\sin(\psi-\hat{\psi})d\hat{s}}$$

Second-Order Dispersion

$$x''+(k_1+\kappa^2)x=0$$

First order in x, x'

$$x''+K\cdot x=f_1 \qquad f_1(\delta)=\kappa\delta$$

First order in x, x', δ

$$x''+K\cdot x=\kappa\delta+f_2(x,x',\delta)$$

Second order in x, x', δ

$$x_\delta(\delta)=D\delta+D_2\delta^2+O^3(\delta)$$

Trajectory of a particle that starts as designed, but has relative energy deviation δ.

$$(D''+K\cdot D)\delta+(D_2''+K\cdot D_2)\delta^2=\kappa\delta+f_2(D,D',1)\delta^2$$

First-order dispersion:

$$D_2''+K\cdot D_2=f_2(D,D',1) \quad \Rightarrow \quad D_2=\int_0^s \hat{f}_2(\hat{D},\hat{D}',1)\sqrt{\beta\hat{\beta}}\sin(\psi-\hat{\psi})d\hat{s}$$

Second-Order Achromats

$$x''+K\cdot x=\kappa\delta+f_2(x,x',\delta)$$

Second order in x, x', δ

$$f_2=-\kappa(\delta^2-\tfrac{1}{2}x'^2-2\kappa x\delta+\kappa^2x^2)+k_1x(\delta-2\kappa x)-\tfrac{1}{2}k_2x^2=f_2(x,x',\delta)$$

Dominant parts: $\kappa D\le 10^{-3},\ D'\le 10^{-1}$

$$f_2(D,D',1)\approx-\kappa+k_1D\delta-\tfrac{1}{2}k_2D^2$$

First and second-order dispersion: $D=\int_0^s\hat\kappa\sqrt{\beta\hat\beta}\sin(\psi-\hat\psi)d\hat s$

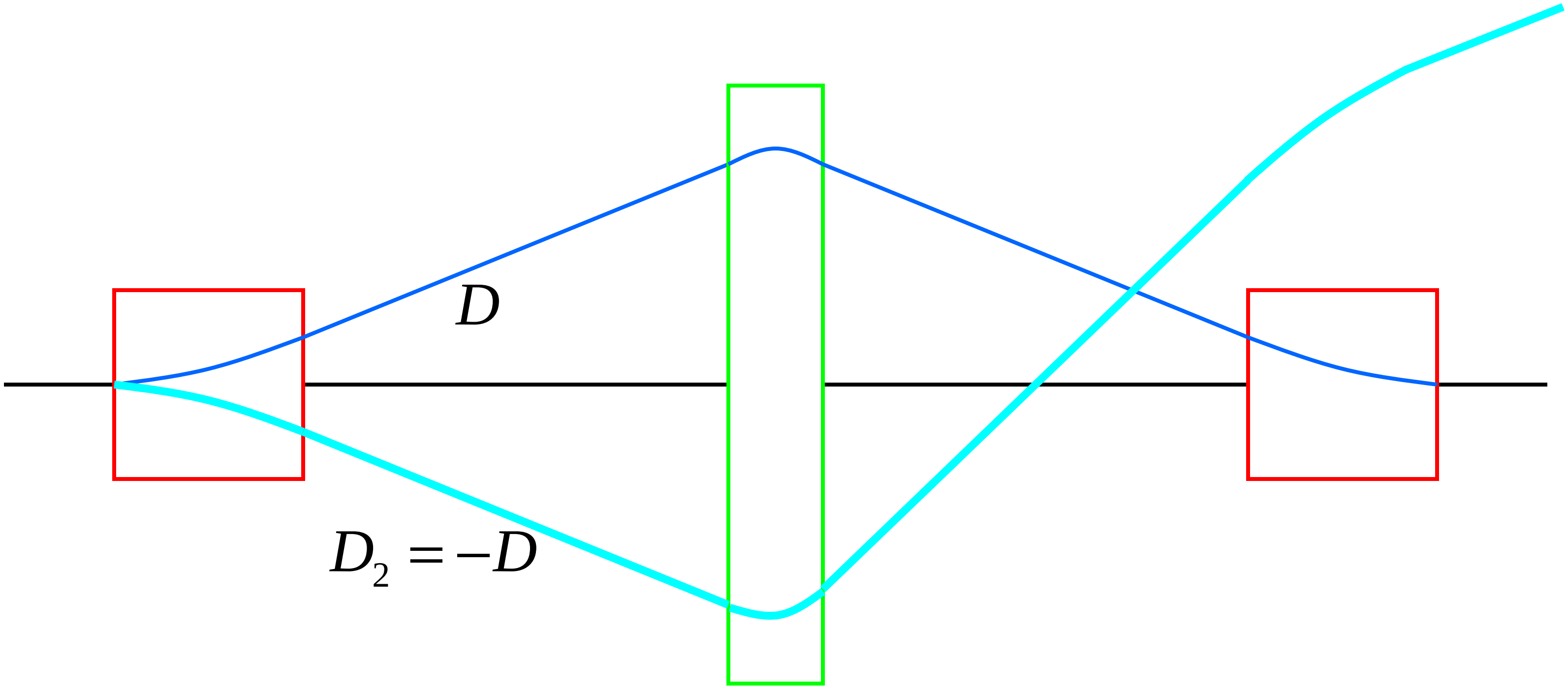
$$D_2=\int_0^s\hat f_2(\hat D,\hat D',1)\sqrt{\beta\hat\beta}\sin(\psi-\hat\psi)d\hat s$$

