Exercise 1 (Undulator radiation)
An undulator with 22mm undulator wave length is operated with a 5GeV electron beam. 
(a) Assuming its pole field is \( B_y(y = \frac{g}{2}) = 2 \text{T} \) and its vertical gap is \( g = 5 \text{mm} \), what is the wavelength of the first harmonic, what is its photon energy? 
(b) Assume its vertical magnetic field at the pole is a step function between +2T and -2T. How strong is the amplitude of the vertical magnetic field in the mid-plane that oscillates with the undulator wave length? How strong is the amplitude of the field components in the mid-plane that oscillate with 1/3 and 1/5 of the undulator wavelength?

Exercise 2 (FEL phase space)
An FEL amplifier has an electrical wave of 20W power, 1mm\(^2\) cross section, and 100nm wavelength in an undulator with 5cm undulator period and \( K = 0.5 \). 
(a) What is the resonant energy for which energy transfer to the wave is maximal? 
(b) How high is the FEL separatrix, i.e. what is the maximal \( \Delta \gamma \) for which electrons move inside the separatrix? For this show that \( \Delta \gamma^2 - A \cos \Psi_+ \) is a constant for appropriately chosen \( A \). Determine \( A \) and the maximal \( \Delta \gamma \) for the separatrix. Note that the energy spread of the electron beam has to be significantly less than the height of the separatrix.
(c) Compute a limit on how much power an electron beam of 1mA can add to the wave of the FEL in the weak amplification approximation.
(d) Estimate how much power a continuous 1mA beam radiates in the undulator per length after it is bunched in the ponderomotive phase.
(e) Estimate how much power a 1mA beam radiates in the undulator per length after it is bunched, if it consists of 1ps long bunches with 100kHz repetition rate.