Accelerator Physics for an ERL x-Ray Source
Homework 6

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Exercise 1 (Guns)
(a) How many particles are in a bunch of 100pC? What is the particle density when the bunch has a volume of 1mm^3? How does this density compare to the gas density in one atmosphere pressure at room temperature? How does it compare to the gas density in the vacuum pipe of a storage ring for a typical pressure of 10^{-9} Torr.
(b) Compute the normalized thermal emittance of a beam that has a Gaussian profile of electrons being emitted, i.e. the rate of electron emission is proportional to $\exp(-\frac{x^2+y^2}{2\sigma^2})$. The normalized emittance ϵ_n is the emittance ϵ for a beam with momentum p, multiplied by p/mc. Assume that the velocities of the electrons have a classical Boltzmann distribution so that the likelihood of velocity component v_x is proportional to $\exp(-\frac{m(v_x^2+v_y^2)}{2k_BT})$ for $T = 300$K.
(c) In a photo-emission gun, a 1W laser with 500nm wavelength leads to the emission of 10mA of electrons. What is the quantum efficiency QE of the cathode?

Exercise 2 (Space Charge):
(a) Derive the relativistic Child-Langmuir law. For this you have to review the classical derivation and replace $v = \sqrt{\frac{2qV}{m}}$ by $v = c\sqrt{1-(\frac{mc^2}{mc^2+qV})^2}$. You might want to use a symbolic math program to solve differential equations.
(b) How much charge can be extracted in a short pulse from a circular spot of 1cm radius when the field at the cathode is 5MV/m.

(c) Derive the linearized equation of motion in a solenoid field. For this use that the magnetic field to first order has to have the form $\vec{B} = B(z)\vec{e}_z - \frac{1}{2}B'(z)(x\vec{e}_x + y\vec{e}_y)$ in order to satisfy $\vec{\nabla} \cdot \vec{B} = 0$. I recommend working in x and y coordinates.

(d) Transform the equation of motion into a rotating coordinate system that rotates with appropriate speed to decouple the equations in x and y. They will then have the form $\ddot{x} = -g^2 x$ and $\ddot{y} = -g^2 y$. What is the rotation speed of the rotating coordinate system and what is g^2. Compare g^2 to the focal length that was specified in class (page 18 on February 27). Is there a reasonable resemblance?