In the first and last dipole of an achromat: $D_2=-D$

In the first and last quadrupole: $D_2''+2k_1D_2\approx 0$

In sextupoles: $\Delta D_2'\approx-\tfrac{1}{2}k_2L_2D^2$

Example of 2nd Order Achromat Design

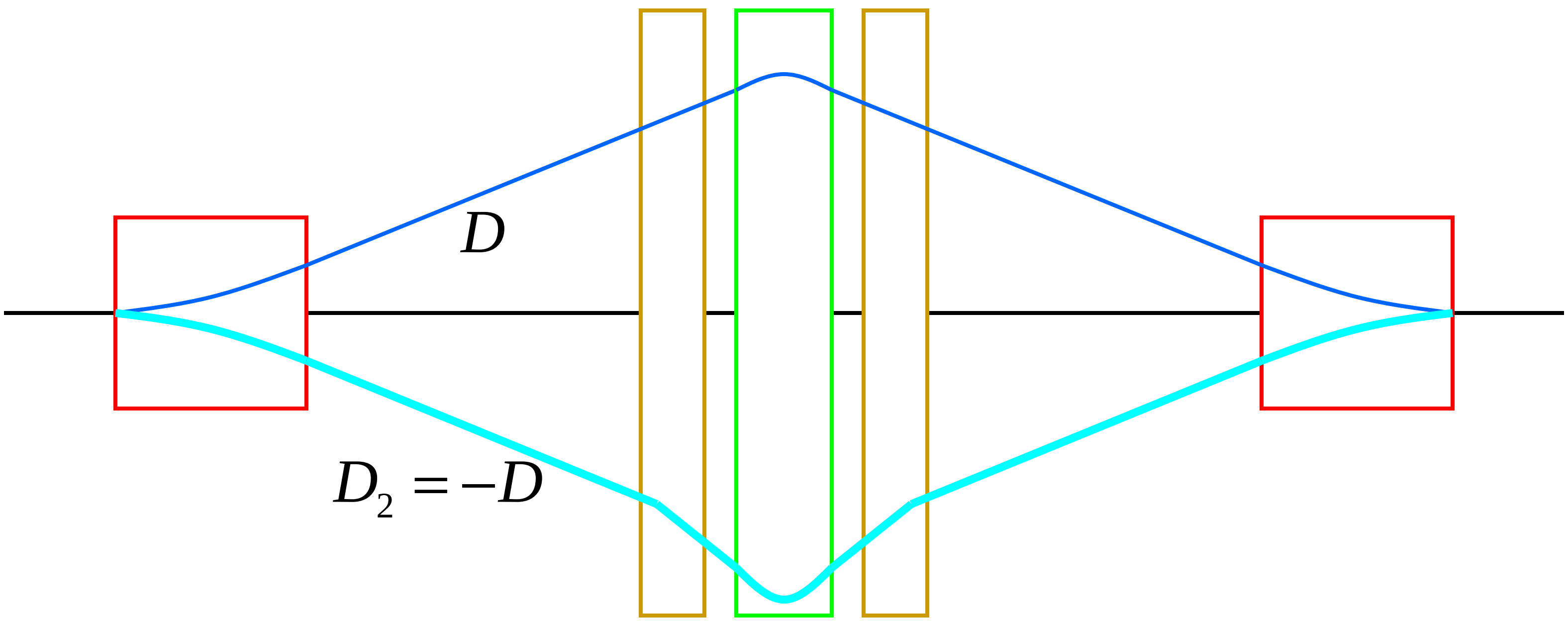


In the first and last dipole of an achromat: $D_2 = -D$

In the first and last quadrupole: $D_2'' + 2k_1 D_2 \approx 0$

In sextupoles: $\Delta D_2' \approx -\frac{1}{2} k_2 L_2 D^2$

Example of 2nd Order Achromat Design



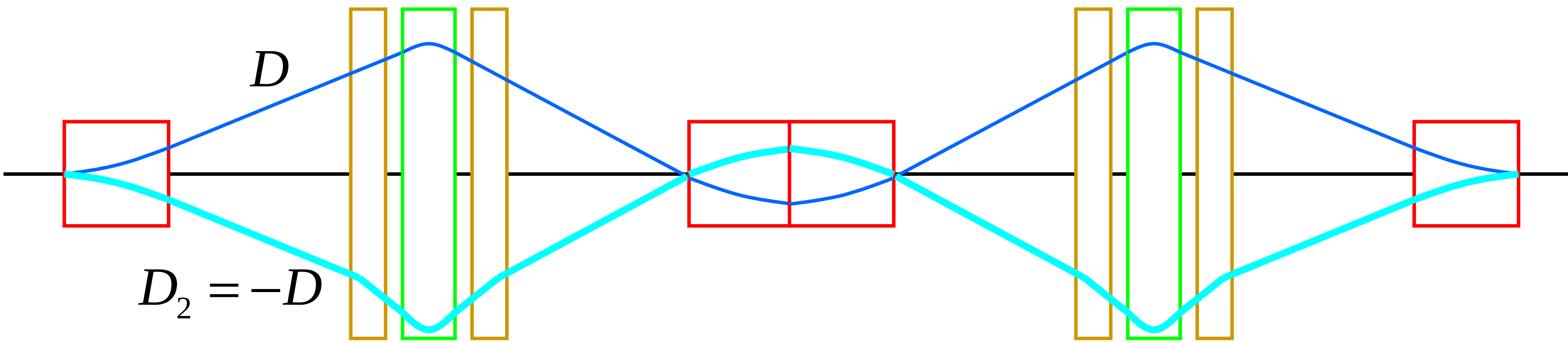
A second order achromat generally needs two sextupoles, for making $D_2 = 0, D_2' = 0$

