Accelerator Physics for an ERL x-Ray Source
Homework 5

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Exercise 1 (Transport Matrices)
(a) Compute the 6×6-dimensional transport matrix of a quadrupole that is focusing in x-direction.
(b) Compute the 6×6-dimensional transport matrix of a dipole magnet for which the entrance and exit pole faces are perpendicular to the beam direction, i.e. for a sector bend magnet.
(c) Derive the equation of motion for Twiss parameters, $\alpha' + \gamma = K(s)\beta$ with $K = \kappa^2(s) + k(s)$ from the linearized equation of motion $x'' = -K(s)x$. Use $x = \sqrt{2J\beta(s)}\sin(\Psi(s) + \Psi_0)$, $\alpha = -\frac{1}{2}\beta'$ and $\Psi'(s) = 1/beta$.

Exercise 2 (Phase space distribution):
(a) Given the Twiss parameters α, β, γ: specify the transformation from the amplitude and phase variables J and Ψ to the Cartesian phase space variables x and x'.
(b) Specify the inverse transformation.
(c) Given the Gaussian beam distribution in amplitude and phase variables, $\rho(J, \phi) = \frac{1}{2\pi\epsilon}e^{-\frac{J}{\epsilon}}$. What is the projection $\rho(x)$ of this distribution on the x axis. Check that the rms width of this distribution leads to $\sqrt{< x^2 >} = \sqrt{\beta\epsilon}$.

Exercise 3 (Twiss parameters):
Find the general form of the beta function in a drift:
(a) by solving the differential equation for $\beta(s)$ with the initial conditions

$\beta(0) = \beta_0$ and $\alpha(0) = \alpha_0$.

(b) by solving the differential equation for $x(s)$ and $x'(s)$. The initial x_0 and x'_0 have to be expressed in terms of the initial Twiss parameters, and the Twiss parameters at s will then be expressed in terms of the solutions $x(s)$ and $x'(s)$.

(c) Find the general form of a beta function in a quadrupole of focusing strength k in terms of the distance s along the quadrupole and in terms of the initial Twiss parameters before the quadrupole.