Accelerator Physics for an ERL x-Ray Source
Homework 4

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Exercise 1 (Undulator radiation)
An undulator with 22mm undulator wave length is operated with a 5GeV electron beam.
(a) Assuming its pole field is $B_y(y = \frac{g}{2}) = 2$T and its vertical gap is $g = 5$mm, what is the wavelength of the first harmonic, what is its photon energy?
(b) Assume its vertical magnetic field at the pole is a step function between +2T and -2T. How strong is the amplitude of the vertical magnetic field in the mid-plane that oscillates with the undulator wave length? How strong is the amplitude of the field components in the mid-plane that oscillate with 1/3 and 1/5 of the undulator wavelength?

Exercise 2 (FEL phase space)
An FEL amplifier has an electrical wave of 20W power, 1mm^2 cross section, and 100nm wavelength in an undulator with 5cm undulator period and $K = 0.5$.
(a) What is the resonant energy for which energy transfer to the wave is maximal?
(b) How high is the FEL separatrix, i.e. what is the maximal $\Delta\gamma$ for which electrons move inside the separatrix? For this show that $\Delta\gamma^2 - A\cos\Psi_+$ is a constant for appropriately chosen A. Determine A and the maximal $\Delta\gamma$ for the separatrix. Note that the energy spread of the electron beam has to be significantly less than the hight of the separatrix.

(c) Compute a limint on how much power an electron beam of 1mA can add to the wave of the FEL in the weak amplification approximation.
(d) Estimate how much power a continuous 1mA beam radiates in the undulator per length after it is bunched in the ponderomotive phase.
(e) Estimate how much power a 1mA beam radiates in the undulator per length after it is bunched, if it consist of 1ps long bunches with 100kHz repetition rate.