In the first and last dipole of an achromat: $D_2 = -D$

In the first and last quadrupole: $D_2'' + 2k_1 D_2 \approx 0$

In sextupoles: $\Delta D_2' \approx -\frac{1}{2} k_2 L_2 D^2$

2^{nd} Order Isochronous Achromats



$$x'' = -x\,(\underbrace{\kappa^2 + k}_{K}) + \delta\kappa$$

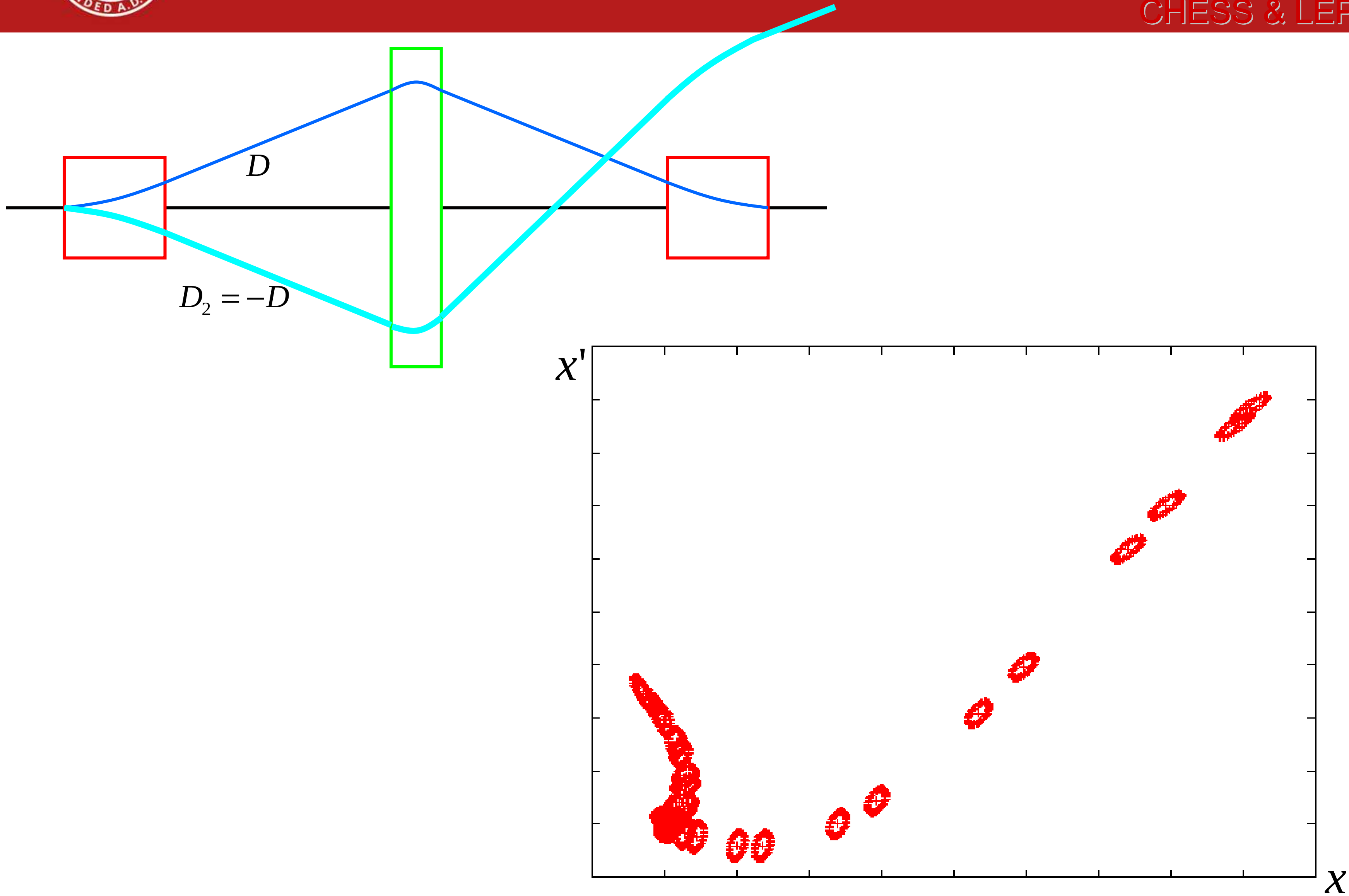
$$y'' = y\,k \quad , \quad \tau' = -\kappa\, x$$

Time of flight for a particle with energy deviation:

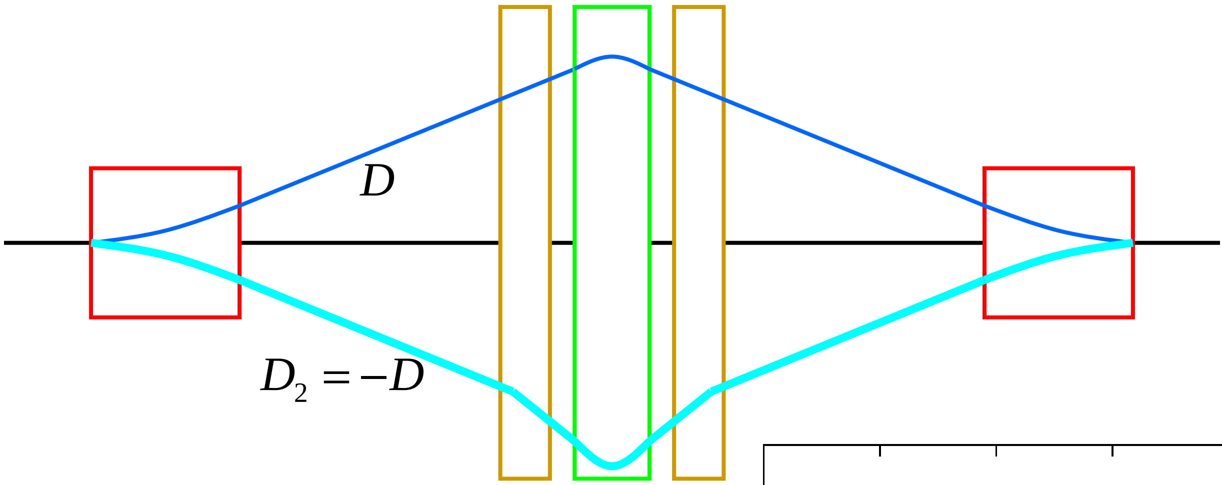
$$\tau = M_{56}\delta + T_{566}\delta^2 = -\int_0^s \hat{\kappa}(\hat{D}\delta + \hat{D}_2\delta^2)d\hat{s}$$

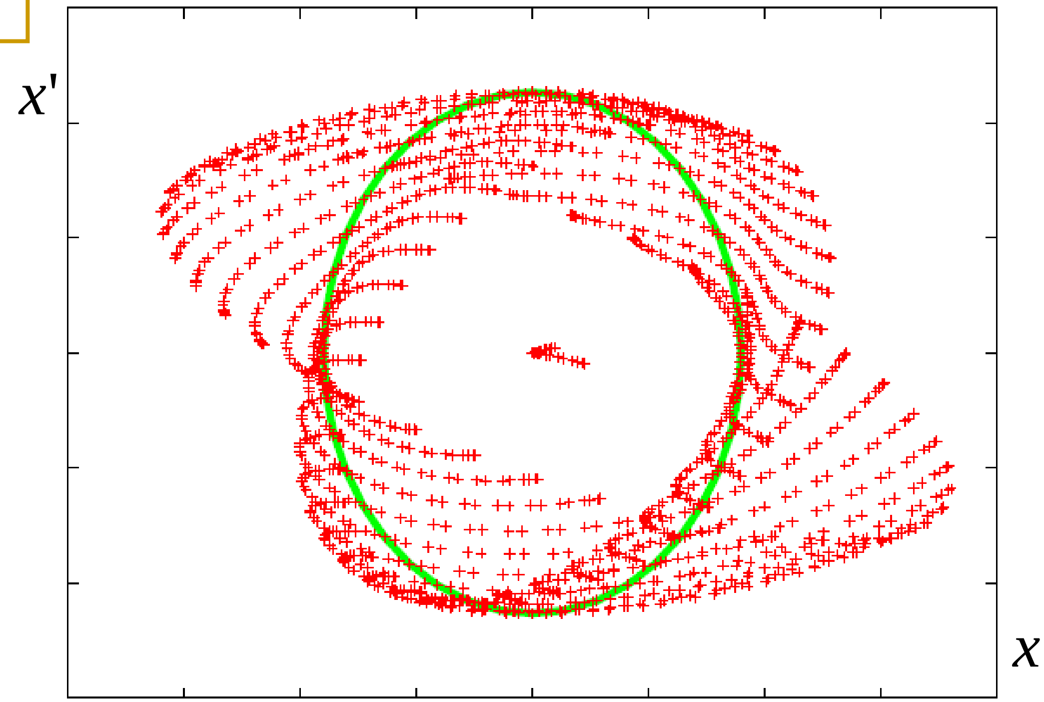
A second order isochronous achromat generally needs two sextupoles, for making $D_2 = 0, D_2' = 0$

Phase space transport without achromat



Phase space transport with achromat





Energy Dependent Phase Advance

$$\Delta\psi = \Delta k l(\hat{s})\hat{\beta}\sin^2(\psi - \hat{\psi})$$

ξ = energy dependence of betatron phase
often called chromaticity

$$\upsilon(\delta) = \upsilon + \tfrac{\partial\upsilon}{\partial\delta}\delta + \dots$$

$$\xi = \tfrac{\partial\upsilon}{\partial\delta} \quad \text{with} \quad \upsilon = \tfrac{\psi}{2\pi}$$

Natural chromaticity ξ_0 = energy dependence of phase advance due to k1 only

$$f_2 = -\kappa(\delta^2 - \tfrac{1}{2}x'^2 - 2\kappa x\delta + \kappa^2 x^2) + k_1 x(\delta - 2\kappa x) - \tfrac{1}{2}k_2 x^2 \approx k_1 x\delta - k_2 D x\delta$$

$$\xi_{x0} = -\frac{1}{2\pi}\oint \beta_x(\hat{s})k_1(\hat{s})\sin^2(\psi - \hat{\psi})d\hat{s}$$

Particles with energy difference oscillate around the periodic dispersion leading to a quadrupole effect in sextupoles that also shifts the tune:

$$\xi_x = \frac{1}{2\pi}\oint \hat{\beta}_x(-k_1 + D_x k_2)\sin^2(\psi - \hat{\psi})d\hat{s}$$

Energy Dependent Beta Function

$$\frac{\Delta\beta}{\beta} = -\Delta k l(\hat{s})\hat{\beta}\sin 2(\psi - \hat{\psi})$$

$$\beta_x(\delta) = \beta_x + \tfrac{\partial\beta_x}{\partial\delta}\delta + \ldots$$

The energy dependent part of the beta function is often called the chromatic beta beat.

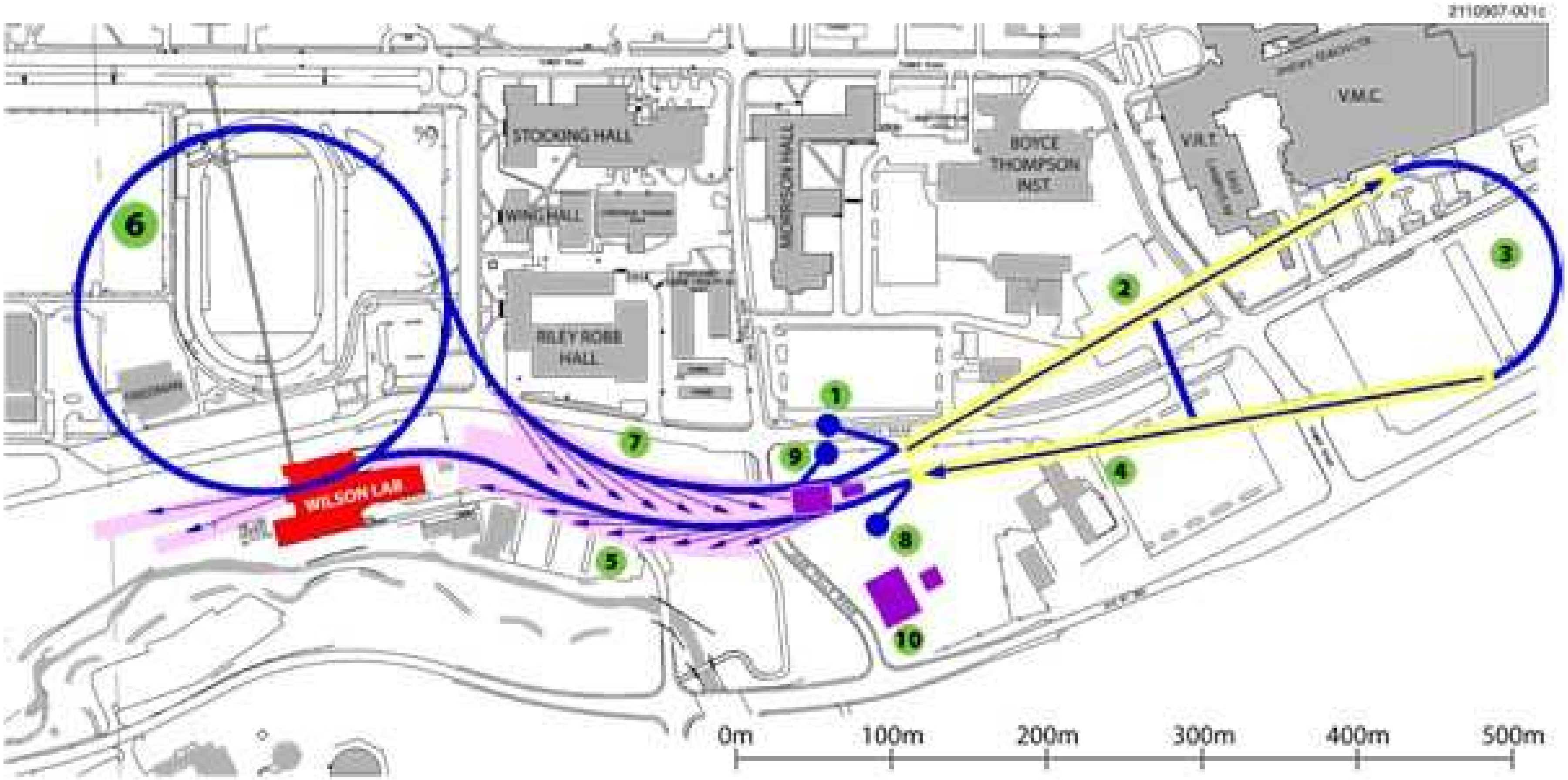
$$f_2 = -\kappa(\delta^2 - \tfrac{1}{2}x'^2 - 2\kappa x\delta + \kappa^2 x^2) + k_1 x(\delta - 2\kappa x) - \tfrac{1}{2}k_2 x^2 \approx k_1 x\delta - k_2 D x\delta$$

Chromatic beta beat:

$$\frac{1}{\beta_x}\frac{\partial\beta_x}{\partial\delta} = \oint \hat{\beta}_x (k_1 - D_x k_2)\sin 2(\psi - \hat{\psi})d\hat{s}$$

… leading to an x-Ray ERL

CHESS & LEPP



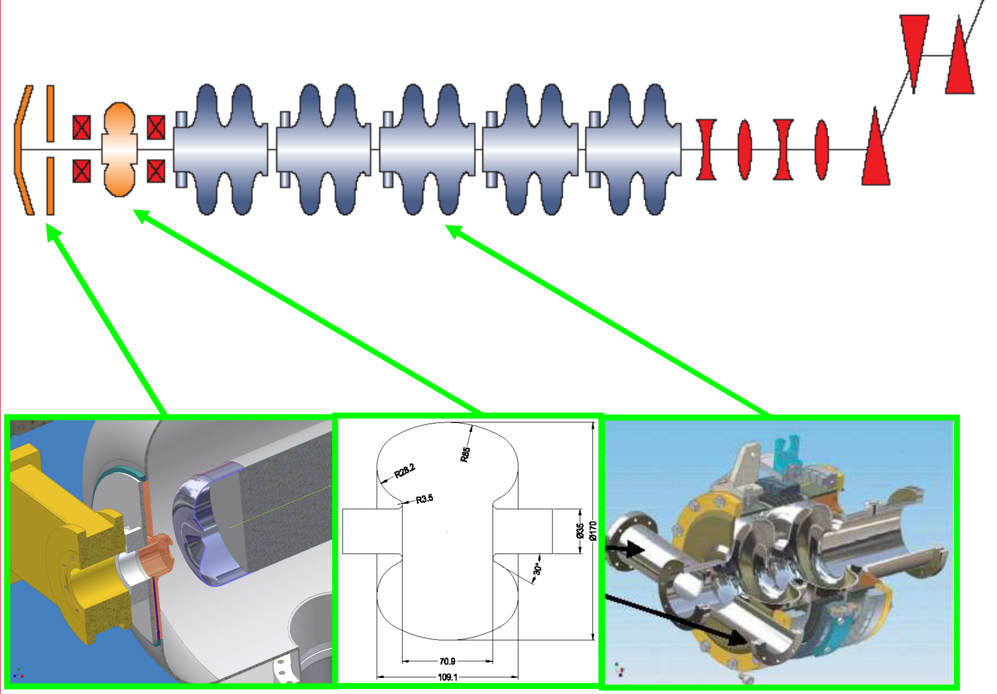
Key: 1) injector, 2) north linac, 3)turn-around arc, 4) south linac, 5) south x-ray beamlines, 6) CESR turn-around, 7) north x-ray beamlines, 8) 1st beam dump, 9) 2nd beam dump and 10)distributed cryoplant. Tunnel cross-section of 12' ID shown on lower right.

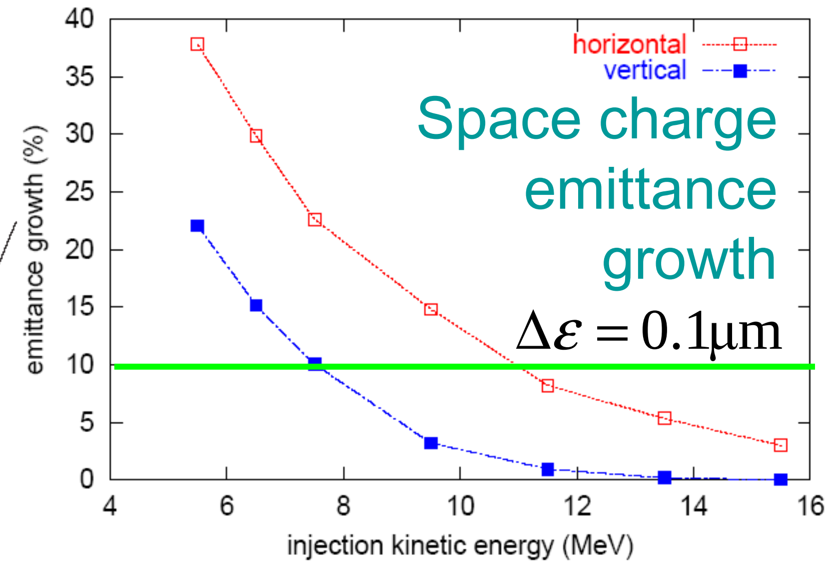
Multivariable optimization to worlds highest electron brightness

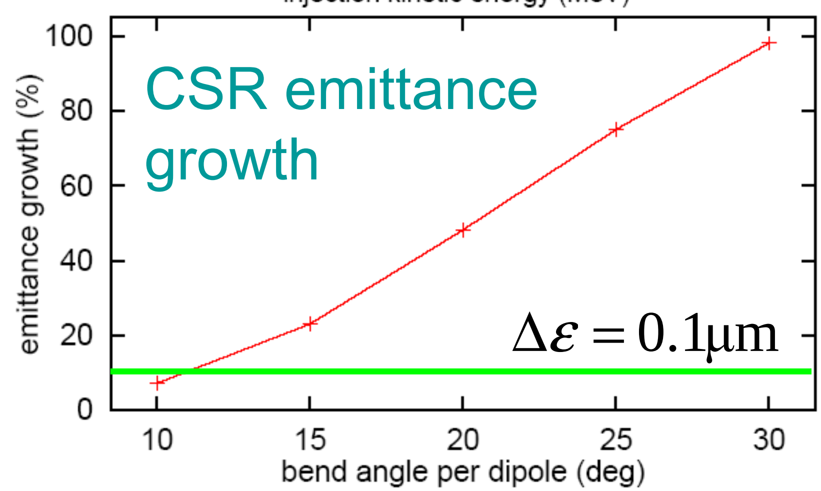
HIGH BRIGHTNESS, HIGH CURRENT INJECTOR DESIGN FOR THE CORNELL ERL PROTOTYPE*

2003 Particle Accelerator Conference

I.V. Bazarov† and C.K. Sinclair



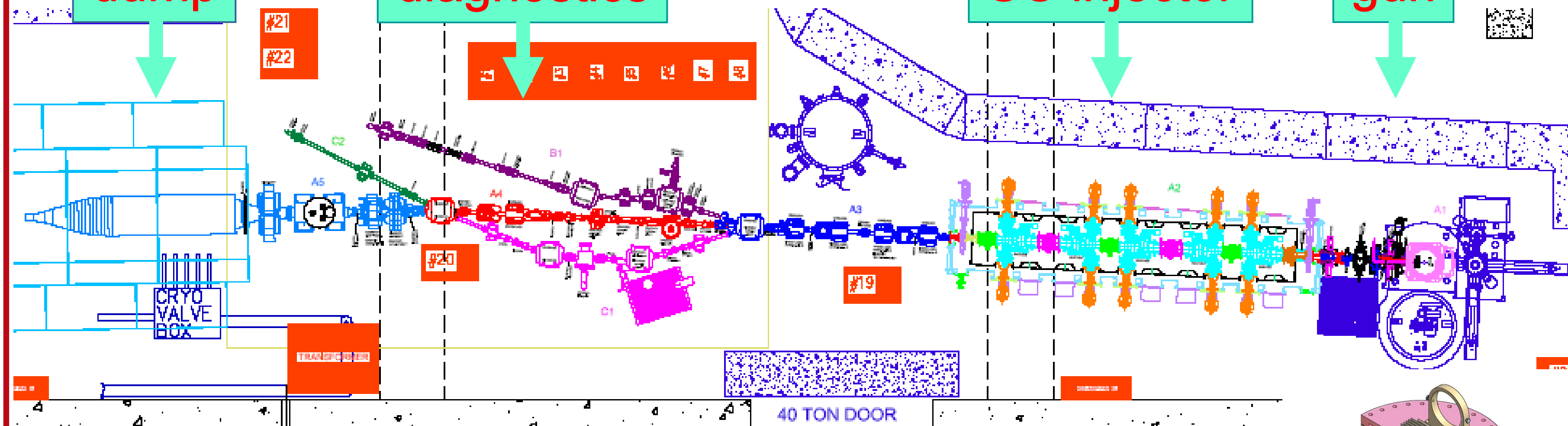




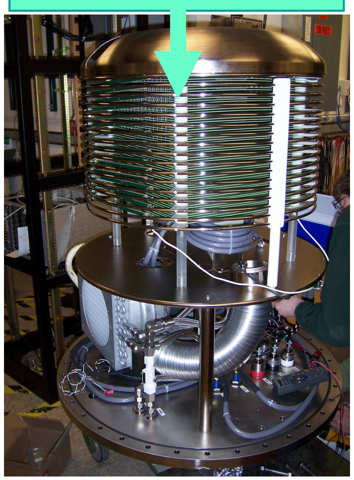
Cornell Injector prototype: Verification of beam production

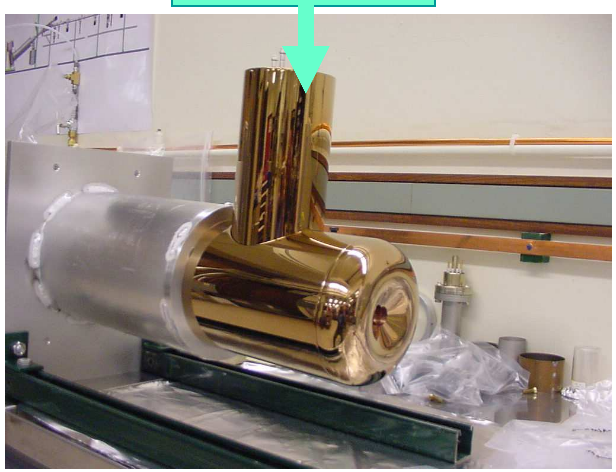
CHESS & LEPP

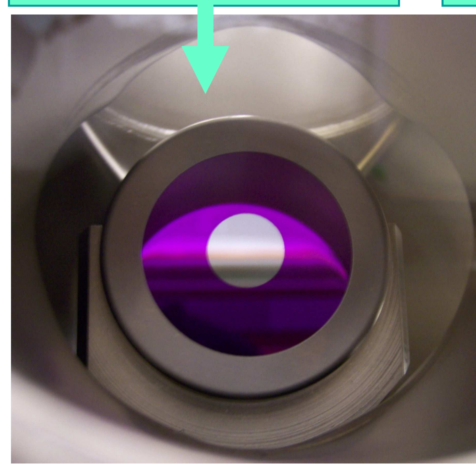
dump | diagnostics | SC injector | gun

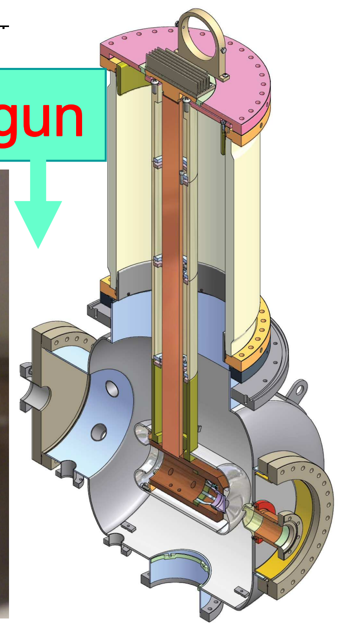


Power sup. | Cathode | Photo emitter | gun









Assembly of the injector accelerator

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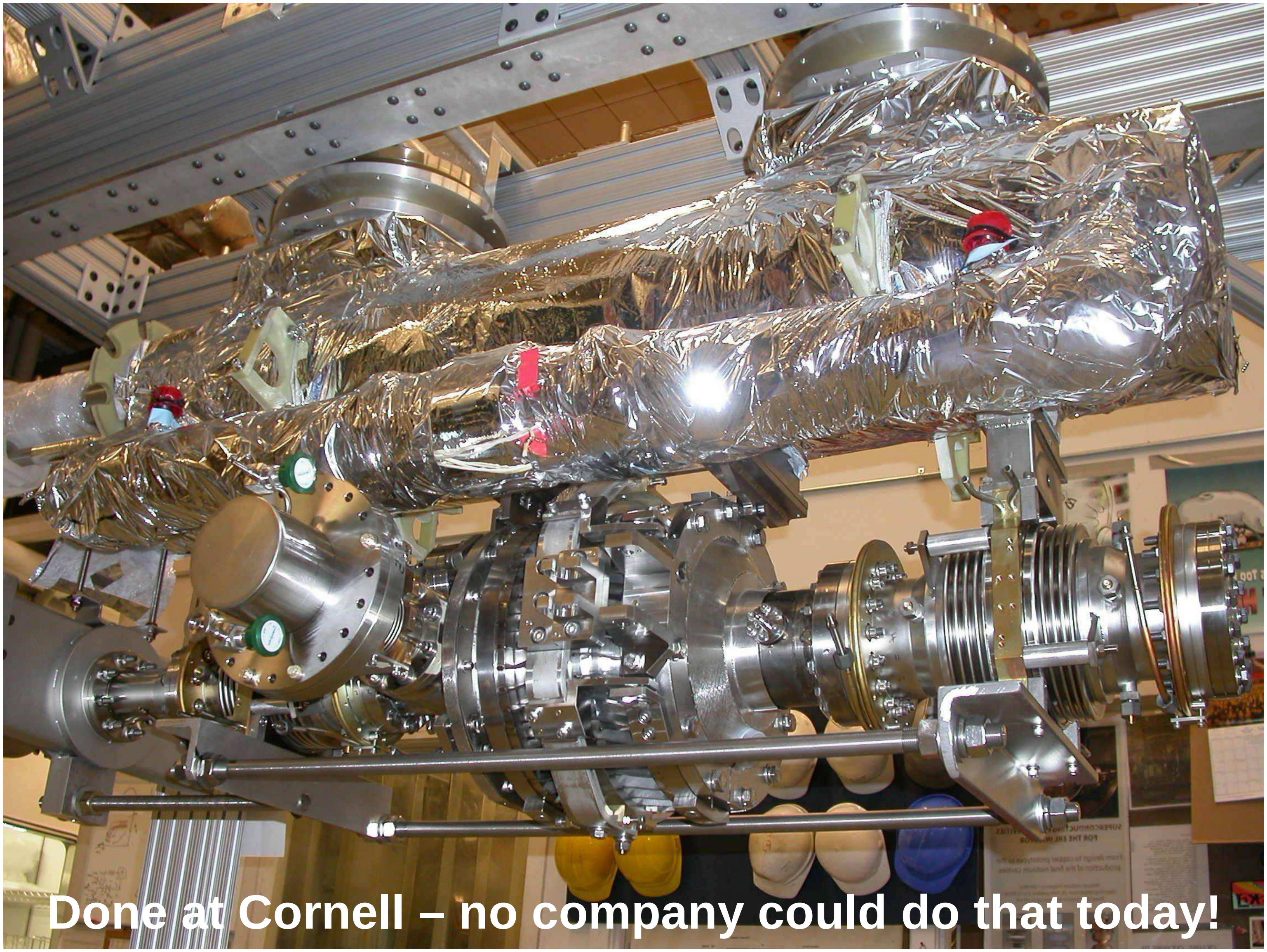
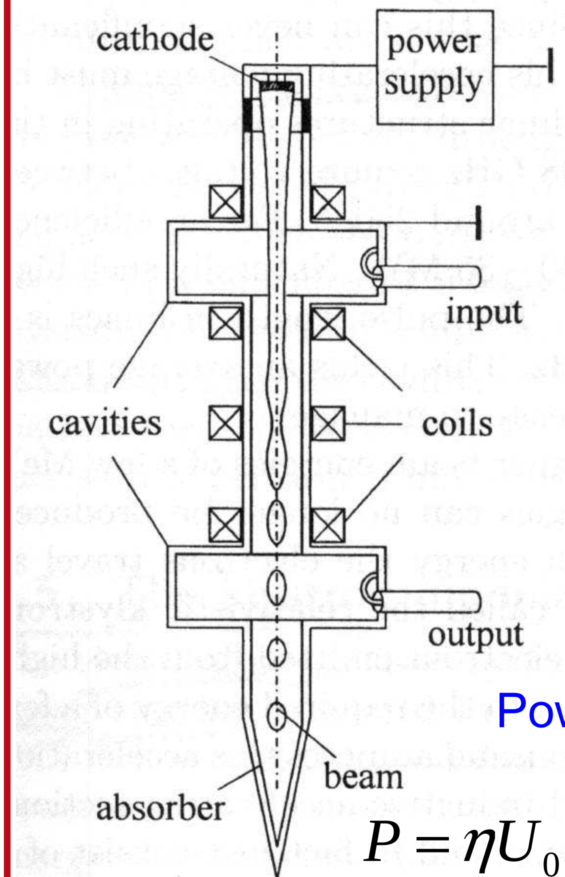


Done at Cornell – no company could do that today!

The Klystron as Power Source



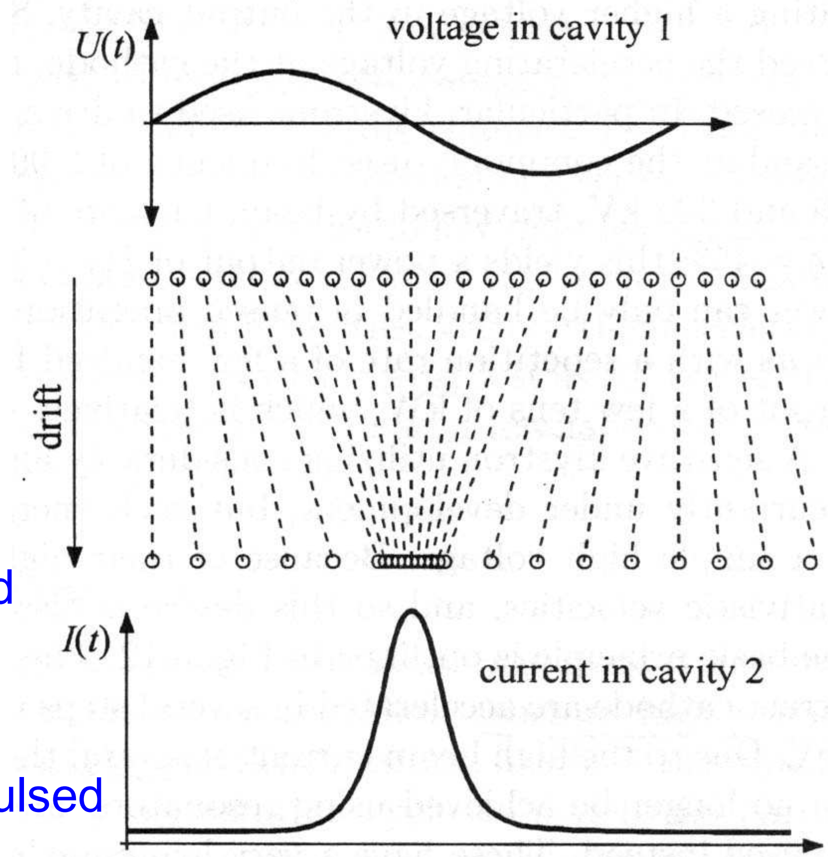
I up to > 10A

Power < 1.5MW

Power < 40MW pulsed

$$P = \eta U_0 I_{\text{beam}} \,, \quad \eta \le 65\%$$

Time of flight bunching

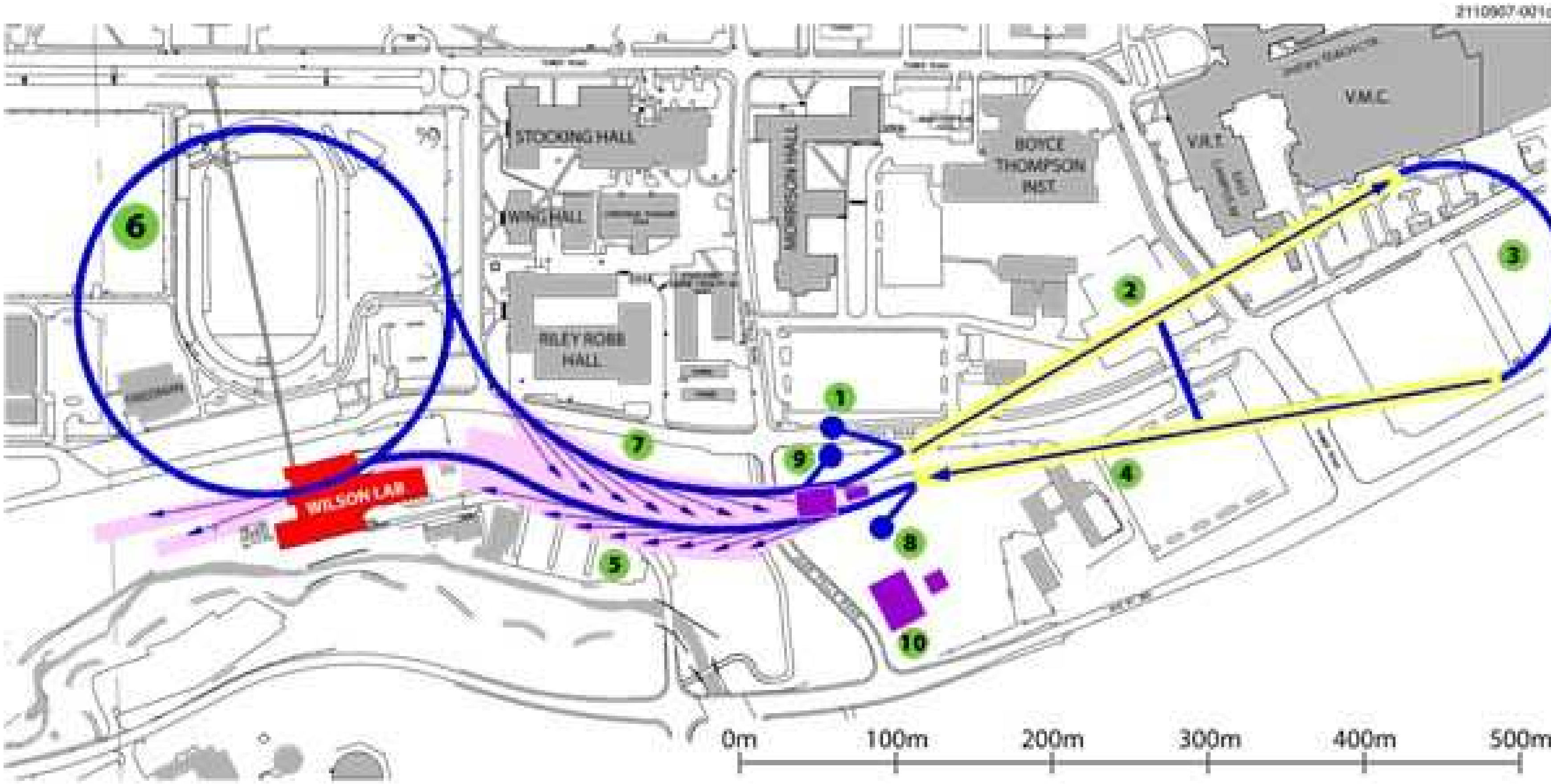


- DC acceleration to several 10kV, 100kV pulsed
- Energy modulation with a cavity
- Time of flight density modulation
- Excitation of a cavity with output coupler

Only works for non-relativistic electrons

Good Luck to you and to the ERL Project

SS & LEPP



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